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ТОЧНЫЕ РЕШЕНИЯ В КОСМОЛОГИЧЕСКИХ МОДЕЛЯХ С НЕМИНИМАЛЬНОЙ СВЯЗЬЮ СКАЛЯРНОГО ПОЛЯ И КРУЧЕНИЯ*

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Рассматриваются модели космологической инфляции со скалярными полями на основе телепараллельной гравитации с неминимальной связью скалярного поля и кручения. Представлен класс точных решений уравнений космологической динамики для произвольного вида параметра Хаббла. Также показана процедура верификации модели с потенциалом скалярного поля $V(\phi) = \frac{m^2}{2}\phi^n$, соответствующего хаотической инфляции с массивным скалярным полем, по наблюдательным ограничениям на значения параметров космологических возмущений.

Ключевые слова: телепараллельная гравитация, кручение, скалярные поля, точные решения, хаотическая инфляция.

EXACT SOLUTIONS IN COSMOLOGICAL MODELS WITH NON-MINIMAL COUPLING OF SCALAR FIELD AND TORSION

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Cosmological inflation models with scalar field based on teleparallel gravity with non-minimal coupling of scalar field and torsion are considered. Solution type obtained in this paper is justified for any Hubble parameter and scalar field evolution. Also, the model with scalar field potenital $V(\phi) = \frac{m^2}{2}\phi^n$, relating to chaotic inflation with massive scalar field, is verified by modern restrictions on values of cosmological perturbations parameters.

Keywords: teleparallel gravity, torsion, scalar fields, exact solutions, chaotic inflation.

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Introduction

Currently, the cosmological inflation has proven to be the most consistent theory for construction early universe models based on metric gravity theories. However, some gravity theories alternative to General Relativity (GR) lead to the same cosmological effects [1], [2]. An example of such theory is teleparallel equivalent of general relativity, or teleparallel gravity (TG) [3]. Despite the local Lorentz symmetry breaking, it has attracted great interest in study of cosmological perturbations and cosmological inflation. Thus, TEGR and GR investigation is essential objective of researches in modern cosmology.

In the context of Modified Teleparallel Gravity (MTG) theories, it is crucial to analyse how local Lorentz symmetry breaking affects the nature of cosmological perturbations evolution [4].

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In this paper we consider exact solutions construction method in cosmological inflation models with non-minimal coupling of torsion and scalar field. Also, we propose generalised solutions, which are justified for arbitrary Hubble parameter and scalar field evolution. Finally, we consider model verification with chaotic inflation potential of scalar field.

1. Non-minimal coupling of torsion and scalar field: exact solutions and inflationary model verification

Teleparallel Gravity (TG) is the case of metric affine gravity, in which spin connection $\omega^a_{b\mu}$ corresponds to Levi-Civita connection of General Relativity (GR), and tetrad field $\mathbf{e}_a(x^\mu)$ connects the spacetime metric $g_{\mu\nu}$ with Minkowski tangent space metric $\eta_{ab} = diag(-1,1,1,1)$ through $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$.

For inflationary model analysis and construction we use corresponding action [4] in units $8\pi G = c = 1$

$$S = \int d^4xe\left[f(T,\phi) + \omega(\phi)X\right], \tag{1.1}$$

where $f(T,\phi)$ is arbitrary function of scalar field ϕ and torsion T, $\omega(\phi)$ – kinetic function, $X = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ – kinetic energy of scalar field and $e = det\left(e^a_\mu\right) = \sqrt{-g}$.

In order to describe inflationary dynamics, we use spatially flat Friedmann-Robertson-Walker metric $ds^2 = dt^2 + a^2\delta_{ij}dx^idx^j$, where $a = a(t)$ is the scale factor. Then we choose corresponding tetrad $e^a_\mu = diag(1,a,a,a)$ such that spin connection is equal to zero.

