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ГИПЕРБОЛИЧЕСКИ-ЛОГАРИФМИЧЕСКАЯ МОДЕЛЬ НЕЛИНЕЙНОЙ ЭЛЕКТРОДИНАМИКИ*

Грошев Д. Е.[a,1], Спасов Д. А.[a,2]

[a] Казанский Федеральный Университет, г. Казань, 420008, Россия

В этой работе мы рассматриваем новую модель нелинейной электродинамики - *"Гиперболически-логарифмическую"*. Эта модель содержит в себе три параметра и описывается лагранжианом следующего вида: $\mathcal{L} = -\mathcal{F} - \frac{A}{\beta} arth(\beta\mathcal{F}) - \frac{C}{2\beta}[\ln(1+\beta\mathcal{F}) + \ln(1-\beta\mathcal{F})]$, где $\mathcal{F} = \frac{1}{4}F_{ik}F^{ik}$. Мы показываем, что в рамках данной модели нарушается дуальная симметрия. Также мы доказываем, что электрическое поле точечного заряда становится несингулярным, а энергия электрического поля - конечной. Мы вычисляем ожидаемую величину параметров теории, опираясь на характеристики электрона, а также на идею Абрахама и Лоренца о том, что вся масса электрона имеет электромагнитную природу. Нами находятся компоненты канонического и симметризованного тензора энергии-импульса.

Ключевые слова: Нелинейная электродинамика, Тензор энергии-импульса, Энергия точечного заряда.

HYPERBOLIC-LOGARITHMIC MODEL OF NONLINEAR ELECTRODYNAMICS

Groshev D. E.[a,1], Spasov D. A.[a,2]

[a] Kazan Federal University, Kazan, 420008, Russia

In this paper we consider a new model of nonlinear electrodynamics - *"Hyperbolic-logarithmic"*. This model contain a three parameters and describe by following Lagrangian: $\mathcal{L} = -\mathcal{F} - \frac{A}{\beta} arth(\beta\mathcal{F}) - \frac{C}{2\beta}[\ln(1+\beta\mathcal{F}) + \ln(1-\beta\mathcal{F})]$, where $\mathcal{F} = \frac{1}{4}F_{ik}F^{ik}$. We show, that in this model dual symmetry is broken. Also we proved that the electric field of a point-like charge becomes non-singular in this framework, static electric energy of this charge is finite. We calculate a theory parameters values guided by electron parameters and Abraham - Lorentz idea about a pure electromagnetic nature of electron mass. We find the canonical and symmetrical Belifante energy momentum tensors.

Keywords: Nonlinear electrodynamics, Energy-momentum tensor, Point-like charge energy.

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Introduction

It is well known that strong electromagnetic field has strong connection with nonlinear theory. QED one-loop quantum corrections give a non-linear terms in classical Maxwell's Lagrangian [1]. Nonlinear electrodynamics uses in a wide range of contexts :cosmology and astrophysics [2], high power laser technologies and plasma physics [3], nuclear physics [4].

On the other hand, some models of nonlinear electrodynamics can solve the problem of Coulomb singularity, namely infinite energy of a point-like charge, which comes out in Maxwell's electrodynamics. The most famous example is Born-Infield electrodynamics [5]. In arcsin-electrodynamics [6] and generalized ModMax electrodynamics [7] this singularity is also absent.

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[1]E-mail: groshevdmitri@mail.ru
[2]E-mail: spasov.01@mail.ru

Considerated model contains finite energy of a point-like charge as well. This fact makes Hyperbolic-logarithmic model more attractive for deep investigation.

We use the Heaviside-Lorentz system with $\hbar = c = \epsilon_0 = \mu_0 = 1$, Latin letters run from 0 to 3.

1. The model

In this paper we introduce a new model of nonlinear electrodynamics witch given by Lagrangian:

$$\mathcal{L} = -\mathcal{F} - \frac{A}{\beta} arth(\beta\mathcal{F}) - \frac{C}{2\gamma}\left[\ln(1+\gamma\mathcal{F}) + \ln(1-\beta\mathcal{F})\right]. \tag{1.1}$$

Here $\mathcal{F} = \frac{1}{4}F_{ik}F^{ik} = \frac{1}{2}(\mathbf{B}^2 - \mathbf{E}^2)$ - invariant of electromagnetic field, A, B - dimensionless parameters, β - parameter with dimension $\text{Ł}^{\frac{1}{4}}$. First pair of Maxwell equations is following

$$\partial_i \left(F^{ik} + \frac{A - \beta C \mathcal{F} F^{ik}}{1 - (\beta\mathcal{F})^2} \right) = 0\,. \tag{1.2}$$

Second pair of Maxwell equations, of course, remains unchanged:

$$\partial_i F^{*ik} = 0\,. \tag{1.3}$$

The electric displacement can be calculate with the help of expression $\mathbf{D} = \partial\mathcal{L}/\partial\mathbf{E}$:

$$\mathbf{D} = \left(1 + \frac{A - \beta C\mathcal{F}}{1 - (\beta\mathcal{F})^2}\right)\mathbf{E}\,. \tag{1.4}$$

