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НЕСИНГУЛЯРНАЯ КОСМОЛОГИЧЕСКАЯ МОДЕЛЬ С КОНЕЧНЫМ ДЕЙСТВИЕМ

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Формулировка квантовой космологии на языке функционального интегрирования подразумевает конечность действия. Вместе с тем, класс моделей с конечным действием чрезвычайно узок и. на первый взгляд, исключает модели вечной инфляции, а также открытые и плоские вселенные и требует существования начальной и финальной сингулярностей. В данной работе, мы обсуждаем возможность построения космологической модели с конечным действием, но свободной от сингулярностей.

Ключевые слова: Космология, действие, сингулярность, модель Деситтера.

NON-SINGULAR COSMOLOGICAL MODEL WITH FINITE ACTION

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The formulation of quantum cosmology in the language of functional integration implies the finiteness of the action. At the same time, the class of models with finite action is extremely narrow and, at first glance, excludes models of eternal inflation, as well as open and flat universes and requires the existence of initial and final singularities. In this paper, we discuss the possibility of constructing a cosmological model with finite action, but free of singularities.

Keywords: Cosmology, action, singularity, De Sitter model.

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Introduction

Although cosmology is only one of the branches of physics (and astronomy), it has one fundamental feature: it is cosmology that defines the background space-time in which all other physical systems are "immersed". This means that the set of consistency requirements imposed on cosmological models must be much more stringent than for other areas of physics.

We begin with the little-used in the literature condition of finiteness of the action [1], [2]. This is an extremely powerful constraint on cosmological models, perhaps more powerful than any other known constraint. The "philosophy"and ideology of this approach are described in detail in the cited works. In short, the action must be finite, when quantized by the functional integral method, otherwise there is no sense in these expressions, and therefore there is no meaningful quantum theory. For example, all reasoning about symmetries is formulated in the language of the action, which also implies its finiteness.

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A perfect example is instantons, which ex definition require a finite action. The finiteness of the action is also important in classical field theory to obtain the equations of motion by varying the action, or Noether currents (which already implies the fulfillment of the Lagrange-Euler field equations), it is necessary to integrate by parts and discard surface terms, which makes sense only when the contributions disappear at infinity (more precisely, on an infinitely distant hypersurface). In other words, the action must be finite. Strictly speaking, this is not a purely mathematical, but rather a physical condition. For example, if we consider a purely homogeneous electric field, then the Lagrangian, which is a Lorentz scalar

$$F_{\mu\nu}^2 \sim \vec{E}^2 - \vec{B}^2 = E^2,$$

turns out to be a constant and the action diverges if integrated over the entire infinite space. But such integration does not make sense in reality, since a real homogeneous electric field is located, say, in a capacitor, between two plates in a battery and occupies a finite volume. In other words, homogeneous fields filling the entire infinite volume are purely formal solutions of Maxwell's equations that cannot be obtained by varying the action, and, at the same time, have no physical meaning. This example clearly demonstrates why the finiteness of the action is a real (and often obvious) physical principle. Here is another wonderful illustrative case. The Davy-Stewartson equations [3] (a two-dimensional integrable generalization of the NLS equation) can be obtained from the principle of least action, as Minore Omote [4] showed in his time and derived an infinite number of conservation laws from Noether's theorem, for DS-1. However, DS allows exact solutions in the form of plane solitons a la solitons of the Kadomtsev-Petviashvili (KP) equations and other solutions that do not tend asymptotically to zero at spatial infinity in all directions on the plane, so, strictly speaking, these solutions should not be associated with the Lagrangian formalism. The only exceptions are the dromions found by Boiti-Leon-Pempinelli [5] and independently by Salle, Lebleu and Yurov [6]. An even more visual demonstration is shown in the example obtained in the works [7], [8] related to the BLP equation . In these works, it is shown that the BLP admits a one-dimensional reduction to the dissipative Burgers equation (which allows one to develop a certain procedure for "dressing"the Burgers equation and even to speak of two-dimensional integrable dissipative structures). But here comes a strange thing: the BLP was derived by the authors (Boiti, Leon and Pempinelli) as a two-dimensional generalization of the sine-Gordon equation, in which there is no dissipation! Moreover, in the work [9] the Hamiltonian structure is described for the BLP. How to reconcile these two facts, which clearly contradict each other? The answer was given in the articles cited above: it's all about the boundary conditions. In particular, Garagash derived the Hamiltonian equations under the assumption that the fields (there are two of them) decay at infinity. The solutions associated with dissipative structures have unlimited level lines, i.e. the Hamiltonian for them diverges. If we pass from the Hamiltonian to the Lagrangian formalism, then these two cases differ precisely in the magnitude of the action. The "dissipative"solutions of the BLP equations correspond to the fact that the action calculated for them diverges. And this is a wonderful illustrative example of the physical meaningfulness of the condition of finiteness of action: for dissipative structures, the action is obvious and should not have meaning, and the fact that it diverges here shows that the principle of finiteness of action is indeed physically meaningful. The divergence of action on specific, formally seemingly correct solutions is the "litmus test"that Lagrangian formalism has ceased to have meaning!

