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АНИЗОТРОПНЫЕ КОСМОЛОГИЧЕСКИЕ МОДЕЛИ ТИПА БИАНКИ I, V, IX В ТЕОРИИ ГРАВИТАЦИИ ХОРНДЕСКИ*

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В данной работе мы анализируем поведение анизотропии в космологических моделях типа Бианки I, V, IX в теории гравитации Хорндески. Выводя исходные уравнения из действия Хорндески, мы получаем обобщенную систему уравнений. Далее мы численно анализируем решения различных скалярно-тензорных теорий гравитации в рамках теории Хорндески, рассматривая анизотропию на ранней и поздней стадиях эволюции Вселенной.

Ключевые слова: модифицированные теории гравитации; скалярно-тензорные теории гравитации; инфляция.

ANISOTROPIC COSMOLOGICAL MODELS OF BIANCHI TYPE I, V, IX IN HORNDESKI GRAVITY THEORY

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In this work we analyze behavior of anisotropy in cosmological models of Bianchi type I, V, IX in Horndeski gravity theory. Deriving background equations from Horndeski action, we obtain generalized system. Further, we numerically analyze solutions of diverse scalar-tensor gravity theories within Horndeski theory framework, considering the anisotropy on early and late stage of universe evolution.

Keywords: modified gravity theories; scalar-tensor gravity theories; inflation.

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Introduction

Due to rapid development of observational cosmology in recent decades some astronomical observations tell us [1], [2], [3], [4], [5], that general relativity needs some modification. One of the ways to do that is to include additional degree of freedom. In that case scalar field can play a good role in modification of general relativity describing laws of gravity and providing interesting results. Recently there are enough scalar-tensor theories that describe the universe evolution [6]. However there exists a generalized one called Horndeski theory [7], it proposes ghost-free and not higher of second order equations of motions. In this work we consider three cosmological theories within Horndeski theory of gravity: Kinetic Gravity Braiding [8], Non-minimal Kinetic Coupling [9], [10], [3] and theory of $G_5 \neq 0$ [12].

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It is also known, that homogeneous isotropic Friedmann universe includes three models with different values of curvature $\kappa = 0, -1, +1$, flat, open and closed correspondingly [13]. Recent observations show us that Cosmic Microwave Background is anisotropic, so we can assume that on early stage of universe evolution it was anisotropic. Luigi Bianchi proposed classification of three dimensional spaces [14], and space-times of Bianchi type I, V, IX in isotropic case correspond to flat, open and closed Friedmann universes. We will focus on these three Bianchi types models. So that we can analyze the behavior of anisotropy on early and late stages.

1. Horndeski theory of gravity

Horndeski theory is the most generalized scalar-tensor theory of gravity, and its action is following:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right), \tag{1}$$

where Horndeski Lagrangian is a sum of four terms $\mathcal{L}_i$ and yield:

$$\begin{aligned}
\mathcal{L}_2 &= G_2(\phi, X)\,,\\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi\,,\\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\\
\mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}\,\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\times\\
&\quad\times\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2\left(\nabla_\mu\nabla_\nu\phi\right)^3\right],
\end{aligned} \tag{2}$$

here $G_{\alpha\phi} \equiv \partial G_\alpha/\partial\phi$ and $G_{\alpha X} \equiv \partial G_\alpha X/\partial X$, $X = -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ is kinetic invariant, $\Box\phi = \nabla^\mu\nabla_\mu\phi$ is d'Alambertian of scalar field, and we used here following notation $(\nabla_\mu\nabla_\nu\phi)^2 = \nabla_\mu\nabla_\nu\phi\nabla^\nu\nabla^\mu\phi$ и $(\nabla_\mu\nabla_\nu\phi)^3 = \nabla_\mu\nabla_\nu\phi\nabla^\nu\nabla^\rho\phi\nabla_\rho\nabla^\mu\phi$. Finally R and $G_{\mu\nu}$ are Ricci scalar and Einstein tensor correspondingly. So, choosing auxiliary functions $G_\alpha(\phi, X), \alpha = \{2, 3, 4, 5\}$ we can get various specific scalar-tensor theories.