The magnetic field is given by $\mathbf{H} = -\partial\mathcal{L}/\partial\mathbf{B}$:

$$\mathbf{H} = \left(1 + \frac{A - \beta C\mathcal{F}}{1 - (\beta\mathcal{F})^2}\right)\mathbf{B}\,. \tag{1.5}$$

We can represented equations (1.4) and (1.5) in tensor form:

$$D_i = \epsilon_{ij}E^j, \quad H_i = \mu^{-1}_{ij}B^j\,, \tag{1.6}$$

where ϵ_{ij} and μ^{-1}_{ij} - electric permittivity and inverse magnetic permeability tensors, respectively. We can write components of these tensors:

$$\epsilon_{ij} = \mu^{-1}_{ij} = \epsilon\delta_{ij}, \quad \epsilon = \left(1 + \frac{A - \beta C\mathcal{F}}{1 - (\beta\mathcal{F})^2}\right)\,. \tag{1.7}$$

First pair of Maxwell's equation can be rewritten in the $\mathbf{D} - \mathbf{H}$ framework:

$$\nabla\mathbf{D} = 0, \quad \frac{\partial\mathbf{D}}{\partial t} - \nabla\times\mathbf{H} = 0\,. \tag{1.8}$$

Second pair of Maxwell's equation has the standard form:

$$\nabla\mathbf{B} = 0, \quad \frac{\partial\mathbf{B}}{\partial t} + \nabla\times\mathbf{E} = 0\,. \tag{1.9}$$

In order to see the dual symmetry of this model, from equations (1.4) - (1.6) we can obtain :

$$\mathbf{DH} = \epsilon^2\mathbf{EB} \neq \mathbf{EB}\,. \tag{1.10}$$

In other words, dual symmetry is broken [8] . In case ($A = 0, \quad C = 0 \to \epsilon = 1$) we arrive at classical electrodynamics with dual symmetry.

2. Electrostatics

In this section we consider electrostatic case (**B**=**H**=0). Assume that a point-like charge q located at $r=0$. Equation for this charge is given by

$$\nabla \mathbf{D}_0 = q\delta(\mathbf{r}) \tag{2.1}$$

with the solution:

$$\mathbf{D}_0 = \frac{q}{4\pi r^3}\mathbf{r}\,. \tag{2.2}$$

Taking into account (1.4), we obtain

$$E_0\left(1+\frac{A+\frac{\beta C E_0^2}{2}}{1-\frac{\beta^2 E_0^2}{4}}\right) = \frac{q}{4\pi r^3}\,. \tag{2.3}$$

Solution of this equation at $r\to 0$ is given by:

$$E_0 = \sqrt{\frac{2}{\beta}}\,. \tag{2.4}$$

So, we can see what in this model maximum electric field is finite. Coulomb singularity is absent like in Born-Infeld theory.

3. Energy-momentum tensor

In this section we find canonical and Belinfante energy-momentum tensors. The expression of canonical energy-momentum tensor is:

$$T^{ik} = -(\partial^i A^m)\left(F^k_{\cdot m} + \frac{(A-\beta C\mathcal{F}))F^k_{\cdot m}}{1-(\beta\mathcal{F})^2}\right) - g^{ik}\mathcal{L}\,. \tag{3.1}$$

This tensor is conserved $\partial_i T^{ik}=0$ but is not symmetrical and gauge-invariant. So, we obtain symmetrical Belinfante tensor by the relation [9] :

$$T^{ik}_{(B)} = T^{ik} + \partial_l X^{lik}\,, \tag{3.2}$$

where

$$X^{ilk} = \frac{1}{2}\left(\Pi^{ls}\delta^{ik}_{sp} - \Pi^{is}\delta^{lk}_{sp} - \Pi^{ks}\delta^{li}_{sp}\,,\right) \tag{3.3}$$

and

$$\Pi^{ik} = \frac{\partial \mathcal{L}}{\partial(\partial_i A_k)} = \epsilon F^{ik}\,. \tag{3.4}$$

Obviously, $X^{lik}=-X^{ilk}$ that give give rise to $\partial_l\partial_i X^{lik}=0$. So, using (3.1)-(3.4) we can obtain:

$$T^{ik}_{(B)} = -F^{im}\left(F^k_{\cdot m} + \frac{(A-\beta C\mathcal{F}))F^k_{\cdot m}}{1-(\beta\mathcal{F})^2}\right) - g^{ik}\mathcal{L}\,. \tag{3.5}$$

The trace of Belinfante tensor is

$$T^k_k = -4\left(\frac{A-\beta C\mathcal{F}}{1-(\beta\mathcal{F})^2}\right) - \frac{4A}{\beta}arth(\beta\mathcal{F}) + \frac{2C}{\beta}[\ln(1+\beta\mathcal{F}) + \ln(1-\beta\mathcal{F})]\,. \tag{3.6}$$

If $A=0$ and $C=0$ we arrive at Maxwell electrodynamics. The trace of energy-momentum tensor (3.6) becomes zero.