If we take the principle of finite action (hereinafter PFA) seriously, we need to analyze other cases as well. For example, what about stationary solutions of the constant type? Consider, for example, a scalar field with a polynomial potential of the type ϕ^4 or a quadratic mass term. In this case, the equations of motion obviously admit only a trivial solution. A non-trivial constant solution corresponds to the case of spontaneously broken symmetry, say with the potential

$$V \sim (\phi^2 - c^2)^2,$$

but the exact solutions in the form of constants are obviously equal here $\phi = \pm c$ (well, and the trivial solution $\phi = 0$), and when they are substituted, the action is zeroed. There is no mystery in constant solutions in mechanics either (say, a particle lying on a table with constant potential and zero kinetic energy): in mechanics, the Lagrange function is integrated in a finite interval, with the condition that variations disappear at the ends of the interval, so that the action there is finite (of course, for non-pathological Lagrange functions). Now let's turn to cosmology.

1. PFD in cosmology

As Barrow and Tipler showed, the total action consists of three parts: the gravitational action, the matter field action, and the boundary term. The latter can be eliminated if and only if the action is finite. The gravitational Lagrangian is the scalar curvature and should be multiplied by the root of the minus determinant of the metric tensor (we use the signature $(+,-,-,-)$). The matter field Lagrangian is determined depending on the type of matter. Say, for an ideal fluid with the energy-momentum tensor

$$T_{\nu}^{\mu} = (p+\rho)u^{\mu}u_{\nu} - p\delta_{\nu}^{\mu},$$

the Lagrangian is proportional to the EMT hole, i.e. the density minus three pressures, i.e. 4 minus three multiplied by the adiabaticity parameter and all this multiplied by the density. But for the electromagnetic field, such a Lagrangian is equal to zero (conformal invariance leads to the tracelessness of the EMT) and therefore it is necessary to use the usual Lagrangian - F^2. It is important to remember in the future that the action is finite if both terms are finite (we have already discarded the surface term).

The first important observation made by Barrow and Tipler is that the validity of the principle of finite action (PFA) for (approximately) homogeneous and isotropic models means that the Friedmann universe must be closed! We will call this "Proposition 1". The reason is obvious - if the universe is not closed (in mathematical language, it is more correct to speak of non-compactness, i.e. in the class of Friedmann cosmologies it is either a 3-sphere or a torus), then the spatial integral of the Lagrange function diverges. The conclusion follows from this: both the stationary universe models of Bondi, Gold, Hoyle and Narlikar and the inflationary models of Gott, Guth, Linde and Vilenkin are incorrect! That is, the model of eternal inflation is immediately rejected by this principle! This is of course a very unpleasant conclusion, and not for inflation, but for PFD: inflation is too well confirmed by observations to reject it because of one principle, even if it sounds very plausible. However - and this is one of the key points of our article - below we will show that there is one remarkable loophole here, allowing us to reconcile the eternally inflating multiverse and PFD, and moreover, there is some experimental evidence that this loophole is realized in nature! Looking ahead - this evidence is the presence of a positive cosmological constant, so if our hypothesis is correct (see below), then it is precisely this that is NECESSARY for reconciling eternal inflation and PFD. That is, the cosmological constant turns out to be not an object that destroys the theory, as is commonly believed, but on the contrary - an object necessary for the theory. Moreover, it is precisely the cosmological constant, and not the quintessence (or even a phantom). Here and below we will refer to our model as "consistent".

