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# ЛИНЕЙНЫЕ ВОЗМУЩЕНИЯ В МОДЕЛИ ТЕМНОЙ МАТЕРИИ С ОСЦИЛЛИРУЮЩИМ СКАЛЯРНЫМ ПОЛЕМ<sup>\*</sup>

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Осциллирующее скалярное поле ведет себя как холодная темная материя в расширяющейся Вселенной. Возмущения скалярного поля рассматриваются в иерархии временных и пространственных масштабов, порождаемых отношением H/m. Показано, что скалярное поле воспроизводит свойства холодной темной материи на всех значимых космологических масштабах и на всем протяжении эволюции Вселенной с момента начала осцилляций. Эффективное обрезание коротковолновых возмущений происходит и на стадии преобладания материи, и на стадии преобладания излучения.

Ключевые слова: Аксион, темная материя, осциллирующее скалярное поле.

# LINEAR PERTURBATIONS IN THE OSCILLATING SCALAR FIELD DARK MATTER MODEL

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An oscillating scalar field acts like cold dark matter in the expanding Universe. Perturbations of the scalar field are considered through a hierarchy of time and spatial scales generated by the H/m ratio. The scalar field reproduces the CDM scenarios on all relevant cosmological scales and throughout the evolution of the Universe from the beginning of the oscillations. An effective cutoff of short wavelength perturbations occurs in both matter-dominated and radiation-dominated epochs.

Keywords: Axion, dark matter, oscillating scalar field.

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#### Introduction

The first evidence in favor of cosmic dark matter (DM) was obtained almost a century ago. Current observational data clearly indicate that nonbaryonic DM contributes about a quarter of the energy density in the Universe [1]. A number of theories provide us with various candidates for DM that are not included in the Standard Model of particle physics [2].

A preferred DM model is a cold DM (CDM) formed by collisionless weakly interacting massive particles (WIMP) being nonrelativistic already from the moment of decoupling from a thermal bath [3]. CDM is in the best agreement with the most reliable observations, and despite some problems, it is a reference model in modern cosmology and astrophysics. Alternative versions of DM have to reproduce the main characteristics and properties of CDM.

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Primarily, CDM is pressureless since it consists of nonrelativistic particles. The next important feature is that CDM provides a fast growth of spatial inhomogeneities in the matter-dominated (MD) stage. The density contrast  $\delta$  for subhorizon perturbations increases as scale factor,  $\delta \propto a$ , leading to the formation of modern large-scale structures [5].

A DM model, which provides similar properties, is based on a massive (pseudo)scalar field when its mass m and the Hubble expansion rate H satisfy the condition  $H \ll m$ .

Although scalar fields have extensive implementations in the inflation theory (see, for example [], and references therein), this model is primarily motivated by axions and axion-like particles (ALPs) as DM candidates [6–9]. Features of this model are well studied for the MD epoch [5, 10–15] when the relation mentioned above holds even for ultralight bosons with masses  $m \sim 10^{-22}$  eV.

In this letter, an approach that can be applied to all relevant cosmological scales and throughout the evolution of the Universe including the radiation-dominated (RD) and  $\Lambda$ -dominated ( $\Lambda$ D) epochs, is briefly presented.

## 1. Description of the approach

We consider DM as a non-interacting massive scalar field minimally coupled to gravity in the flat Friedmann-Lemaître-Robertson-Walker spacetime. The field equation reads as

$$\ddot{\psi} + 3\frac{\dot{a}}{a}\dot{\psi} + M^2\psi = 0, \qquad (1)$$

This equation is conveniently thought in terms of dimensionless quantities, so that the dot denotes a derivative with respect to the dimensionless comoving cosmological time  $H_{\rm s}t \rightarrow t$ , where  $H_{\rm s}$  is the initial value of the Hubble parameter for the respective epoch, and  $M = m/H_{\rm s} \gg 1$  is a natural large parameter, which determines a ratio between time scales in the scalar field evolution.

The oscillating scalar field can be examined using an asymptotic expansion based on this ratio [16]. This method provides an appropriate way to separate inhomogeneities according to the spatial scales, giving opportunities for a more detailed description of the scalar field perturbations. Besides, the equations are solved without averaging techniques. This approach reproduces the results previously obtained for the background scalar field and its subhorizon perturbations in the MD epoch [10–15], as well as offers a description for the RD and  $\lambda D$  epochs.

A solution to Eq. (1) can be obtained as a two-time dependent uniformly valid asymptotic expansion in powers of 1/M

$$\psi(t) = u(t)\cos T + O\left(\frac{1}{M^2}\right),\tag{2}$$

where the slow time t corresponds to a cosmological time scale while the fast time T = Mt, and  $u(t) \propto a^{-3/2}(t)$ . The scale factor evolves according to which entity dominates the epoch under consideration. Details of solutions for subdominant terms also depend on the epoch.

