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ЭФФЕКТ НЕНУЛЕВОЙ КОСМОЛОГИЧЕСКОЙ ПОСТОЯННОЙ В СУПЕР-ПУАНКАРЕ-ИНВАРИАНТНОЙ ВСЕЛЕННОЙ

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Суперпространство Минковского $SM(4,4|\lambda,\mu)$ определено в работе [1] как инвариант супергруппы Пуанкаре суперпреобразований, удовлетворяющих суперуравнениям Киллинга. В данной статье формулы суперримановой геометрии В. П. Акулова и Д. В. Волкова [2] используются для вычисления суперсвязности и суперкривизны суперпространства Минковского. Показано, что кривизна суперпространства Минковского отлична от нуля, и суперметрика Минковского является решением суперуравнений Эйнштейна с ненулевой правой частью, при этом 8-мерная искривленная супер-Пуанкаре-инвариантная вселенная $SM(4,4|\lambda,\mu)$ поддерживается чисто фермионным супертензором энергии-импульса с двумя свободными вещественными параметрами λ, μ и, более того, обладает определяемой этими параметрами ненулевой космологической постоянной $\Lambda = 12/(\lambda^2-\mu^2)$, что могло бы дать новый взгляд на проблему космологической постоянной.

Ключевые слова: суперсимметрия, суперпространство Минковского, суперуравнения Эйнштейна, космологическая постоянная.

NON-VANISHING COSMOLOGICAL CONSTANT EFFECT IN SUPER-POINCARE-INVARIANT UNIVERSE

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In [1] we defined the Minkowski superspace $SM(4,4|\lambda,\mu)$ as the invariant of the Poincare supergroup of supertransformations, which is a solution of Killing superequations. In the present paper we use formulae of super-Riemannian geometry developed by V. P. Akulov and D. V. Volkov [2] for calculating a superconnection and a supercurvature of Minkowski superspace. We show that the curvature of the Minkowski superspace does not vanish, and the Minkowski supermetric is the solution of the Einstein superequations, so the eight-dimensional curved super-Poincare invariant superuniverse $SM(4,4|\lambda,\mu)$ is supported by purely fermionic stress-energy supertensor with two free real parameters λ, μ, and, moreover, it has non-vanishing cosmological constant $\Lambda = 12/(\lambda^2-\mu^2)$ defined by these parameters that could mean a new look at the cosmological constant problem.

Keywords: supersymmetry, Minkowski superspace $SM(4,4|\lambda,\mu)$, Einstein superequations, cosmological constant.

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Introduction

The consistent supersymmetry approach to the theory of gravitation according to which supergeometry must be determined by the properties of supersymmetry requires the developing of group-invariant methods in the supergravity. In this direction not only there were absent concrete results but in many cases the very concepts that must be basic for the connection between the supergeometry and supersymmetry have not been developed. In [1] an attempt was made to fill in this gap.

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We consider the supersymmetry as an automorphism of a supergeometric structure and, in part, as an infinitesimal supertransformation preserving the metric of the superspace, the metric itself is defined as the invariant of the corresponding supergroup of transformations, in the spirit of Klein's approach where the notion of symmetry, or group of transformations, is a fundamental notion determining the geometry of the space. Following this program we derived in [1] the Lie superderivative of a supermetric and defined the Minkowski superspace $SM(4,4|\lambda,\mu)$ as an invariant of the Poincare supergroup of supertransformations, which is a solution of Killing superequations. The found supermetric contains two real parameters λ, μ and becomes degenerate only if $\lambda = \pm\mu$. As a result we obtained the two-parameter family of the Minkowski superuniverses that is being studied in this article.

The article is organised as follows. In the first section we introduce basic definitions and give a brief review of earlier results. In the second section we discuss the super-Poincare-invariant Minkowski superspace. The third and fourth sections are devoted to calculating a superconnection and a supercurvature of the Minkowski superspace by using the apparatus of super-Riemannian geometry developed by V. P. Akulov and D. V. Volkov? [2]. In the fifth section we write and analyze the Einstein superequations for Minkowski superspace.