The second observation is as follows: models without beginning and end in time (like, for example, the static Einstein model) are also excluded by the PFD. This will be "Statement 2". Let's say that the static Einstein model is closed (i.e., "Statement 1"works), but this condition is necessary but not sufficient, since both Lagrangians and roots of the metric (it is simpler to talk about such products as Lagrangian densities) turn out to be constants and the integral over time obviously gives a divergence, which violates "Statement 2". The conclusion, or reformulation of "Statement 2"is as follows: the PFD is true if and only if the universe exists for a finite time, i.e. it must have a beginning and an end, and this means the presence of traditional singularities. In other words, Barrow and Tipler literally state the following: "if you want to avoid singularities in the scalar curvature, you will inevitably have to pay the price of singularities in the action. And vice versa: you can get rid of the divergence (singularity) of the action at the price of introducing singularities at the boundary of spacetime"(see [10] for details). Note that the convergence of the action only for closed Friedmann cosmologies with an ideal fluid can be fully investigated simply by monitoring the scale factor. Let us demonstrate this non-obvious statement. Obviously, both contributions to the action around the singularity, which, for convenience, is achieved at $t = 0$ (this can always be done by a linear transformation), are proportional to t^s, where $s = -1 + 2/\gamma$, and $\gamma = w + 1$ is the adiabaticity parameter (we are of course talking about Friedmann cosmologies with an ideal fluid and a barotropic equation of state). The only difference is in the numerical factor: for the gravitational action it is equal to $\gamma - 4/3$, and for matter fields it does not depend on the adiabaticity parameter. At $\gamma = 2$ (the extremely rigid equation of state) another functional dependence arises: $\log t$, à $\gamma = 0$, and this is either dS or AdS. In addition, in the vicinity of the singularity, the scale factor is obviously $a \sim t^m$ with $m = 2/(3\gamma)$. The value $s > 0$ at $0 < \gamma < 2$, so at $t \to 0$, the action remains finite at an adiabaticity parameter lying in this interval (or $-1 < w < 1$). This completely excludes singularities in the phantom zone (we will dwell on this in more detail later) and in the region $w > 1$, and this is the behavior of ekpyrotic cosmologies, so that these models [11] are completely excluded (in agreement with [12]), along with the phantom singularity of the Big type Rip . When $\gamma = 2$ the logarithm gives a divergence of the action at zero (by the way, this excludes chaos of the mixed world model type, since there, with the Kasner vacuum, the asymptotes have the form $a \sim t^{1/3}$, $m = 1/3$ hence $\gamma = 2$, and we excluded this case (see the details in the two cited papers [1], [2])). Thus, the most chaotic Taub and BHL singularities are excluded by the PFD. And by the way, the "Omega point"in the Tipler form is excluded! But approaching zero of the scale factor with $w = -1/3$ ($\gamma = 2/3$) gives $s = 2$. That is, the PFD allows such a version of the "Omega point" [13] The PFD allows, but remember that the scalar curvature diverges there, as $1/t^2$.

Now let's see why the flat and open models are excluded. The flat one is the simplest. There, the dependence $S(t)$ and $a(t)$ are determined by simple power functions with exponents s and m not approximately, around the singularity, but exactly. But then, in flat cosmology, the condition $s > 0$, which guarantees the convergence of the action at the singularity point, guarantees divergence at $t \to \infty$. The story is similar in open cosmology. At large times, the term with curvature dominates in the Friedmann equation, which is inversely proportional to the square of the scale factor. It follows that at large times, $a \sim t$, i.e. $m = 1$ and $\gamma = 2/3$ (i.e. $w = -1/3$). By the way, this is obvious from the Rauchaudhuri equation: since the scale factor tends to a linear function, the second derivative tends to zero, and therefore the combination tends to zero $\rho + 3p$. This means that the action behaves as a power function with exponent $s = 2$ (we know this) and diverges at $t \to \infty$.

So, only closed cosmology remains. Once again, this happens: for two reasons. First, the integral over spatial variables is finite (and the action does not diverge because of this), second, such a universe expands for a finite time and therefore the integral over time converges.

Barrow analyzed many interesting examples (both scalar fields and anisotropic models, sudden singularity), but we refer the reader to his work and will now deal with our "consistent"model.