2. Bianchi classification

Further to set our framework, we will use Bianchi classification [14] of 3-dimensional spaces, that provide us with an anisotropic metric in following form:

$$ds^2 = -dt^2 + a_1^2(t)\omega^1\otimes\omega^1 + a_2^2(t)\omega^2\otimes\omega^2 + a_3^2(t)\omega^3\otimes\omega^3, \tag{3}$$

where $a_i(t)$ are scale factors and ω^i are 1-forms with $i = \{1, 2, 3\}$. In order to get specific type of Bianchi space it is necessary to know the commutation relation for 1-forms. In this work, we consider only Bianchi types I, V and IX, these models defined by following commutation relations:

$$\begin{aligned}
\text{Bianchi I:}&\qquad d\omega^i = 0,\\
\text{Bianchi V:}&\qquad d\omega^i = \omega^i\wedge\omega^1,\\
\text{Bianchi IX:}&\qquad d\omega^i = -2\varepsilon^{ijk}\omega^j\wedge\omega^k,
\end{aligned} \tag{4}$$

where $\wedge$ is a wedge product, and ε^{ijk} is Levi-Civita symbol. So, with these commutations relations it easy to get background equations varying action.

K^T_μ	K^I_μ	K^V_μ	K^{IX}_μ
K^T_0	0	$-\frac{1}{a_1^2}$	$\frac{1}{3}\left[\frac{2}{a_1^2}+\frac{2}{a_2^2}+\frac{2}{a_2^3}-\frac{a_1^2}{a_2^2a_3^3}-\frac{a_2^2}{a_1^2a_3^2}-\frac{a_2^2}{a_1^2a_3^2}\right]$
K^T_i	0	$-\frac{1}{a_1^2}$	$-\frac{3a_i^2}{a_j^2a_k^2}+\frac{a_j^2}{a_i^2a_k^2}+\frac{a_k^2}{a_i^2a_j^2}-\frac{2}{a_i^2}+\frac{2}{a_j^2}+\frac{2}{a_k^2}$

Table 1. Curvature values in components of Einstein tensor and background equations in Bianchi type models I, V, IX, where T defines the type of the model. For Bianchi IX model $\{i,j,k\}$ get following values $\{1,2,3\},\{2,3,1\},\{3,1,2\}$.

3. Anistoropic cosmological models

So let us vary this action (1) with respect to metrics $g_{\mu\nu}$, then we get following modified background equations for gravitational field. The first one is tt-component equation:

$$\begin{aligned}
&G^0_0(\mathcal{G}-2G_{4X}\dot\phi^2-2G_{4XX}\dot\phi^4+2G_{5\phi}\dot\phi^2+G_{5X\phi}\dot\phi^4)\\
=\;& G_2-G_{2X}\dot\phi^2+G_{3\phi}\dot\phi^2-3HG_{3X}\dot\phi^3+6H\dot\phi\left(G_{4\phi}+G_{4X\phi}\dot\phi^2\right)-H_1H_2H_3\dot\phi^3\left(5G_{5X}+G_{5XX}\dot\phi^2\right)\\
-\;& G_{5X}\dot\phi^3\left(H_1K^T_1+H_2K^T_2+H_3K^T_3\right)+3K^T_0\left(2G_{4X}\dot\phi^2+2G_{4XX}\dot\phi^4-2G_{5\phi}\dot\phi^2-G_{5X\phi}\dot\phi^4\right),
\end{aligned}\tag{5}$$

and a linear combination of space part equations give us following:

$$\begin{aligned}
&\mathcal{G}(G^i_i+K^T_i)-(H_j+H_k)\frac{d\mathcal{G}}{dt}-K^T_i\mathcal{R}=G_2-\dot\phi\frac{dG_3}{dt}\\
+\;& 2\frac{d}{dt}(G_{4\phi}\dot\phi)-\frac{d}{dt}(G_{5X}\dot\phi^3H_jH_k)-G_{5X}\dot\phi^3H_jH_k(H_j+H_k),
\end{aligned}\tag{6}$$

where values $\{i,j,k\}=\{1,2,3\},\{2,3,1\},\{3,1,2\}$. Here we also denote values K^T_μ that vary with respect to Bianchi type: Then, the Einstein tensor components are following