We recall that a linear space g is called $\mathbb{Z}_2$-graded if it is represented in the form of a direct sum of two spaces: $g = g^0 \oplus g^1$; elements of g^0 and g^1 are called homogeneous elements, even and odd, respectively. The dimension of g is a pair (p,q), where p is dimension of even subspace and q is dimension of odd subspace. The fact that $x \in g^i,\ \ i \in \mathbb{Z}_2\ \ (i = 0,1)$ is written in the form $\sigma(x) = i$ and $\sigma(x)$ is called the *parity* of the element x.

The Lie superalgebra is a $\mathbb{Z}_2$-graded linear space with a fixed parity $g = g^0 \oplus g^1$, on which a bilinear operation $[x,y]$ (the supercommutator) is given so that for any homogeneous elements $x,y,z \in g$ there hold the following identities:

$$\sigma([x,y]) = \sigma(x) + \sigma(y),$$
$$[x,y] = (-1)^{\sigma(x)\sigma(y)+1}[y,x],$$
$$[x,[y,z]](-1)^{\sigma(x)\sigma(z)} + [z,[x,y]](-1)^{\sigma(z)\sigma(y)} + [y,[z,x]](-1)^{\sigma(y)\sigma(x)} = 0,$$

Let $\{e_A\}$ be a standard homogeneous basis in g: $\{e_A\} \in g^0$ for $1 \le A \le p$ and $\{e_A\} \in g^1$ for $p+1 \le A \le p+q$, further we use the notation

$$\sigma(A) = \sigma(e_A).$$

The structure constants C^D_{AB} of the Lie superalgebra are defined by the expansion $[e_A,e_B] = C^D_{AB}e_D$.

Let Λ_q be the Grassmannian algebra, i.e. an associative algebra with the unit, where there exists a system of q generators $\theta^1,\ldots,\theta^q$ satisfying the relations

$$\theta^\alpha\theta^\beta + \theta^\beta\theta^\alpha = 0,\ (\theta^\alpha)^2 = 0 \quad (\alpha,\beta = 1,\ldots,q). \tag{1}$$

If a multiplication of elements of the Lie superalgebra g by elements of Λ_q from the left is defined and for homogeneous elements $\mu,\nu \in \Lambda_q$ and $x,y \in g$ there holds

$$[\mu x,\nu y] = \mu\nu[x,y](-1)^{\sigma(\nu)\sigma(x)},$$

then g is called the Lie superalgebra with Grassmannian structure.

The Lie superalgebras and there Grassmannian spans can be realized as an algebra of differential operators of the form

$$(Xf)(z) = \sum_A X^A(z)\frac{\partial}{\partial z^A}f(z), \tag{2}$$

where $X^A(z)$ and $f(z)$ belong to the local supermanifold, i.e. to the algebra $\Lambda_{p,q}(U)$ of functions defined on $U \subset \mathbb{R}^p$ with their values in Grassmannian algebra Λ_q. Elements of the algebra $\Lambda_{p,q}(U)$ can be written in the form

$$f = f(x,\theta) = \sum_{k\ge 0}\sum_{\alpha_1,\ldots,\alpha_k} f_{\alpha_1\ldots\alpha_k}(x^1,\ldots,x^p)\theta^{\alpha_1}\ldots\theta^{\alpha_k}, \tag{3}$$

where $x^1,\dots,x^p$ are coordinates in U, $\theta^1,\dots,\theta^q$ are generators of Λ_q, and $(z^1,\dots,z^{p+q}) \equiv (x^1,\dots,x^p,\theta^1,\dots,\theta^q)$ are the homogeneous (i.e. even (x) and odd (θ)) generators of $\Lambda_{p,q}(U)$. In the case of odd generator $z^A=\theta^A$, i.e. when $\sigma(z^A)=1$, the symbol $\partial/\partial z^A$ in (2) denotes the left derivative which is computed by carrying θ^A out from the product $\theta^\alpha\dots\theta^\beta$ to the left according to rules (1) and then deleting.

The set $D_{p,q}(U)$ of operators (2) is $\mathbb{Z}_2$-graded. An operator X is homogeneous if its coefficients X^A are homogeneous, and the sum $\sigma(X^A)+\sigma(z^A)$ does not depend on A. In this case, the parity of the operator X is equal to $\sigma(X)=\sigma(X^A)+\sigma(z^A)$.