2. Inflation and complementarity

As already noted, the conclusions obtained in [1] contradict the idea of eternal inflation. In principle, one can try to save inflation by considering a model with $\Omega > 1$, which Linde did, relying on data on the absence of correlations at low multipoles [14]. The general result of his research: such a model can be made, but it will differ from the familiar chaotic or new inflation in two fundamental respects: homogeneity and isotropy must be considered not as consequences of inflation, but as consequences of the fact that the universe arose due to quantum tunneling with suppressed probability, and it is necessary to adjust the parameters with an accuracy of up to 10%. Normal inflation, as a rule, eternal, demonstrates a huge number of e -folds and this leads to an infinite universe (on a hypersurface inside the packet universe, along which the inflaton and (or) matter fields are homogeneous). Thus, we obtain a multiverse as an inevitable consequence of inflation, or rather two multiverses: the first and second according to Tegmark's classification [15]. But the corresponding action (even for multiverse 1, i.e. for one package universe) will diverge. And this will happen due to eternal inflation, and does not depend on the closedness or openness of the model.

Indeed, let us take the famous Vilenkin model - the birth of a multiverse from nothing [16]. A closed bubble is born, full of metastable vacuum, which inflates exponentially. Fractal regions constantly arise in it, in which inflation ends and energy is pumped into frozen fluctuations, which gives secondary heating and the birth of 10^{80} elementary particles. These regions expand at the speed of light, but are separated by an inflating metastable vacuum. The fact that the expansion goes on forever (inflation goes on forever) means that these regions are infinite from the INSIDE for their observers. The action diverges with any method of calculation. For example, from the point of view of an internal observer, his universe is spatially infinite, even if in the future it collapses (see [17], [18]), which means the action diverges. And if we think of an "external"observer, the action will diverge because dS eternal expansion takes place. Then the contribution to the action (the first or second term - it doesn't matter) will be determined by the integral of $a^3 \sim \exp(3Ht)$ $(H = const)$ over time with an infinite upper limit, so it will diverge, despite the fact that the spatial integral converges (if this is not clear, then, say, in the gravitational part there is a scalar curvature, and in the dS regime it is a constant; similarly in matter fields). And by the way, it is for this reason that the principle of finite action excludes AdS models. There are no real singularities on the boundary of space-time (as for dS), and the number of oscillations is infinite, so the action diverges. To limit ourselves to one oscillation means the presence of some new boundary condition determined by physics unknown to us. In general, this is a question for specialists in string theory, AdS / CFT correspondence, etc. Well, those who assume that AdS geometry is possible must remember that the geometry of an open world is certainly realized there, so the action will diverge according to spatial variables, as we have already discussed above.

Let's return to dS , i.e. to our universe. So far everything looks disappointing, but have we taken everything into account? No, not everything. We have not taken into account the most important cosmological horizon. Its presence changes everything!

The horizon requires the inclusion of Susskind's ingredient: the complementarity principle (CP) [19], [20]. This is both good and bad. Let's start with the good. From the point of view of an observer inside, the universe ends at the horizon. The spatial volume of the universe is finite, and it is not 40 billion light years (taking inflation into account), but 13.7 billion. That is, 10^{28}cm. From the point of view of the internal observer, the horizon is covered with a "quantum flame"whose thickness is the same as the stretched horizon (the Planck thickness). The fact of the horizon's presence and the application of CP gives the convergence of the spatial integral when calculating the action, and this does not depend on whether the universe is closed, open, or flat, as long as the horizon is inside (in a closed universe) or simply "is"in a flat and open universe. This is the good news. The bad news is that the action for a stationary (dS expanding) universe will still diverge in time. But this will not be the case if the expansion inside the bubble is replaced by compression.

A natural question is: why should this happen, if the universe is dominated by positive vacuum energy? In response, we propose to look at things from the other side. As already noted, if it is undesirable to have divergent action, then one should use divergent scalar curvature (and the Ricci and Riemann tensor). This makes more sense, argue Barrow and Tipler, because singularities at the edge of spacetime are simply the edge of spacetime. Perhaps one should not fight them and expect quantum gravity to be a theory that will "kill"these singularities, but accept them as a fact, accept them as something that saves us from a much worse problem - singularities in action. This is really worse, because with divergent action there is no hope of constructing quantum gravity. This is the point of view of Barrow and Tipler.