$$\begin{aligned}
G^0_0 &= -\left(H_1H_2+H_2H_3+H_3H_1+3K^T_0\right),\\
G^i_i &= -\left(H_j^2+H_k^2+\dot H_j+\dot H_k+H_jH_k+K^T_i\right),
\end{aligned}\tag{7}$$

where $H_1=\dot a_1/a_1, H_2=\dot a_2/a_2, H_3=\dot a_3/a_3$ are Hubble parameters with respect to scale factor a_1,a_2,a_3. And here we also used following notation

$$\mathcal{G}=2G_4-2G_{4X}\dot\phi^2+G_{5\phi}\dot\phi^2,\qquad \mathcal{R}=2G_4-G_{5\phi}\dot\phi^2-G_{5X}\ddot\phi\dot\phi^2.\tag{8}$$

Further, varying action (1) we get background equation for scalar field. We rewrite it in compact form

$$\frac{1}{\mathrm{a}^3}\frac{d}{dt}\left(\mathrm{a}^3\mathcal{J}\right)=\mathcal{P},\tag{9}$$

Values of $\mathcal{J}$ and $\mathcal{P}$ yield

$$\begin{aligned}
\mathcal{J} &= \dot\phi\Big(G_{2X}-2G_{3\phi}+3H\dot\phi(G_{3X}-2G_{4X\phi})+G^0_0(-2G_{4X}-2G_{4XX}\dot\phi^2+2G_{5\phi}+G_{5\phi X}\dot\phi^2) \qquad (10)\\
&+ H_1H_2H_3\dot\phi(3G_{5X}+G_{5XX}\dot\phi^2)+G_{5X}\dot\phi(H_1K^T_1+H_2K^T_2+H_3K^T_3)+3K^T_0\dot\phi^2(-2G_{4XX}+G_{5\phi X})\Big),\\
\mathcal{P} &= G_{2\phi}-\dot\phi^2(G_{3\phi\phi}+G_{3\phi X}\ddot\phi)+G_{4\phi}R+2G_{4\phi X}(3H\ddot\phi\dot\phi-\dot\phi^2G^0_0)+G_{5\phi\phi}G^0_0\dot\phi^2\\
&+ G_{5\phi X}\dot\phi^3H_1H_2H_3-3K^T_0(2G_{4\phi X}\dot\phi^2+G_{5\phi X}\ddot\phi\dot\phi^2). \qquad (11)
\end{aligned}$$

In this work we also use following parametrization in order to separate isotropic and anisotropic parts:

$$a_1=\mathrm{a}e^{\beta_++\sqrt{3}\beta_-},\quad a_2=\mathrm{a}e^{\beta_+-\sqrt{3}\beta_-},\quad a_3=\mathrm{a}e^{-2\beta_+},\tag{12}$$

$\mathcal{K}$	Linear combinations	I	V	IX
$\mathcal{K}^T$	K_0^T	0	$\frac{K^V}{a^2}$	$\frac{\mathcal{K}^{IX}}{a^2}$
$\mathcal{K}_+^T$	$K_1^T + K_2^T - 2K_3^T$	0	0	$\frac{3}{a^2}\frac{\partial \mathcal{K}^{IX}}{\partial \beta_+}$
$\mathcal{K}_-^T$	$K_1^T - K_2^T$	0	0	$\frac{\sqrt{3}}{a^2}\frac{\partial \mathcal{K}^{IX}}{\partial \beta_-}$

Table 2. Linear combinations of curvature values for Bianchi type models I, V, IX, where T stand to Bianchi type.