In that case, cosmological dynamic equations according to action (1.1) can be obtained as follows [4]

$$f(T,\phi) - \omega(\phi)X - 2Tf_{,T} = 0, \tag{1.2}$$

$$f(T,\phi) + \omega(\phi)X - 2Tf_{,T} - 4\dot{H}f_{,T} - 4H\dot{f}_{,T} = 0, \tag{1.3}$$

$$-\omega_{,\phi}X - 3\omega(\phi)H\dot{\phi} - \omega(\phi)\ddot{\phi} + f_{,\phi} = 0. \tag{1.4}$$

Let us propose exact solutions of equations (1.2)-(1.4), which can be written as

$$f(T,\phi) = -K\sqrt{T} - G(\phi)\sqrt{T} - V(\phi), \tag{1.5}$$

$$\omega(\phi) = -\frac{1}{3}\frac{G_{,\phi}}{V(\phi)}, \quad \dot{\phi} = \frac{\sqrt{6}V(\phi)}{G_{,\phi}}, \tag{1.6}$$

where K is arbitrary constant, $G(\phi)$ defines non-minimal interaction between scalar field and torsion, $f = f(T,\phi)$ sets gravitation theory type.

It is easy to verify the solution, if one substitutes (1.5)-(1.6) into equations (1.2)-(1.4). As can be seen from (1.5) and (1.6) proposed solution is justified for any Hubble parameter and scalar field evolution, which corresponds to different inflationary scenario and perturbation parameters as well.

It should be noted that there are 2 types of conditions for inflationary dynamics analysis: $V(\phi) < 0$ and $G_{,\phi} > 0$ or $V(\phi) > 0$ and $G_{,\phi} < 0$ accordingly. The case $\omega > 0$ corresponds to phantom scalar field absence [5].

Now we can choose arbitrary $H(t)$ and $\phi(t)$. Let us consider the second type of exact solutions for chaotic inflation with massive scalar field [6]:

$$V(\phi) = V_0\phi^n, \quad H(t) = \frac{m}{t} + \lambda, \quad \phi(t) = e^{\phi_0 t}. \tag{1.7}$$

where $V_0 = \frac{m^2}{2}$, m is scaclar field mass. Observe that in case of $m^2 > 0$ there is no tachyonic instability in current cosmological model [7].

Thus, we get other parameters for considered model, from eq. (1.5)–(1.6):

$$G(\phi) = -\frac{\phi^n\sqrt{6}V_0}{n\phi_0}, \quad \omega(\phi) = \frac{\sqrt{6}}{3\phi_0\phi}. \tag{1.8}$$

$$f(T,\phi) = -K\sqrt{T} + \frac{\phi^n\sqrt{6}V_0}{n\phi_0}\sqrt{T} - V_0\phi^n, \quad (1.9)$$

Let us turn to cosmological perturbations parameters, which verification is crucial for estimating viability of cosmological models. Notice that in papers [4], [8] verification of cosmological inflation models with non-minimal coupling of scalar field and torsion was considered.

Main parameters of cosmological perturbations are tensor-to-scalar ratio r, scalar indices of scalar n_S and tensor n_T perturbations, as well as power spectrum of scalar perturbations $\mathcal{P}_S$. At the horizon crossing ($k = aH$) in current inflation model with non-minimal coupling of T and ϕ they are defined by the relations: [4]

$$Q_s = \frac{\omega(\phi)X}{H^2} = -\frac{V(\phi)}{H^2G_{,\phi}}, \quad Q_T = -\frac{1}{2}f_{,T} = \frac{1}{4\sqrt{T}}(K+G(\phi)) \quad (1.10)$$

$$\mathcal{P}_S = \frac{H^2}{8\pi^2Q_S} = -\frac{H^4G_{,\phi}}{8\pi^2V(\phi)}, \quad \mathcal{P}_T = \frac{H^2}{2\pi^2Q_T} = -\frac{2H^2\sqrt{T}}{\pi^2(K+G(\phi))}, \quad (1.11)$$

$$n_S - 1 = -2\epsilon - \eta + 2\eta_{\mathcal{R}}, \quad n_T = -2\epsilon - \delta_{f,T}, \quad (1.12)$$