The supercommutator (the super Lie bracket)

$$[X,Y]=XY-(-1)^{\sigma(X)\sigma(Y)}YX,$$

which defines a superanalog of the Lie derivative L_XY turns $D_{p,q}(U)$ into the Lie superalgeba called the Lie superalgebra of (local) vector fields. For homogeneous operators $X,Y\in D_{p,q}(U)$ we have

$$[X,Y]^A=\sum_C\left(X^C\frac{\partial}{\partial z^C}Y^A-(-1)^{\sigma(X)\sigma(Y)}Y^C\frac{\partial}{\partial z^C}X^A\right).$$

1. The super-Poincare-invariant Minkowski superspace.

Let M_0 be a p-dimensional differentiable manifold. We assign to each open subset $U\subset M_0$ the algebra $\Lambda_{p,q}(U)$ of infinitely diferentiable functions (3) on U with values in Grassmannian algebra Λ_q. The manifold M_0 with the sheaf of algebras $\Lambda_{p,q}(U)$ is a supermanifold.

Consider a supermanifold with coordinates $z^A=(x^a,\theta^\alpha)$, where x^a $(a=0,1,2,3)$ are bosonic (even), and θ^α $(\alpha=1,2,3,4)$ are fermionic (odd) coordinates. Then $\sigma(x^a)=0$, $\sigma(\theta^\alpha)=1$, where σ is the parity operator.

A superspace is said to be Riemannian one or super-Riemannian space if on this superspace there is given a non-degenerate metric form ([3], p. 22, and [2])

$$ds^2=dz^Bdz^Ag_{AB}(z)\equiv\tilde g_{AB}dz^Adz^B,\quad g_{AB}(z)=(-1)^{\sigma(A)\sigma(B)}g_{BA}(z),\tag{4}$$

where differentials dz^A and dz^B possess the same parity as the corresponding coordinates do, and

$$\tilde g_{AB}=(-1)^{\sigma(A)+\sigma(B)+\sigma(A)\sigma(B)}g_{AB}.$$

The contravariant metric tensor g^{AB} is determined by the relation

$$g^{AC}g_{CB}=(-1)^{\sigma(A)\sigma(B)}\delta_B{}^A.\tag{5}$$

Thanks to the signature factor $(-1)^{\sigma(A)\sigma(B)}$ in (5) we have

$$g^{AC}g_{CA}=(-1)^{\sigma(A)}\delta_A{}^A=0.\tag{6}$$

Minkowski superspace was defined in [1] as a superspace endowed with a supermetric invariant with respect to transformations belonging to the Poincare's supergroup. Then the Minkowski suppermetric must satisfy super Killing's equations

$$L_Xg_{AB}\equiv Xg_{AB}+(-1)^{\sigma(A)\sigma(X)}\left(\partial_AX^C\right)g_{CB}+(-1)^{\sigma(B)[\sigma(A)+\sigma(X)]}\left(\partial_BX^C\right)g_{CA}=0,$$

where L_Xg_{AB} is the Lie derivative L_X of a metric form g_{AB} with respect to X [1], X are generators of the Poincare's supergroup realized in the following infinitesimal transformations:

$$P_a=\partial_a,\quad M_{ab}=x_a\partial_b-x_b\partial_a+\frac{1}{2}\theta^\alpha\left(\gamma_{ab}\right)_\alpha{}^\beta\partial_\beta,\quad Q_\alpha=\partial_\alpha-\theta^\beta\left(\gamma^aC^{-1}\right)_{\beta\alpha}\partial_a,$$

hereinafter lowercase Latin indices $a, b, \ldots$ take the values $0, 1, 2, 3$, lowercase Greek indices $\alpha, \beta, \ldots = 1, 2, 3, 4$, γ^a are the Dirac gamma-matrices, $\gamma^{ab} = (1/2)(\gamma^a\gamma^b - \gamma^b\gamma^a)$, and $C = i\gamma^0\gamma^2$ is the matrix of charge conjugation.

