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КОСМОЛОГИЧЕСКАЯ ДИНАМИКА И АККРЕЦИЯ НА ЧЁРНЫЕ ДЫРЫ В МОДЕЛИ $f(R)$-ГРАВИТАЦИИ ПО СТАРОБИНСКОМУ

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Путем анализа ключевых космологических параметров, включая постоянную Хаббла, параметр замедления и параметр рывка, рассматривается эволюция Вселенной в рамках метрической $f(R)$-гравитации по Старобинскому. Согласно полученным результатам, в настоящее время Вселенная находится в фазе ускоренного расширения. Более того, анализ параметра рывка показывает, что скорость расширения монотонно увеличивается с течением времени. Далее в рамках метрической $f(R)$-гравитации по Старобинскому исследовалась динамика черной дыры, а именно аккреция: поглощение черными дырами энергии и вещества из окружающей их среды. Полученные результаты указывают на то, что более высокая скорость аккреции приводит к увеличению массы и времени жизни черной дыры. Анализ также показывает, что на определенном красном смещении некоторые черные дыры могут испаряться полностью, независимо от их начальной массы.

Ключевые слова: метрическая $f(R)$-гравитация, модель Старобинского, космологическая модель со степенной зависимостью масштабного фактора, космологический параметр, испарение черной дыры.

COSMOLOGICAL DYNAMICS AND BLACK HOLE ACCRETION IN STAROBINSKY-TYPE $f(R)$ GRAVITY

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In this study, we investigate the evolution of the universe by analyzing key cosmological parameters, including the Hubble parameter, deceleration parameter, and jerk parameter, within the framework of Starobinsky-type metric $f(R)$ gravity. Our findings indicate that the universe is currently undergoing an accelerated phase of expansion. Again, our analysis on jerk parameter reveals that the rate of accelerated expansion increases monotonically over time. Moreover, we extend our study to explore black hole dynamics within the metric $f(R)$ gravity under Starobinsky's assumption by considering accretion: absorption of surrounding's energy-matter by the black holes. Our results suggest that a higher accretion rate leads to an increase in black hole mass and extends its lifespan. Furthermore, our analysis indicates that certain black holes may completely evaporate at a specific redshift, irrespective of their initial mass at formation.

Keywords: metric $f(R)$ gravity, Starobinsky Model, power-law cosmology, cosmological parameter, black hole evaporation.

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Introduce

The observed accelerated expansion [1, 2] of the universe poses a significant theoretical challenge in modern cosmology. Two primary approaches have been proposed to explain this phenomenon. The first involves the introduction of dark energy [3, 4], a hypothetical component with negative pressure that modifies the energy-momentum tensor on the right-hand side of Einstein's field equations. The second approach entails modifications to the theory of gravity [5, 6], altering the geometric structure on the left-hand side of Einstein's equations. Although dark energy models have been relatively successful in explaining the present accelerated expansion, their fundamental nature remains elusive. The simplest candidate, the cosmological constant (Λ) [1, 7], when interpreted as the vacuum energy of quantum fields, leads to a discrepancy of approximately 123 orders of magnitude between theoretical predictions and observational data. To address this issue, alternative dynamical models [8] such as Quintessence [9] and K-essence [10] have been proposed. Additionally, Phantom energy [11, 12], which violates the dominant energy condition, and modified matter models like the Chaplygin gas [13] have been explored. However, these models face challenges in providing a comprehensive explanation of cosmological phenomena, including the coincidence problem [14] and early-universe inflation [15, 16]. As an alternative to dark energy, various modified gravity theories have been investigated. These models extend the Einstein–Hilbert action by incorporating arbitrary functions of curvature invariants, such as the Ricci scalar (R) [17–19], the trace of the energy-momentum tensor (T) [20], or the Gauss-Bonnet term (G) [21]. Such extensions are motivated by low-energy effective actions in quantum gravity and string theory, offering a unified framework to address both early-time inflation and late-time cosmic acceleration. Among these, $f(R)$ gravity, formulated within the metric formalism [22,23], represents one of the simplest yet effective modifications of General Relativity.