But we propose to ask the question differently: is it possible to get rid of both singularities at once, both the singularity in action and the singularity at the edge of space-time? Is it possible? Answer: it is possible in a contracting flat universe filled only with positive vacuum energy density (i.e. dS). Firstly, in such a universe there is no "ordinary"singularity, because the scalar curvature is constant (we emphasize this, in general, obvious circumstance, because there may be a wrong impression that the singularity is simply transferred to infinity, but not eliminated. In this case, there would be no particular difference from ordinary singularities, which are also "located"at the edge of space-time. However, there is no singularity at all in our "consistent"model). Secondly, in such a universe, the scale factor $a \sim \exp(-Ht)$, and therefore the gravitational action, converges. And the action with the matter fields is also finite, and this despite the fact that we integrate from t_0 to $+\infty$. But here the question immediately arises: the horizon in such a universe is located at infinity, i.e. it no longer exists, which means that now the action diverges again due to the spatial integral. It looks like a complete fiasco, but let's not rush.

Let us postpone this problem until the next section and try to consider extremely general models. We already know that if the universe contracts asymptotically, then both singularities can be avoided simultaneously (if we forget about spatial infinity). We have already considered the exponential, the second option is power-law compression. Such models appeared in the works of Pedro Gonzalez-Diaz and colleagues [21], [22] studying the dynamics of the universe after the hypothetical Big Bang Trip through the exploded molehill and it is not accidental. Obviously, this is only possible for phantoms, with a constant (negative) γ, since if at large t $a \sim t^{-n}$, with $n = -m > 0$, then it means $\gamma < 0$. But such models are excluded, because if they are admissible, then somewhere in the multiverse there must be a domain (in fact, infinitely many domains) where the singularity Big appears Rip , which, as we have already said, is excluded by the PFD. It is easy to show why. Let, $a = C(T-t)^{-n}$, where T is the moment of the singularity of the big break, $n > 0$. The scalar curvature in the Friedmann universe is given by the formula

$$R = 6\frac{a\ddot{a} + \dot{a}^2 + k}{a^2}, \tag{1}$$

Substitute the expression for a and set $k = 0$, then multiply by a^3 and integrate. This will be the gravitational part of the action and we find that it is proportional to $(T-t)^{-3n-1}$. Thus, Big Rip is excluded by the PFD condition, since expression 1 obviously diverges at $t \to T$.

For completeness, consider the asymptotic contraction according to the law $a \sim t^{-n}$, $n > 0$. Then, obviously, the scalar curvature behaves in the flat case as $1/t^2$, i.e. disappears, which shows the absence of a singularity at infinity. The gravitational action behaves as t^{-1-3n}, i.e. also converges, but the presence of Big Rip spoils the whole picture.

Thus, all hope lies in the remaining "consistent"model, in which both singularities are absent, if the problem of the divergence of the spatial integral is solved. Of course, other models can be investigated, for example of the type $\exp(-t^2)$, which we have already considered earlier [23], but it must be clearly understood that there is no physical reason to do so.

3. Contracting dS Universe and Instanton Tunneling

Now let's turn to the horizon problem. Does the horizon disappear during compression? A meaningful answer to this question depends on what is (or rather will be, or even more precisely, can be) the reason that expansion will be replaced by compression. The first possible answer is the Vilenkin-Garrigy mechanism, in which the scalar field slowly creeps down. Accordingly, the value of the effective cosmological constant decreases, is zeroed out, becomes negative, and an uncontrollable collapse begins. By the way, here we must keep in mind the following: this does not mean that inflation is not eternal and that the universe is not infinite, although at first glance it seems that this is exactly the case: after all, we know that it is eternal inflation that gives the amazing effect of spatial infinity of the "pocket"universe for an observer from the inside - the infinite time of expansion for the "external"observer turns out to be infinite space for the internal one. And if it collapses, does this mean that there is only finite time for the external one? No. It does not. Here is the same effect as with the decay of the metastable vacuum in Green's pictures: we draw a hyperbola (or another spatially similar hypersurface of constant time) and approximate the dynamics with a cellular automaton. In the first step, we paint the square above the hyperbola in the center black - this is the area where the compression began. In the second step, we move one cell up and paint two squares on the sides of the black one black, the central one even blacker. One step up, two more black ones on the edges, the two black ones on the previous one become two even blacker, and the central one black-black-black. The hypersurface is drawn through squares of constant color. In order not to get confused with colors, you can use numbers: one cell with number 1, all the rest with zeros. Above it there are already three non-zero cells: the central one with number 2 and two with number 1 on the edges. The rest are zeros. Above, there are already five non-zero ones: the central one with the number 3, its neighbors are two with the numbers 2, and two close with the numbers 1. Then above there are already 7, etc. Let's assume that the collapse occurs in cells with numbers equal to 100. Then we stop at this point, but the number of cells with numbers less than 100 will be infinite and all cells with the number N <100 form an infinite space with a clock showing N. Simply, the number of such infinite hypersurfaces will be N (starting from zero). We recommend that the reader perform these simple actions on a piece of paper in a box and verify the validity of our statement.