To consider small inhomogeneous perturbations of the scalar field in the expanding Universe we use the Newtonian gauge for general scalar metric perturbations so that the line element has the form

$$ds^{2} = (1+2\Phi) dt^{2} - a^{2}(t)(1-2\Phi)\delta_{ij} dx^{i} dx^{j}, \qquad (3)$$

The scalar field is now described by the sum

$$\psi(t, \mathbf{x}) = \psi_0(t, T) + \varphi(t, T, \mathbf{x}), \qquad (4)$$

where  $\varphi \ll \psi_0$ , and the homogeneous background  $\psi_0(t, T)$  is represented by the leading term in (2). The fast time can now depend on spatial inhomogeneities,  $T = M(t + S(t, \mathbf{x}))$ , where a linear perturbation of the phase S is taken into account in the same order as  $\varphi$ .

The scalar field perturbations obey the equation

$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} - \frac{1}{a^2}\nabla^2\varphi + M^2\varphi - M\frac{\partial_T\psi_0}{a^2}\nabla^2S + 2M^2\partial_T^2\psi_0\dot{S} - 2\left(\ddot{\psi}_0 + 3\frac{\dot{a}}{a}\dot{\psi}_0\right)\Phi - 4\dot{\psi}_0\dot{\Phi} = 0, \quad (5)$$

where the dot implies  $\partial_t + M \partial_T$  for the two-time dependent functions and is reduced to the derivative with respect to t for quantities depending only on the slow cosmological time. Eq. (5) is supplemented with the equations for metric perturbations in the respective epoch.

Since Eq. (5) includes the rapidly oscillating background functions, we also search solutions for perturbations as an asymptotic expansion in powers of 1/M:

$$\varphi = \varphi_0(t, T, \mathbf{x}) + \frac{1}{M} \varphi_1(t, T, \mathbf{x}) + \frac{1}{M^2} \varphi_2(t, T, \mathbf{x}) + \cdots .$$
(6)

The two time scales give rise to a hierarchy in the wavelength scales of the spatial inhomogeneities. To properly include perturbations with different wavelength scales in the expansion (6), we introduce a parameter Q identifying inhomogeneity wavelengths with respect to the Hubble scale so that the perturbed functions are taken to depend on the rescaled coordinate  $X^i = Q x^i$ .

As we will see below, superhorizon perturbations, as well as perturbations entering the Hubble scale, are described by Q = 1. These modes are related to cosmic structure formation in both the RD and MD epochs. The value of  $Q = M^{1/2}$  corresponds to fluctuations evolving deep inside the horizon. Perturbations of this kind are most responsible for the galaxy formation. When Q = M, the equations describe perturbations of the Compton wavelength scale, which are deep below the Jeans length and thus are out of interest.

Substituting (6) into (5), we find for the leading term

$$\varphi_0(t, T, \mathbf{X}) = u(t) \left[ v(t, \mathbf{X}) \cos T + w(t, \mathbf{X}) \sin T \right].$$
(7)

This solution is valid for all considered scales and for arbitrary behaviors of the scale factor. However, the requirement not to produce secular terms in the solutions leads to different equations for the amplitudes from (7).

### 2. Applications to the post-inflationary epochs

Considering the spatial inhomogeneities well inside the Hubble scales in the MD stage, we obtain the well-known equation for the density contrast  $\delta = 2v$  [13]. For a plane wave perturbation with rescaled wavenumber  $K = Q^{-1}k$ , this equation reads as

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} + \left(\frac{K^4}{4a^4} - \frac{3}{4}u^2\right)\delta = 0.$$
(8)

Perturbations with large wavenumbers do not grow but oscillate. The dividing point between these behaviors is the Jeans wavenumber  $k_{\rm J}$  when the expression in the parentheses is zero. In dimensional terms, this corresponds to  $k_{\rm J} = (16\pi G \rho m^2 a^4)^{1/4}$ .

For  $K \to 0$ , Eq. (8) gives a growing solution  $\delta \propto a$ . However, contrary to what is argued in [13], the equation cannot be applied at all scales. To describe perturbations outside and near the horizon the asymptotic expansions with the rescaling factor Q = 1 have to be used.

Neglecting the decaying mode, we find for the density contrast

$$\delta = -2\phi_0 \left(1 + \frac{K^2 a}{3}\right). \tag{9}$$

where  $\phi_0$  is a time-independent value of the gravitational potential. The same relation takes place for the perfect fluid CDM [5]. On the superhorizon scales,  $K \to 0$ , the density contrast is constant in time, while fluctuations entering the horizon are governed by the second term and grow linearly with the scale factor.