In this study, we explore the evolution of the universe within the framework of Starobinsky-type $f(R)$ gravity [24, 25], a modified metric gravity model proposed by Alexei Starobinsky. This approach extends General Relativity by incorporating an additional quadratic curvature term in the gravitational field equations. Specifically, we consider a Lagrangian density of the form $f(R) = R + \frac{R^2}{6M^2}$, where M^2 is a phenomenological constant [26] with the same dimension as R. The inclusion of the R^2 term introduces significant cosmological implications, particularly in driving inflationary dynamics and affecting the late-time evolution of the universe. To analyze the cosmological impact of these modifications, we investigate the existence of exact power-law solutions [23] during different cosmic evolution phases dominated by a perfect fluid. Such solutions are crucial as they often correspond to asymptotic or intermediate states within the dynamical phase space governing the full spectrum of Friedmann-Robertson-Walker (FRW) cosmologies. Our study aims to describe the complete cosmic evolution by analyzing the variation of cosmological parameters as a function of redshift. Cosmological parameters play a fundamental role in characterizing the evolution of the universe. The Hubble parameter (H) [27, 28] quantifies the universe's expansion rate at any given epoch, providing insights into its age, expansion history, and large-scale dynamics. The deceleration parameter (q) [29,30] determines whether the cosmic expansion is accelerating or decelerating; a positive indicates deceleration, whereas a negative q signifies acceleration. Additionally, the jerk parameter (j) [31,32] measures the rate of change of cosmic acceleration and helps in distinguishing different cosmological models. These parameters, which are directly linked to the scale factor $a(t)$, serve as crucial observational probes for testing and constraining theoretical models of the universe's evolution. In our calculation we considered the value of phenomenological constant (M^2) as $\frac{0.6538}{{t_0}^2}$ [30], where t_0 is the present age of universe.

Again, beyond cosmological expansion, black holes represent another profound aspect of gravitational physics. These extreme astrophysical objects, characterized by intense gravitational fields, are believed to have formed by the end of the radiation dominated era. In this study, we employ the Starobinsky-type metric gravity framework to investigate both the expansion dynamics of the universe and the evolution of Schwarzschild black holes across different cosmic epochs. We assume that following the inflationary phase, the universe transitioned into a radiation-dominated era, persisting until a characteristic time, after which it entered matter-dominated phase. During the early stages of the matter-dominated era, the universe underwent decelerated expansion, which subsequently transitioned into an accelerated phase. Within this cosmological framework, we analyze the evolution of Schwarzschild black holes [33–35] by considering two key processes: Hawking radiation and the accretion of energy-matter from the surrounding environment. By incorporating both mechanisms, we aim to explore their combined influence on black hole dynamics and their implications for cosmic evolution.

1. Mathematical Paradigm

The field equations of the gravity f(R) model yield the first Friedmann equation for a spatially flat Friedmann-Robertson-Walker (FRW) universe (k=0) as follows

$$H^2 = -\frac{1}{6f'}\Big[6H\dot{R}f'' - Rf' + f - 16\pi G\rho\Big], \tag{1}$$

where $f' = \frac{\partial f(R)}{\partial R}$, $f'' = \frac{\partial^2 f(R)}{\partial R^2}$, $f''' = \frac{\partial^3 f(R)}{\partial R^3}$, $H = \frac{\dot{a}(t)}{a(t)}$ is the Hubble Parameter and $R = 12H^2 + 6\dot{H}$ is the Ricci Scalar, with $\dot{H} = \frac{dH}{dt}$ and $\dot{R} = \frac{dR}{dt}$. For a perfect fluid in a spatially homogeneous and isotropic universe having density ρ, the energy conservation equation can be given by