It is important to understand exactly how everything happens. We have talked about cellular automata, but this can be confusing and create the wrong impression that cells with the same numbers sequentially generate their offspring. This is absolutely not true! We used automata only to make the algorithm obvious. In fact, the Vilenkin-Garrighi mechanism is not a sequence of causally connected regions where expansion is replaced by contraction, not a history where cells generate new ones. In fact, these cells **are not causally connected to each other at all**. This is a complete analogy with the boundary of packet universes, which constantly advances on the inflating sea and represents constantly occurring Big Bangs. But these Bangs are causally unconnected and are located on spatially similar hypersurfaces. Our cells with identical numbers also lie on spatially similar surfaces, and not on isotropic ones, as it may seem, and the impression of the coherence of the entire dynamics is connected only with the fact that there is a global scalar field with a certain pattern, which changes very slowly in space and therefore seems to "lead in a dance"different cells in an apparent common rhythm, a rhythm which creates the impression that these cells are causally connected to each other, since they dance so harmoniously. But this is - let us repeat - an illusion.

Unfortunately, such a mechanism of changing expansion by compression does not suit us, because as soon as the cosmological constant becomes zero, the saving horizon disappears. The disappearance of the horizon seems to be an absolute violation of relativistic physics, but this is not so: the transition to the compression phase is not consistent, causal, since it occurs on spatially similar hypersurfaces. Imagine our cellular pattern once again: for an observer located in one of the cells corresponding to the collapse, there is no horizon anymore, since the horizon is associated with acceleration, and for such an observer, all hypersurfaces on which the cells with the number zero lie, lie in the past. This means that the universe is already spatially infinite from the point of view of the internal observer, i.e. **it is no longer possible to hide behind the saving PD** and the action inevitably diverges, violating the PFD.

We need another process, in which, from the point of view of the internal observer, a finite bubble of the "new phase"is formed, in which compression takes place, and the walls of the bubble of the new phase expand at the speed of light inside the dS volume. This bubble very quickly fills almost the entire dS volume, but can never catch up with the horizon, which means that the horizon exists all the time, reliably hiding the collapsing universe from the environment and the spatial region remains finite. So, we have what we were looking for: a collapsing dS bubble, but finite because it is eternally surrounded by the dS horizon.

We are not discussing the mechanism of formation of such a bubble now. We are talking about more general things, namely, if such a mechanism exists, does it lead to the required picture, in which both curvature singularities and action singularities are absent? If we think about it a little, it becomes clear that there is one objection: quantum transitions. The point is that our model must exist forever, otherwise it will transform into some other one, but above we have already given arguments in favor of the fact that any other model will either not be physically consistent or will allow one of the two types of singularities. On the other hand, in string theory (and in field theory in general) there is a non-zero probability of instanton tunneling [24] of the universe. If the universe exists forever, then such tunneling will occur with probability one, but it turns out that the horizon saves us again.

For the assessment we will use the approach announced in [25]. Let be A the velocity per unit of 4-volume for the nucleation of a bubble of a new phase, which begins to propagate at the speed of light, destroying everything in its path within the causal future cone of the event - the nucleation of the bubble. This is a description of the decay process of the old vacuum. Don Page introduces the quantity $P(p) = \exp(-AV_4(p))$, where $V_4(p)$ is the 4-volume of the light cone of the past event in the background space-time. Accordingly, $P(p)$ this is the probability that space-time will be preserved, will not decay in the event p. Well, all the truly complex physics "sits"in the numerical value A. Now it is clear how to avoid the black swan of quantum decay. There are two regions: I - the region between the bubble boundary and the horizon and II - the region of the collapsing dS space inside the bubble. Let us consider them separately.