Solving the super Killing equations with respect to g_{AB}, we have obtained the superextension of the Minkowski metric $\eta_{ab} = \mathrm{diag}(1, -1, -1, -1)$ as a supermetric invariant under the Poincare's supergroup

$$ds^2_{SM} = dx^b dx^a \eta_{ab} - 2d\theta^\beta dx^a \theta^\alpha \left(\gamma^a C^{-1}\right)_{\beta\alpha} + \\ d\theta^\beta d\theta^\alpha \left(\theta^\gamma\theta^\delta \left(\gamma^a C^{-1}\right)_{\alpha\gamma} \left(\gamma^a C^{-1}\right)_{\beta\delta} + \lambda \left(C^{-1}\right)_{\alpha\beta} + \mu \left(\gamma_5 C^{-1}\right)_{\alpha\beta}\right). \tag{7}$$

For finding contravariant components of Minkowski supermetric we expand the matrix (g_{AB}) in a series with respect to variables θ and separate the addend of degree zero: $g = \overset{0}{g} + U$, where

$$\overset{0}{g} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & \lambda \left(C^{-1}\right)_{\alpha\beta} + \mu \left(\gamma_5 C^{-1}\right)_{\alpha\beta} \end{pmatrix}, \tag{8}$$

$$U = \begin{pmatrix} 0 & -\theta^\delta \left(\gamma_b C^{-1}\right)_{\alpha\delta} \\ -\theta^\delta \left(\gamma_a C^{-1}\right)_{\beta\delta} & \theta^\gamma\theta^\delta \left(\gamma^a C^{-1}\right)_{\alpha\gamma} \left(\gamma_a C^{-1}\right)_{\beta\delta} \end{pmatrix}.$$

The inverse to g matrix $g^{-1} = \left(g^{AB}\right)$ exists if and only if there exists an inverse to $\overset{0}{g}$ matrix $\overset{0}{g}{}^{-1}$ ([3], p. 91). In this case the inverse matrix g^{-1} can be found with the help of the formula

$$g^{-1} = \left(\sum_{n=0}^{\infty} (-1)^n \left(\overset{0}{g}{}^{-1} U\right)^n\right) \overset{0}{g}{}^{-1} \tag{9}$$

where the series from the right is finite because of nilpotency of the matrix $\left(\overset{0}{g}{}^{-1} U\right)$. It is easy to check that the inverse of the block-diagonal matrix (19) will not exist only in the following two cases: *a*) $\lambda = \mu$, *b*) $\lambda = -\mu$. In all other cases the inverse of the matrix $\overset{0}{g}$ is of the form

$$\overset{0}{g}{}^{-1} = \begin{pmatrix} \eta^{ab} & 0 \\ 0 & \left(\lambda^2 - \mu^2\right)^{-1} \left(\lambda C^{\alpha\beta} - \mu \left(C\gamma_5\right)^{\alpha\beta}\right) \end{pmatrix}.$$

Applying (21) and taking into account (5) and (6) we find that

$$g^{-1} = \left(g^{AB}\right) = \begin{pmatrix} g^{ab} & g^{a\beta} \\ g^{\alpha b} & g^{\alpha\beta} \end{pmatrix},$$

where

$$g^{ab} = \eta^{ab} \left(1 + \theta^\mu\theta^\nu A_{\mu\nu}\right), \quad g^{a\beta} = \theta^\mu \left(L^a\right)_\mu{}^\beta, \quad g^{b\alpha} = \theta^\mu (L^{bT})^\alpha{}_\mu, \quad g^{\alpha\beta} = -B^{\alpha\beta}, \tag{10}$$

and the following notations are used:

$$A_{\alpha\beta} \equiv \frac{1}{\lambda^2 - \mu^2} \left(\lambda (C^{-1})_{\alpha\beta} + \mu (\gamma_5 C^{-1})_{\alpha\beta}\right), \tag{11}$$

$$(L^b)_\alpha{}^\beta = \frac{1}{\lambda^2 - \mu^2} \left(\lambda (\gamma^b)_\alpha{}^\beta - \mu (\gamma_5\gamma^b)_\alpha{}^\beta\right), \tag{12}$$

$$B^{\alpha\beta} = \frac{1}{\lambda^2 - \mu^2} \left(\lambda C^{\alpha\beta} - \mu (C\gamma_5)^{\alpha\beta}\right). \tag{13}$$