$$\dot{\rho} + 3H(1+\gamma)\rho = 0. \tag{2}$$

The equation of state parameter, denoted as γ, characterizes different cosmic epochs, taking the value $\gamma = \frac{1}{3}$ in the radiation-dominated era and $\gamma = 0$ in the matter-dominated era. To maintain a positive and physically meaningful gravitational coupling, we impose the condition $f' > 0$. Furthermore, ensuring the stability of classical solutions to Einstein's field equations necessitates the constraint $f'' > 0$. Using the Starobinsky model approximation, $f(R) = R + \frac{R^2}{6M^2}$ in conjunction with a power-law [23, 36] form of the scale factor, the energy density of the cosmic fluid can be expressed as

$$\rho = \begin{cases} \frac{3}{32\pi G t^2}, t < t_e, \\ \frac{3{\beta_1}^2}{8\pi G t^2}\Big[1 - \frac{3(2\beta_1 - 1)}{M^2 t^2}\Big], t_e < t < t_m, \\ \frac{3{\beta_2}^2}{8\pi G t^2}\Big[1 - \frac{3(2\beta_2 - 1)}{M^2 t^2}\Big], t > t_m. \end{cases} \tag{3}$$

Specifically, for $t < t_e$, the universe is in the radiation-dominated era(r.d.e). The interval $t_e < t < t_m$ corresponds to the early stage of the matter-dominated era (e.m.d.e), characterized by a decelerated expansion. For $t > t_m$, the universe enters the later stage of the matter-dominated era (l.m.d.e), where the expansion transitions into an accelerated phase. Here $\beta_1 = \frac{1}{12}\Big[(11+M^2t^2) - \sqrt{(5 + M^2t^2)^2 - 4M^2t^2}\Big]$ and $\beta_2 = \frac{1}{12}\Big[(11 + M^2t^2) + \sqrt{(5 + M^2t^2)^2 - 4M^2t^2}\Big]$.

Similar to the Standard Model of Cosmology and scalar-tensor theory, the scale factor in this framework also evolves during the radiation-dominated era as

$$a(t) \propto \begin{cases} t^{\frac{1}{2}}, t < t_e, \\ t^{\beta_1}, t_e < t < t_m, \\ t^{\beta_2}, t > t_m. \end{cases} \tag{4}$$

Cosmological redshift [37, 38] refers to the increase in the wavelength of light emitted by distant galaxies due to the expansion of the universe. As space expands, photons traveling through it undergo a corresponding stretch, leading to a redshift that is directly related to the galaxy's distance. This phenomenon serves as a fundamental observational tool for probing the accelerated expansion of the universe, as a higher redshift indicates that galaxies are receding at an increasing rate. The relationship

Table 1. The values of $a(z)$, H, q and J with respect to different values of z

z	$a(z)$	H (in sec^{-1})	q	J
0	1	100.679	−0.55	0.6681
0.5	0.665796	127.437	−0.361	0.2383
1	0.499015	78.817	0.993	2.7516
1.5	0.399346	122.381	0.986	2.8982
2	0.332961	175.695	0.989	2.9479
2.5	0.285499	238.746	0.992	2.9701

between redshift and the cosmic time(t) evolves over different epochs and is expressed as

$$z = \begin{cases} \left(\left(\frac{t_e}{t}\right)^{\frac{1}{2}} \left(\frac{t_m}{t_e}\right)^{\beta_1(t_e)} \left(\frac{t_0}{t_m}\right)^{\beta_2(t_m)}\right) - 1, z < z_e, \\ \left(\left(\frac{t_m}{t}\right)^{\beta_1(t_e)} \left(\frac{t_0}{t_m}\right)^{\beta_2(t_m)}\right) - 1, z_e < z < z_m, \\ \left(\left(\frac{t_0}{t}\right)^{\beta_2(t_m)}\right) - 1, z > z_m. \end{cases} \tag{5}$$

Here, z_e is the redshift during transition of universe from radiation dominated era to matter dominated era and z_m is the transition redshift from decelerated expansion to accelerated expansion of universe.