We will consider a region larger than that given by Page's formula, the entire region I . Let the bubble originate at $t = 0$, in the center dS of the region with the Hubble parameter H, i.e. at $r = 0$ and begin to propagate at the speed of light. Integrating the isotropic radial piece of the interval with these boundary conditions, we obtain that at the moment of time t the coordinate of the boundary of the bubble of the new phase is equal to $r = (1 - \exp(-Ht))/H$. The physical distance is obtained by multiplying this value by the scale factor at the moment of time zero (the entire description is from the point of view of an observer contemplating the appearance of the bubble). That is, $a(0) = \exp(0) = 1$. This means that at the moment of time t in a flat metric the three-dimensional volume of region I $V_3(t) = 4\pi(H^{-3} - r(t)^3)/3$. Now we integrate it from t to infinity and get:

$$V_4(t) = (4/9)\exp(-Ht)\pi(\exp(-2Ht) - (9/2)\exp(-Ht) + 9)/H^4.$$

Thus, for large t, the main contribution is given by the term e^{-Ht} (the rest are higher powers and disappear faster), which means that for the entire region I (for all its points) the probability of survival is

$$P_I(t) = \exp(-\frac{4\pi A}{H^4}\mathrm{e}^{-Ht}),$$

i.e. tends to unity as an exponential to the power of an exponential at large times. The probability of a quantum transition is not negligible only at large times, but it is at these times that the probability of survival turns out to be essentially unity (minus one divided by Google, i.e. essentially zero). This miracle occurs because $V_4(t) \to 0$ "twice exponentially"when growing t in the region I. So in the region I the Black Swan flew away!

And region II ? But there is an asymptotic contraction there! The scale factor $a = \exp(-Ht)$, so the 4-volume of region II from t to infinity is equal to $(1/3)\exp(-3Ht)/H$ and after substitution into Page's formula gives the same effect as region I ! Thus we obtain the third remarkable property (the first two: the absence of singularities in curvature and in action) of the "consistent"model: it is eternal and stable to phase transitions between vacuums. At least in the zeroth approximation of our understanding of quantum-gravitational effects, which is purely rudimentary.

4. Discussion

Here the dramatic question arises: what would cause a bubble of such a complex, unnatural configuration to arise? This is the second most difficult question (after the question of what kind of physics happens when a bubble inflates and enters the region of the stretched horizon, the quantum foam?), but it is not at all one of those to which there is no answer. In fact, there are several answers, but all of them are unpleasantly vague. For example, phase transitions due to quantum tunneling cannot be ruled out, but the characteristic times of such processes are very large, much larger than the time during which the Vilenkin-Garrigy mechanism would lead the universe to collapse (on the order of a trillion years), which, as we know, is not suitable. Hence the conclusion: we need a phase transition in a time of the order of 10 gigayears or less. But what could lead to it? Here are two thoughts. First, the "destruction"of the universe by a bubble of a new phase was calculated by Page to get rid of Boltzmann brains, in the work already cited above. He came up with estimates of the order of 19 gigayears, but he was unable to find an adequate mechanism: both the use of decay in supersymmetric theories and the hypothesis of mechanisms associated with Higgs fields required fine adjustments at a level of at least 0.001 (i.e. 0.1%), which is questionable. However, there are interesting works (see, for example, [26]) that relate possible vacuum instability to Higgs fields in a more plausible way. This is the first direction.

The second thought is connected with the transition to compression after passing through a singularity of the second type according to the classification of N. O. T. [27], the so-called Sudden future singularity , discovered by Barrow [28], which was investigated in [29], where the authors came to the surprising conclusion that we are separated from it by a million to 10 million years! There is also another option, described in our work [18], where we studied the possibility of generating a sudden singularity by a downward creeping inflaton.