If following the authors of the book [6] we assume that the "real"theory of a massless point particle is a supersymmetric theory [6] (Vol. 1, p. 33) then we must replace the action of a classical massless point particle

$$S = \int \eta_{ab} dx^a dx^b$$

by its supersymmetric generalization

$$S = \int ds^2_{SM} \tag{14}$$

where ds^2_{SM} is the supermetric (7) derived from group-theoretic considerations. It is important to note that, given the rule $\int d\theta^\alpha = 0$ ([3], p. 74) the action (14) corresponds to the action

$$S = \int \eta_{ab} \left(dx^a + \theta^\alpha \left(C\gamma^a \right)_{\beta\alpha} d\theta^\beta \right) \left(dx^b + \theta^\mu \left(C\gamma^b \right)_{\nu\mu} d\theta^\nu \right) \tag{15}$$

([6], the formulae (1.3.4), p. 32 and (5.1.5), p. 283) that describes a point particle propagating not in Minkowski space, but in a superspace with coordinates (x^a, θ^α), here θ^α are anticommuting coordinates, transforming as spinors under Lorentz transformations of coordinates x^a.

2. A superconnection of the Minkowski superspace.

V. P. Akulov and D. V. Volkov [2] have defined the Levi-Civita superconnection of the super-Riemannian metric (1) by the formulae

$$\begin{gathered} \Gamma_A{}^B(d) = dz^C \Gamma_{CA}{}^B(z), \\ \Gamma_{AB}(d) = \Gamma_A{}^C(d) g_{CB} = dz^D \Gamma_{D,AB}, \\ \Gamma_{A,BC} = \frac{1}{2}\left(\partial_A g_{BC} + (-1)^{\sigma(A)\sigma(B)} \partial_B g_{AC} - (-1)^{(\sigma(A)+\sigma(B))\sigma(C)} \partial_C g_{AB} \right), \\ \Gamma_{A,B}{}^C = (-1)^{\sigma(D)} \Gamma_{A,BD} g^{DC}, \end{gathered} \tag{16}$$

where the following symmetry properties hold:

$$\Gamma_{A,BC} = (-1)^{\sigma(A)\sigma(B)} \Gamma_{B,AC}.$$

Applying these formulae to Minkowski supermetric (7) we get

$$\Gamma_{a,bc} = \Gamma_{a,b\delta} = \Gamma_{a,\beta c} = \Gamma_{\alpha,bc} = \Gamma_{\alpha,\beta c} = 0,$$

$$\Gamma_{a,\beta\delta} = \Gamma_{\beta,a\delta} = -(\gamma_a C^{-1})_{\beta\delta},$$

$$\Gamma_{\alpha,\beta\delta} = \theta^\nu \left((\gamma_a C^{-1})_{\nu\alpha} (\gamma^a C^{-1})_{\beta\delta} - (\gamma_a C^{-1})_{\nu\beta} (\gamma^a C^{-1})_{\alpha\delta} \right).$$

For $\Gamma_{A,B}{}^C$ we find by using (5) and (10)–(13)

$$\Gamma_{a,b}{}^c = \Gamma_{a,b}{}^\delta = 0,$$

$$\Gamma_{a,\beta}{}^c = \Gamma_{\beta,a}{}^c = -\theta^\mu (M_a{}^c)_{\beta\mu},$$

$$\Gamma_{a,\beta}{}^\delta = \Gamma_{\beta,a}{}^\delta = -(L_a)_\beta{}^\delta,$$

$$\Gamma_{\alpha,\beta}{}^c = \theta^\mu \theta^\nu \left((\gamma^a C^{-1})_{\mu\alpha} (M_a{}^c)_{\beta\nu} - (\gamma^a C^{-1})_{\mu\beta} (M_a{}^c)_{\alpha\nu} \right),$$

$$\Gamma_{\alpha,\beta}{}^\delta = \theta^\nu \left((\gamma^a C^{-1})_{\nu\alpha} (L_a)_\beta{}^\delta - (\gamma^a C^{-1})_{\nu\beta} (L_a)_\alpha{}^\delta \right),$$

where

$$(M_a{}^c)_{\alpha\beta} = \frac{1}{\lambda^2 - \mu^2} \left(\lambda (\gamma_a \gamma^c C^{-1})_{\alpha\beta} + \mu (\gamma_a \gamma_5 \gamma^c C^{-1})_{\alpha\beta} \right).$$