2. Cosmological Parameters

2.1. Hubble Parameter

The Hubble parameter H describes the rate at which the universe expands at a given time. It is mathematically expressed as $H = \frac{\dot{a}(z)}{a(z)}$, where $a(z)$ is the scale factor that characterizes the relative expansion of the universe, and $\dot{a}(z)$ denotes its time derivative. As a function of time, the Hubble parameter plays a fundamental role in understanding the universe's expansion history and cosmic dynamics. It is also directly linked to the cosmological redshift z through the relation

$$H = -\frac{\dot{z}}{1+z}. \tag{6}$$

Here,

$$\dot{z} = \begin{cases} -\frac{(z+1)^3}{2t_m}\left(\frac{t_m}{t_0}\right)^{2\beta_2(t_m)}, z_e < z < z_m, \\ -\frac{\beta_2(t_m)}{t_0}(1+z)^{\beta_2(t_m)+\frac{1}{\beta_2(t_m)}}, z > z_m. \end{cases}$$

2.2. Deceleration Parameter

The deceleration parameter (q) is a fundamental cosmological quantity used to describe the expansion dynamics of the universe. It determines whether the cosmic expansion is decelerating ($q > 0$), accelerating ($q < 0$), or proceeding at a constant rate ($q = 0$). Mathematically, it is defined as, $q = -\frac{\ddot{a}(z)}{H\dot{a}(z)}$, where $\ddot{a}(z)$ denotes, the second time derivative of $a(z)$. Furthermore, the deceleration parameter can be reformulated in terms of the redshift (z) as

$$q = -1 + \left[\frac{\ddot{z}(1+z)}{\dot{z}^2} - 1\right], \tag{7}$$

where,

$$\ddot{z} = \begin{cases} \frac{3(z+1)^5}{4t_m}\left(\frac{t_m}{t_0}\right)^{4\beta_2(t_m)}, z_e < z < z_m, \\ \left(\frac{\beta_2(t_m)}{t_0}\right)^2 \left(\beta_2(t_m) + \frac{1}{\beta_2(t_m)}\right)(1+z)^{2\beta_2(t_m)+\frac{2}{\beta_2(t_m)}-1}, z > z_m. \end{cases}$$

2.3. Jerk Parameter

The jerk parameter (j) characterizes the rate of change of cosmic acceleration, providing crucial insights into the dynamical evolution and acceleration history of the universe. A positive trend in j suggests an increasing rate of cosmic acceleration, whereas a decreasing j indicates a deceleration in this rate. Mathematically, the jerk parameter is defined as $j = -\frac{\dddot{a}(z)}{a(z)H^3}$. Where $\dddot{a}(z)$ represents the third derivative of the scale factor $a(z)$ with respect to cosmic time z. Furthermore, the jerk parameter can be expressed as a function of redshift (z), facilitating its empirical estimation through observational data as

$$j = \dddot{z}\left(\frac{z+1}{\dot{z}}\right)^3 - \frac{6\ddot{z}(z+1)}{\dot{z}^2} + 6, \tag{8}$$

where,

$$\dddot{z} = \begin{cases} -\frac{15(z+1)^7}{8{t_m}^3}\left(\frac{t_m}{t_0}\right)^{6\beta_2(t_m)}, z_e < z < z_m, \\ -\left(\frac{\beta_2(t_m)}{t_0}\right)^3\left(\beta_2(t_m) + \frac{1}{\beta_2(t_m)}\right)\left(2\left(\beta_2(t_m) + \frac{1}{\beta_2(t_m)}\right) - 1\right)(1+z)^{\left(3\left(\beta_2(t_m)+\frac{1}{\beta_2(t_m)}\right)-2\right)}, z > z_m. \end{cases}$$