It would seem that we have abandoned the consideration of inflaton as a source of change of the expansion mode to the compression mode? Not quite: we have indeed abandoned the use of the effect of slowly creeping inflaton, but the appearance of features of the Sudden type is a completely different story and we assume that this story should look exactly as it should - not a global creep and a slow decrease of the cosmological constant on huge scales (much larger than the horizon), but the emergence of a small bubble inside the Hubble volume. The point is that relativistic equations mean the following - time and space scales are one and the same (especially in the natural system of units). Let's say the creep of inflaton calculated by Vilenkin and Garriga has a time scale of a trillion years (i.e. a thousand gigayears), which means the spatial scale of changes is approximately the same. The field itself is smoothed out by inflation on much larger scales, but - and this is interesting - a trillion light years is quite enough for us, since our entire volume inside the horizon is one percent of this scale! But Sudden is characterized by other times, much smaller than 10 gigayears, which means that spatial inhomogeneities corresponding to such time scales should be smaller than the Hubble volume. In Dabrowski et al., this is a million light years, i.e. a bubble the size of the distance between the Milky Way and M31 (Andromeda Nebula), and our estimate was 10 gigayears, i.e. exactly our Hubble volume, which in principle will also work, but we must remember that this is an absolutely qualitative and rough (albeit very witty) estimate. A detailed study can reduce it by orders of magnitude.

Of course, we need to be sure that Sudden does not produce the singularities we are fighting! Barrow himself has considered the sudden singularity for compliance with the PFD. Although the question will have to be carefully studied, it is almost obvious that there are no singularities there that are excluded by the "consistent"model. Indeed, in our model, the second derivative of the scale factor divided by the scale factor is proportional to the delta function (see [18]), so the scalar curvature becomes infinite for one moment, but this is **a soft divergence** through which the geodesic can be continued, as is well known thanks to a whole pool of rigorous mathematical results in this area (see, for example, [30], [31]). And the action remains finite, because we need to multiply, in essence, this delta function by the cube of the scale factor (smooth and continuous) and integrate, which obviously gives just a cube a at this point. The remaining terms in the Gaussian curvature are smooth, without discontinuities. True, in our article a singularity arises during compression, since we used Λ the CDM model with $a \sim (\sinh t)^{2/3}$. To prevent this from happening, we can use an additional condition the essence of which sounds like this: if during compression all dark matter and baryons are transformed into massless radiation, then the density of matter fields must be exactly equal to the "quantum potential"introduced in [32]. Then these contributions are exactly annihilated and one exponential remains, e^{-Ht} leading the universe as prescribed by the "consistent"model.

We are aware that this coincidence may seem far-fetched, but we suggest looking at it this way: the coincidence condition and only it guarantees that both types of singularities are absent. If we consider that singularities are a signal of the destruction of the theory, then such a condition must necessarily be an ingredient of the theory (just as the number of generations of quarks must coincide with the number of generations of leptons in the standard model), just as the dimension 10 turns out to be critical in string theory from the consistency condition. Thus, we can take a risk and make a prediction: in a unified theory (for example, in M-theory) it should turn out that the model is consistent only if (i) the density of matter fields and the "quantum potential"coincide and (ii) the unified theory requires the obligatory presence of Sudden future singularity (that is, a vacuum phase transition in the range of 10-20 gigayears that changes the sign of the Hubble constant).

Conclusion

The model described in this paper is nothing more than a line of thought based on the assumption that a complete theory should be free of both types of singularity. It is probably possible to propose other models with the same property, but it seems that the set of such models will be very limited, and this makes such speculative studies interesting - special models usually predict very specific observational effects. In conclusion, we want to show that a "consistent"model may be able to shed light on the central problem of dS cosmology: how to reconcile Susskind's FD with the holographic principle? If the number of bits in the universe does not exceed the area of the horizon in Planck units, then how can such a description be additional to the external multiverse? The multiverse contains infinitely much information. This means that the description "outside"(even imaginary) cannot be equivalent to the description "inside". But FD requires exactly this: there are two ways to describe physics and they cannot be used simultaneously: either the first or the second. But these are two ways to describe the same physics. These are two equivalent descriptions, in a certain sense. However, it is absolutely obvious that descriptions containing objects with infinitely large information (or entropy) and containing only objects with finite information cannot be equivalent, i.e. complementary!

Now, it is possible that the "consistent"model does not suffer from this conundrum. The point is that it can be shown to satisfy two conditions: (1) the geodesics are finite, (2) the free energy diverges, which means that it processes infinitely many bits. In other words, the answer to the riddle is surprisingly simple: in our model, a single universe contains infinitely much information, so it is equivalent in this sense to the entire multiverse. So Susskind complementarity as applied to cosmological horizons works. And also: it has a well-defined arrow of time and unlimited growth of entropy. But that's another story.

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