3. A supercurvature of the Minkowski superspace.

A curvature supertensor of the Riemannian supermetric (1) is defined by the formula [2]

$$R_{DBA}{}^{C}=(-1)^{\sigma(D)\sigma(B)}\partial_B\Gamma_{DA}{}^{C}-\partial_D\Gamma_{BA}{}^{C}+(-1)^{\sigma(A)\sigma(B)}\Gamma_{DA}{}^{F}\Gamma_{FB}{}^{C}-$$

$$(-1)^{(\sigma(A)+\sigma(B))\sigma(D)}\Gamma_{BA}{}^{F}\Gamma_{FD}{}^{C}$$

and has the symmetry properties

$$R_{DCBA}=-(-1)^{\sigma(A)\sigma(B)}R_{DCAB}=-(-1)^{\sigma(C)\sigma(D)}R_{CDBA},$$

$$R_{DCBA}=(-1)^{(\sigma(A)+\sigma(B))(\sigma(C)+\sigma(D))}R_{BADC},$$

$$R_{DCBA}+(-1)^{\sigma(B)(\sigma(C)+\sigma(D))}R_{BDCA}+(-1)^{(\sigma(C)+\sigma(B))\sigma(D)}R_{CBDA}=0,$$

where

$$R_{DCBA}=R_{DCB}{}^{F}g_{FA}.$$

Using these formulae and the superconnection $\Gamma_{A,B}{}^{C}$ of Minkowski supermetric (7) found in the preceding section we have

$$R_{dba}{}^{c}=R_{dba}{}^{\tau}=0,$$

$$R_{db\alpha}{}^{c}=\frac{2\theta^{\nu}}{\lambda^2-\mu^2}\left(\gamma_{db}\gamma^{c}C^{-1}\right)_{\alpha\nu},$$

$$R_{d\beta a}{}^{c}=-R_{\beta da}{}^{c}=-\frac{\theta^{\nu}}{\lambda^2-\mu^2}\left(\gamma_a\gamma_d\gamma^{c}C^{-1}\right)_{\beta\nu},$$

$$R_{db\alpha}{}^{\delta}=\frac{2}{\lambda^2-\mu^2}\left(\gamma_{db}\right)_{\alpha}{}^{\delta},$$

$$R_{d\beta a}{}^{\delta}=-R_{\beta da}{}^{\delta}=-\frac{1}{\lambda^2-\mu^2}\left(\gamma_a\gamma_d\right)_{\beta}{}^{\delta},$$

$$R_{\delta b\alpha}{}^{c}=-R_{b\delta\alpha}{}^{c}=$$

$$\left(M_b{}^{c}\right)_{\alpha\delta}+\frac{\theta^{\varepsilon}\theta^{\nu}}{\lambda^2-\mu^2}\left(2\left(\gamma^{k}C^{-1}\right)_{\varepsilon\delta}\left(\gamma_{bk}\gamma^{c}C^{-1}\right)_{\alpha\nu}+\right.$$

$$\left.\left(\gamma^{k}C^{-1}\right)_{\varepsilon\alpha}\left(\gamma_k\gamma_b\gamma^{c}C^{-1}\right)_{\delta\nu}\right),$$

$$R_{\delta\beta a}{}^{c}=\left(M_a{}^{c}\right)_{\delta\beta}+\left(M_a{}^{c}\right)_{\beta\delta}+$$

$$\frac{\theta^{\varepsilon}\theta^{\nu}}{\lambda^2-\mu^2}\Big(\left(\gamma^{k}C^{-1}\right)_{\varepsilon\beta}\left(\gamma_a\gamma_k\gamma^{c}C^{-1}\right)_{\delta\nu}+$$

$$\left(\gamma^{k}C^{-1}\right)_{\varepsilon\delta}\left(\gamma_a\gamma_k\gamma^{c}C^{-1}\right)_{\beta\nu}\Big),$$