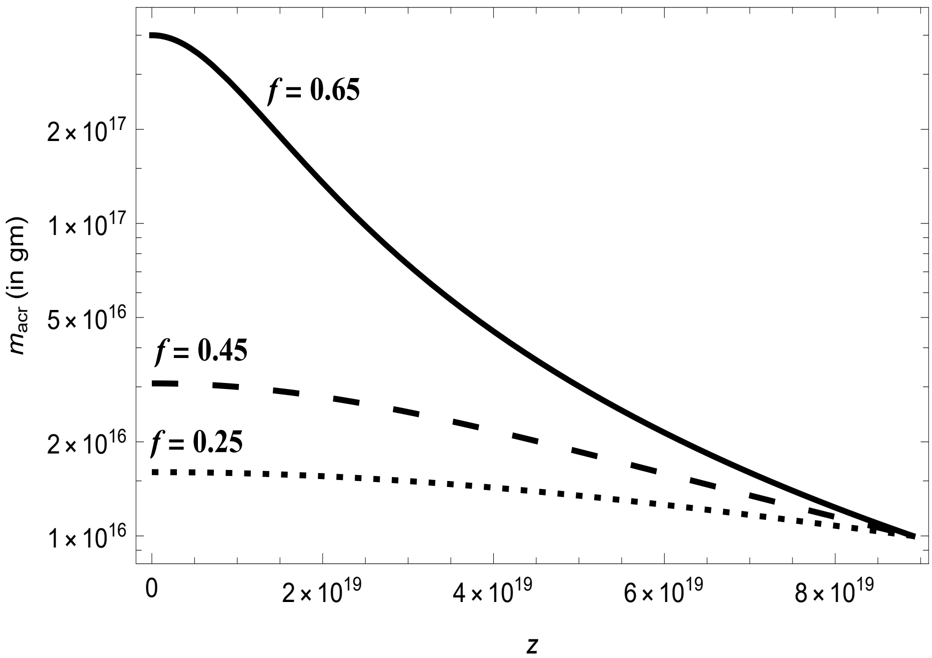

Fig. 1. Variation of accreting mass (m_{acr}) of $m_i = 10^{16}$ gm with redshift (z) for different accretion efficacy (f) in radiation dominated era.

3. Evolution of Black hole

The mass of a black hole evolves through two fundamental processes: accretion [39, 40], where it gains mass by absorbing surrounding matter and energy, and Hawking evaporation [41, 42], a quantum mechanical phenomenon that leads to mass loss near the event horizon. In the context of modified gravity within the metric formalism, these mechanisms are significantly influenced, altering the black hole's growth and evaporation dynamics.

3.1. Black hole Accretion

The mass of a black hole increases through the process of accretion, governed by the following equation

$$\dot{m}_{acr} = 16\pi G^2 m^2 f\rho. \tag{9}$$

Here, f represents the accretion efficiency, which determines the black hole's accretion rate. The solution to the above differential equation in the radiation-dominated era is given by

$$m_{acr} = m_i \left[1 + \frac{3f}{2}\left(\left(\frac{z_{acr}+1}{z_i+1}\right)^2 - 1\right)\right]^{-1}. \tag{10}$$

Fig. 1 illustrates the dependence of black hole accretion mass m_{acr} on redshift (z) for different values of radiation accretion efficiency (f). The results indicate that an increase in accretion efficiency (f) leads to a corresponding growth in black hole mass.

3.2. Black hole evaporation

The mass of a black hole gradually decreases due to Hawking radiation, following the relation [42]

$$\dot{m}_{evp} = -\frac{a_H}{256\pi^3}\frac{1}{G^2 m^2}. \tag{11}$$

Here, a_H is a dimensionless constant associated with the characteristics of the black hole horizon. The solution to the given differential equation in the radiation-dominated era is given by Here, a_H is a dimensionless constant associated with the characteristics of the black hole horizon. The solution to the given differential equation in the radiation-dominated era is given by

$$m_{evp} = \left[m_{acr}^3 - \frac{3t_e a_H}{256\pi^3 G^2}\left(\left(\frac{t_0}{t_m}\right)^{\beta_2(t_m)}\left(\frac{t_0}{t_m}\right)^{\beta_2(t_m)}\right)^2\left(\frac{1}{(z_{evp}+1)^2} - \frac{1}{(z_{acr}+1)^2}\right)\right]^{1/3}. \tag{12}$$