4. Energy of the point-like charge

In this section we study the electric energy of point-like particle - electron. In electrostatic case (**B**=0) the energy density is given by:

$$\rho^E = T^{00}_{(B)} = E^2\left(\frac{1}{2} + \frac{A + \frac{\beta C E^2}{2}}{1 - \frac{\beta E^2}{2}}\right) - \frac{A}{\beta} arcth\left(\frac{\beta E^2}{2}\right) + \frac{C}{2\beta}\left[\ln\left(1 + \frac{\beta E^2}{2}\right) + \ln\left(1 - \frac{\beta E^2}{2}\right)\right]. \quad (4.1)$$

Total energy can be obtain from: $\mathcal{E} = \int \rho_E dV$. Using new variables [6]

$$x = \frac{4\sqrt{2}\pi r^2}{e\sqrt{\beta}}, \quad y = \sqrt{\frac{\beta}{2}}E, \quad (4.2)$$

we can find expression for total energy:

$$\beta^{\frac{1}{4}}\mathcal{E} = \frac{e^{\frac{3}{2}}}{2^{\frac{11}{4}}\sqrt{\pi}}\int_0^\infty \sqrt{x}\left[y^2\left(1 + \frac{2(A + Cy^2)}{1 - y^4}\right) - Aarcth(y^2) + \frac{C}{2}\ln(1 - y^4)\right]. \quad (4.3)$$

Taking into account (2.3) we can write

$$x = \frac{1 - y^4}{y(1 - y^4 + A + Cy^2)}. \quad (4.4)$$

Thus, we can transform integral (4.3) to

$$\beta^{\frac{1}{4}}\mathcal{E} = \frac{e^{\frac{3}{2}}}{2^{\frac{11}{4}}\sqrt{\pi}}\int_0^1 \frac{y^2(1 - y^2 + 2(A + Cy^2)) - Aarcth(y^2)(1 - y^4) + \frac{C}{2}(1 - y^4)\ln(1 - y^4)}{y^{(\frac{5}{2})}(1 - y^4 - A + 2Cy^2)^{\frac{5}{2}}\sqrt{1 - y^4}} \times$$
$$\times \left(4y^4(1 - y^4 + A + Cy^2) + (1 - y^4)(1 - 5y^4 + 3Cy^2 + A)\right) dy. \quad (4.5)$$

With the help of numerical calculation of integral (4.5) we can realize the Abraham and Lorentz idea [10] about pure electromagnetic mass of electron. Let's consider the Heaviside-Lorentz system with $\hbar = c = \epsilon_0 = \mu_0 = 1$. Elementary charge can be found from the expression for fine-structure constant: $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$. We obtain $e = \sqrt{4\pi\alpha} \approx 0.30286$.

On the other hand, we can use quantities of classical electron radius $r_0 = 2.818$ fm and electron mass $0,511$ MeV to obtain dimensionless left part of (4.5): $\beta^{\frac{1}{4}}\mathcal{E} = 1.440$.

Numerical calculus gives that, for, example, this value corresponds to the following quantities of model parameters:

$$A = 0,008 \quad B = 0,0011. \quad (4.6)$$

Conclusion

We introduce a new, *"Hyperbolic-logarithmic"* model of nonlinear electrodynamics with three parameters: dimensionless A, B and dimensional β. After finding the field equations of the theory, we calculated the electric field of a point-like charge and we showed that at the origin - the location of the charge - it takes a finite value, that is $E_0 = \sqrt{\frac{2}{\beta}}$, and is not singular. We showed that the dilatation symmetry is broken due to the nonzero quantities of parameters. We obtain the canonical and symmetric Belinfante energy-momentum tensors. Moreover, we show the example of values of model parameters, witch corresponds the idea about electromagnetic mass of electron.

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Авторы

Грошев Дмитрий Евгеньевич, к.ф.-м.н., ассистент, Казанский Федеральный Университет, ул. Кремлевская, д. 16а, г. Казань, 420008, Россия.
E-mail: groshevdmitri@mail.ru

Спасов Денис Александрович, Казанский Федеральный Университет, ул. Кремлевская, д. 16а, г. Казань, 420008, Россия.
E-mail: spasov.01@mail.ru

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Authors

Groshev Dmitry Evgenevich, Ph.D, Assisnant, Kazan Federal University, Kremlevskaya str., 16a, Kazan, 420008, Russia.
E-mail: groshevdmitri@mail.ru

Spasov Denis Aleksandrovich, Kazan Federal University, Kremlevskaya str., 16a, Kazan, 420008, Russia.
E-mail: spasov.01@mail.ru

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