$$R_{\delta\beta\alpha}{}^{c} = \theta^{\varepsilon}\Big(-2\left(\gamma^k C^{-1}\right)_{\beta\delta}\left(M_k{}^c\right)_{\alpha\varepsilon} + \left(\gamma^k C^{-1}\right)_{\beta\alpha}\left(M_k{}^c\right)_{\delta\varepsilon} +$$

$$\left(\gamma^k C^{-1}\right)_{\delta\alpha}\left(M_k{}^c\right)_{\beta\varepsilon} + \left(\gamma^k C^{-1}\right)_{\varepsilon\delta}\left(M_k{}^c\right)_{\alpha\beta} + \left(\gamma^k C^{-1}\right)_{\varepsilon\beta}\left(M_k{}^c\right)_{\alpha\delta} -$$

$$-\left(\gamma^k C^{-1}\right)_{\varepsilon\alpha}\left(M_k{}^c\right)_{\delta\beta} - \left(\gamma^k C^{-1}\right)_{\varepsilon\alpha}\left(M_k c\right)_{\beta\delta}\Big) +$$

$$\frac{\theta^{\varepsilon}\theta^{\nu}\theta^{\sigma}}{\lambda^2-\mu^2}\Big(\left(\gamma^k C^{-1}\right)_{\varepsilon\delta}\left(\gamma^l C^{-1}\right)_{\nu\beta}\left(\gamma_k\gamma_l\gamma^c C^{-1}\right)_{\alpha\sigma} +$$

$$\left(\gamma^k C^{-1}\right)_{\varepsilon\beta}\left(\gamma^l C^{-1}\right)_{\nu\delta}\left(\gamma_k\gamma_l\gamma^c C^{-1}\right)_{\alpha\sigma} -$$

$$\left(\gamma^k C^{-1}\right)_{\varepsilon\alpha}\left(\gamma^l C^{-1}\right)_{\nu\beta}\left(\gamma_k\gamma_l\gamma^c C^{-1}\right)_{\delta\sigma} -$$

$$\left(\gamma^k C^{-1}\right)_{\varepsilon\alpha}\left(\gamma^l C^{-1}\right)_{\nu\delta}\left(\gamma_k\gamma_l\gamma^c C^{-1}\right)_{\beta\sigma}\Big),$$

$$R_{\delta\beta a}{}^{\tau} = \frac{\theta^{\nu}}{\lambda^2-\mu^2}\Big(\left(\gamma^k C^{-1}\right)_{\nu\beta}\left(\gamma_a\gamma_k\right)_{\delta}{}^{\tau} + \left(\gamma^k C^{-1}\right)_{\nu\delta}\left(\gamma_a\gamma_k\right)_{\beta}{}^{\tau}\Big),$$

$$R_{\delta b\alpha}{}^{\tau} = -R_{b\delta\alpha}{}^{\tau} = \frac{\theta^{\nu}}{\lambda^2-\mu^2}\left(2\left(\gamma^k C^{-1}\right)_{\nu\delta}\left(\gamma_{bk}\right)_{\alpha}{}^{\tau} +\right.$$

$$\left.\left(\gamma^k C^{-1}\right)_{\nu\alpha}\left(\gamma_k\gamma_b\right)_{\delta}{}^{\tau}\right),$$

$$R_{\delta\beta\alpha}{}^{\tau} = -2\left(\gamma^c C^{-1}\right)_{\delta\beta}(L_c)_{\alpha}{}^{\tau} + \left(\gamma^c C^{-1}\right)_{\beta\alpha}(L_c)_{\delta}{}^{\tau} - \left(\gamma^c C^{-1}\right)_{\delta\alpha}(L_c)_{\beta}{}^{\tau} +$$
$$\frac{\theta^{\epsilon}\theta^{\nu}}{\lambda^2-\mu^2}\Big(\left(\gamma^c C^{-1}\right)_{\epsilon\delta}\left(\gamma^a C^{-1}\right)_{\nu\beta}(\gamma_c\gamma_a)_{\alpha}{}^{\tau} + \left(\gamma^c C^{-1}\right)_{\epsilon\beta}\left(\gamma^a C^{-1}\right)_{\nu\delta}(\gamma_c\gamma_a)_{\alpha}{}^{\tau} -$$
$$\left(\gamma^c C^{-1}\right)_{\epsilon\alpha}\left(\gamma^a C^{-1}\right)_{\nu\beta}(\gamma_c\gamma_a)_{\delta}{}^{\tau} - \left(\gamma^c C^{-1}\right)_{\epsilon\alpha}\left(\gamma^a C^{-1}\right)_{\nu\delta}(\gamma_c\gamma_a)_{\beta}{}^{\tau}\Big).$$