Fig. 2 illustrates the dependence of black hole evaporation mass m_{evp} on redshift (z) for different values

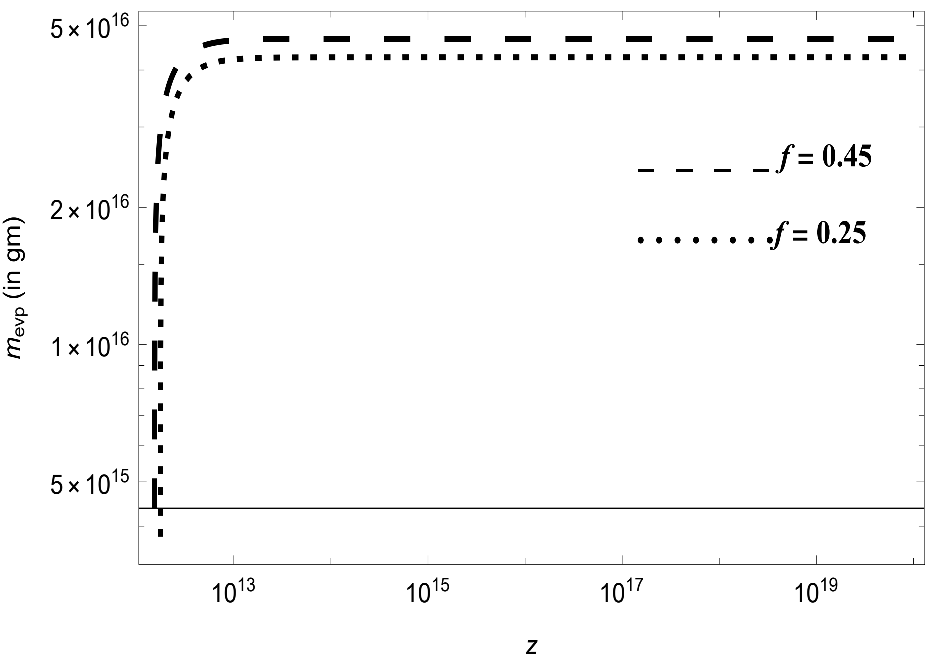

Fig. 2. Variation of evaporating mass (m_{evp}) of $m_i = 10^{16}$ gm with redshift (z) for different accretion efficacy (f) in radiation dominated era.

of radiation accretion efficiency i.e. $f = 0.45$ and $f = 0.25$. The results indicate that an increase in accretion efficiency (f) leads to a corresponding decay in black hole mass.

The redshift corresponding to a given evolutionary period is determined by the following equation.

$$z_{evp} = \left[t_0 \left(\frac{G^2 m_i^3}{3} + t_e \left(\left(\frac{t_m}{t_e} \right)^{\beta_1(t_e)} \left(\frac{t_0}{t_m} \right)^{\beta_2(t_m)} \right)^2 \right)^{-1} \right]^{\beta_2(t_m)} - 1. \quad (13)$$

Conclusion

In this study, we investigate key cosmological parameters namely, the Hubble para-meter(H), deceleration parameter(q), and jerk parameter(j) within the framework of metric $f(R)$ gravity. The analysis is conducted for both decelerated and accelerated expansion phases, employing the Starobinsky model as a theoretical foundation. The behaviour of the jerk parameter j indicates a progressive increase in the rate of acceleration, suggesting that the universe will continue expanding at an accelerated pace into the distant future. In addition to cosmological evolution, we explore black hole dynamics within the metric-based $f(R)$ gravity framework, adhering to Starobinsky's assumptions. Our analysis considers both mass accretion from the surrounding energy-matter and mass loss via Hawking radiation. The results indicate that black hole mass increases with higher accretion efficiency during radiation accretion, while the redshift associated with black hole evaporation decreases with greater formation mass.

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- Author contribution: S.K. Pati wrote the main manuscript text and performed all the analysis. B. Nayak contributed to the idea, figures, tables and results. All authors discussed the results and reviewed the manuscript.

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