Also, we find for the phase variation  $S = \phi_0 t$ , which implies that the oscillation frequency fluctuates according to the perturbations of the gravitational potential. The oscillations become incoherent outside the Hubble scale, however the background evolution of the Universe remains homogeneous since the scale factor is driven only by the amplitude u, which is independent of the fast time. The developed procedure can be applied to scalar field perturbations in the RD universe. However, the approximation  $m \gg H$  now fails for ultralight fields, but it remains valid for more massive (pseudo)scalars, such as QCD axions. The gravitational potential in this epoch is [5]

$$\Phi = -3 \frac{\phi_0(k)}{\chi^2} \left( \cos \chi - \frac{\sin \chi}{\chi} \right) \,, \tag{10}$$

where  $\chi = k\eta/\sqrt{3}$ , and  $\eta$  is the conformal time.

On the Hubble scales, the gravitational potential produces the energy density and the phase perturbations of the scalar field

$$\delta = \delta_0 + 9\phi_0 \left(\operatorname{Ci} \chi - \log \chi - \frac{\sin \chi}{\chi} - \frac{\cos \chi}{\chi^2} + \frac{\sin \chi}{\chi^3}\right), \qquad S = \frac{9\phi_0}{K^2} \left(1 - \frac{\sin \chi}{\chi}\right). \tag{11}$$

The long wavelength perturbations,  $K \to 0$ , behave similarly to those in the MD stage. The metric and the energy density perturbations are time-independent. The phase fluctuations are determined through the gravitational potential as  $S = \phi_0 t$ , implying that the scalar field becomes incoherent at the superhorizon scales.

For perturbations entering the horizon ( $\chi > 1$ ), the phase fluctuations become time-independent and vanish for large wavenumbers, while the density contrast increases logarithmically, providing an early growth of inhomogeneities in the same manner as for non-relativistic matter.

Deep inside the horizon, the solution for the density contrast is given by

$$\delta = 2v_0 \cos\left(\frac{K^2}{2}\log\chi\right) + 2w_0 \sin\left(\frac{K^2}{2}\log\chi\right) - \frac{9\phi_0}{K^2} \int_{\chi_0}^{\chi} j_1(s) \sin\left(\frac{K^2}{2}\log\frac{\chi}{s}\right) \mathrm{d}s.$$
(12)

The density perturbations driven by the gravitational potentials are described by the last term in (12). The corresponding solutions describe behaviors providing an effect similar to the Jeans instability in the MD universe.

When  $K \to 0$ , the integral gives

$$\delta = 9\phi_0 \left( \operatorname{Ci}\chi - \operatorname{Ci}\chi_0 + \left(1 - \log\frac{\chi}{\chi_0}\right) \frac{\sin\chi_0}{\chi_0} - \frac{\sin\chi}{\chi} \right) \,. \tag{13}$$

This solution describes the logarithmic growth of the long wavelength subhorizon perturbations and relates (12) to the perturbations at the Hubble scales represented in (11).

When K increases, the scalar field energy density perturbations stop growing and develop into an oscillating regime. However, contrary to the ordinary Jeans effect, there is no definite value of K dividing growing and non-growing modes. Nevertheless, it follows from these solutions that there is an effective short wavelength cutoff for the growing perturbations in the RD epoch. Scaling the cutoff length to the characteristic size of dwarf galaxies shows that their production is reduced for the scalar field DM with  $m \sim 10^{-13}$  eV.

Considering of the scalar field perturbations in the  $\Lambda D$  stage, we find that the energy density perturbations do not grow on all relevant scales.

## Conclusions

As a result, it is clear that there are more overlaps between the oscillating scalar fields and the CDM than discussed before [13–15]. The inhomogeneities of the scalar fields well outside the Hubble radius remain constant in time in both the RD and MD stages while the inhomogeneities increase entering the horizon scale. There is a logarithmic growth in the RD stage and a linear one in the MD stage. The inhomogeneities stop growing in the  $\Lambda D$  stage. These results exactly reproduce the standard CDM scenario.

and these variations result in decoherence of the oscillations outside the Hubble scale.

A valuable virtue of the scalar field as a DM model is that it provides a density cusp avoidance in the halo, as well as a natural mechanism to reduce the abundance of small halos for the particular field masses. The latter property was a motivation to study ultralight bosons with  $m \sim 10^{-22}$  eV as DM candidates. A similar opportunity is also revealed for light scalars with  $m \sim 10^{-13}$  eV.

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