4. The Einstein superequations.

The Ricci tensor and the scalar curvature of the Riemannian supermetric (1) are defined by the equations [2]

$$R_{BA} = (-1)^{\sigma(F)(\sigma(F)+\sigma(A))} R_{BFA}{}^{F},$$

$$R = g^{AB} R_{BA}.$$

From here using the formulae of the section IV and the equations (10)–(13) we obtain Ricci curvature components of the Minkowski supermetric (7)

$$R_{ab} = \frac{4}{\lambda^2-\mu^2} g_{ab},$$

$$R_{b\alpha} = \frac{4}{\lambda^2-\mu^2} g_{b\alpha},$$

$$R_{\beta a} = \frac{4}{\lambda^2-\mu^2} g_{\beta a},$$

$$R_{\alpha\beta} = \frac{4}{\lambda^2 - \mu^2} g_{\alpha\beta} + 4A_{\alpha\beta}$$

(where $A_{\alpha\beta}$ is given by the equation (11)) and finally a scalar curvature

$$R = -\frac{16}{\lambda^2 - \mu^2}.$$

By writing the Einstein equations with cosmological constant

$$R_{AB} - \frac{1}{2} R g_{AB} + \Lambda g_{AB} = T_{AB} \qquad (17)$$

we find that the Minkowski supermetric (7) is the solution of these equations with cosmological constant

$$\Lambda = -\frac{12}{\lambda^2 - \mu^2}$$

and stress-energy supertensor

$$T_{AB} = \begin{pmatrix} 0 & 0 \\ 0 & 4(\lambda^2 - \mu^2)^{-1} \left(\lambda (C^{-1})_{\alpha\beta} + \mu (\gamma_5 C^{-1})_{\alpha\beta} \right) \end{pmatrix} \qquad (18)$$

with zero bosonic and nonzero fermionic parts. It is important to note that cosmological constant is nonzero for any values of real parameters λ, μ, $\lambda \neq \pm\mu$ of the Minkowski superspace-time family.

Conclusion

We follow earlier proposed in [1] scheme of obtaining supergeneralizations of classical solutions of General Relativity. On the first stage, this scheme includes constructing supersymmetric expansions of corresponding space-time symmetry groups, then finding solutions of supersymmetric Killing equations and analyzing corresponding Einstein superequations. In this paper the scheme is realized for the Minkowski space whose symmetry group is the Poincare group and its supersymmetric expansion is the Poincare supergroup. The supergeneralization of the Minkowski metric is the Minkowski supermetric $SM(4, 4|\lambda, \mu)$ (7) depending on two real parameters λ, μ obeying the only condition of nondegeneracy of the supermetric $\lambda \neq \pm\mu$. Using the V. P. Akulov and D. V. Volkov's formulae of super-Riemannian geometry [2] we calculated a superconnection and a supercurvature of the Minkowski superspace. It was shown that the curvature of the Minkowski superspace does not vanish, and the Minkowski supermetric is the solution of the Einstein superequations (17), so the eight-dimensional curved super-Poincare invariant superuniverse $SM(4, 4|\lambda, \mu)$ is supported by purely fermionic stress-energy supertensor (18) with two free real parameters λ, μ, and, moreover, it has non-vanishing cosmological constant $\Lambda = 12/(\lambda^2 - \mu^2)$ defined by these parameters that could mean a new look at the cosmological constant problem. See, for example, [7] where, in the frame of a unified approach based on affine geometry [8], the torsion induced 4-fermion interaction is considered and it is discussed how this interaction affects the cosmological term, supposing that a quark condensation occurs during the quark-gluon/hadron phase transition in the early universe.

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