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_S} \approx 4\frac{Q_{SK}}{Q_{TK}} = -16\frac{16\sqrt{6}V(\phi)}{G_{,\phi}(K+G_{,\phi})}. \quad (1.13)$$

The cosmological inflation analysis is carried out in terms of slow-roll parameters, which have the form

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \delta_{\omega X} = -\frac{\omega X}{2H^2f_{,T}}, \quad \delta_{f,T} = \frac{\dot{f}_{,T}}{Hf_{,T}}, \quad (1.14)$$

$$\delta_{f\dot{H}} = \frac{f_{,TT}\dot{T}}{Hf_{,T}}, \quad \delta_{fX} = \frac{f_{,T\phi}\dot{\phi}}{Hf_{,T}}, \quad \eta_{\mathcal{R}} = \delta_{f,T}\left[1 + \frac{\delta_{f,T}}{\delta_{f\dot{H}}}\left(1 + \frac{\delta_{fX}}{\delta_{\omega X}}\right)\right], \quad (1.15)$$

According to the last cosmological restrictions, cosmological perturbations parameters are constrained as follows: [9, 10]

$$r < 0.032 \quad n_S = 0.9663 \pm 0.004, \quad P_S = 2.1 \cdot 10^{-9}. \quad (1.16)$$

Let us receive the inverse dependence of average e-fold number during inflation ($N = 60$) versus cosmic time

$$N = \int H(t)dt = -m\ln(t) + \lambda t => t(N) = exp\left[\frac{1}{m}\left[-mLambertW\left(\frac{\lambda e^{\frac{m}{N}}}{m}\right) + N\right]\right]. \quad (1.17)$$

where $LambwertW$ is ω-Lambert function. In the same token, cosmological perturbations parameters are defined from equations (1.10)–(1.15). numerical calculations of (1.12)-(1.13), we receive values of arbitrary constants, which are shown in table 1, for $n = 2$ in eq. (1.7).

Table 1. Cosmological perturbations parameters and values of arbitrary constants.

r	n_s	P_s	m	λ	ϕ_0	k	V_0
0.03	0.9663	2.1×10^{-9}	5.3	2×10^{-4}	3.3459×10^{-7}	2.18×10^{-4}	1.1481×10^{-11}
0.03	0.9667	2.1×10^{-9}	5.3	2×10^{-4}	3.3459×10^{-7}	2.3×10^{-4}	1.1482×10^{-11}
0.02	0.9663	$2. \times 10^{-9}$	5.3	2×10^{-4}	3.3459×10^{-7}	3.35×10^{-4}	1.1945×10^{-11}
0.02	0.9667	2.1×10^{-9}	5.3	2×10^{-4}	8.1340×10^{-7}	3.47×10^{-4}	2.2793×10^{-11}

As we see from table 1, the inflationary model considered corresponds to observational constrains on perturbation parameters (1.16). It should be observed that inflation energy scale (here, V_0) was estimated in [11].

Conclusion

We have considered exact solutions construction method in cosmological models with non-minimal coupling of scalar field and torsion, which are the modifications of teleparallel equivalent of general relativity (TEGR).

In addition, we propose exact solutions, which are justified for arbitrary Hubble parameter $H(t)$ and scalar field evolution $\phi(t)$.

Also, we define two types of solutions in terms of scalar field potential $V(\phi)$ and non-minimal interaction function $G(\phi)$. Finally, the model with $V(\phi)$ and $(G_{,\phi}) < 0$ was considered as an example for chaotic type inflation with massive scalar field. The selected model satisfies observational constraints on values of cosmological perturbations parameters as 95% C.L.

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