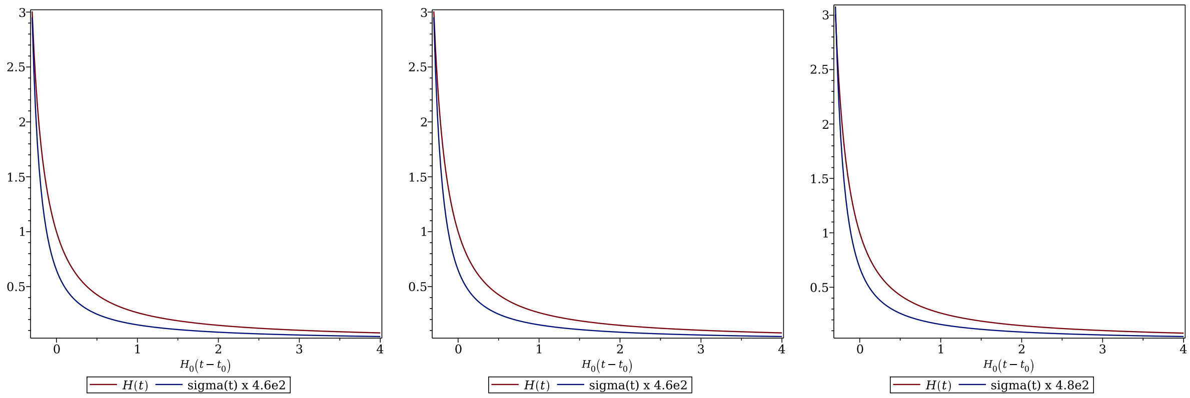

Fig. 1. Left panel: Bianchi type I model. Middle panel: Bianchi type V model. Right panel: Bianchi type IX model. Behaviour of Hubble parameter (red line) and anisotropy (blue line) with respect to cosmic time.

where $\mathrm{a} = \mathrm{a}(t)$ is a mean scale factor and $\beta_\pm = \beta_\pm(t)$ are anisotropy parameters. According to this framework we get background equations with respect to mean Hubble parameter $3\dot{\mathrm{a}}/\mathrm{a} \equiv 3H = H_1 + H_2 + H_3$, mean scale factor a and anisotropy parameters $\beta_\pm$. Here we also introduce new notations for $\mathcal{K}^V$ and $\mathcal{K}^{IX}$:

$$\mathcal{K}^V = -e^{-2\beta_+ - 2\sqrt{3}\beta_-}, \tag{13}$$

$$\mathcal{K}^{IX} = -\frac{1}{3}e^{-8\beta_+}\left(4e^{6\beta_+}\cosh^2(\sqrt{3}\beta_-) - 1\right)\left(4e^{6\beta_+}\sinh^2(\sqrt{3}\beta_-) - 1\right). \tag{14}$$

3.1. Kinetic Gravity Braiding theory

Theory of Kinetic Gravity Braiding can be obtained choosing following auxiliary functions in the form:

$$G_2 = K(\phi, X), \quad G_3 = G(\phi, X), \quad G_4 = \frac{1}{16\pi}, \quad G_5 = 0. \tag{15}$$

We substitute these functions into generalized background equations (5), (6) and using parametrization (12) we obtain equations:

$$3(H^2 - \sigma^2 + \mathcal{K}^T) = 8\pi\left[-K + K'_X\dot{\phi}^2 - G'_\phi\dot{\phi}^2 + 3HG'_X\dot{\phi}^3\right], \tag{16}$$

$$2\dot{H} + 3H^2 + 3\sigma^2 + \mathcal{K}^T = 8\pi\left[-K + \dot{\phi}\dot{G}\right], \tag{17}$$

$$\frac{1}{\mathrm{a}^3}\frac{d}{dt}\left(\mathrm{a}^3\dot{\beta}_+\right) = \frac{1}{6}\mathcal{K}_+^T, \tag{18}$$

$$\frac{1}{\mathrm{a}^3}\frac{d}{dt}\left(\mathrm{a}^3\dot{\beta}_-\right) = \frac{\sqrt{3}}{6}\mathcal{K}_-^T, \tag{19}$$

and background equation for scalar field:

$$\frac{1}{\mathrm{a}^3}\frac{d}{dt}\left[\mathrm{a}^3\dot{\phi}\left(K'_X - 2G'_\phi + 3H\dot{\phi}G'_X\right)\right] = K'_\phi - \dot{\phi}^2(G''_{\phi\phi} + G''_{\phi X}\ddot{\phi}), \tag{20}$$

where $f'_X = df/dX, f'_\phi = df/d\phi$, a $K = K(\phi, X), G = G(\phi, X)$. The first one equation (16) can give us initial condition for scalar field ϕ. Also for Bianchi I and V, we have $\mathcal{K}^T_\pm = 0$, then the equations for anisotropy parameters (18), (19) can be easily integrated, so they have the following behavior:

$$\dot{\beta}_\pm = \frac{C_\pm}{\mathrm{a}^3}, \tag{21}$$

where $C_\pm$ are constants of integration. Unfortunately for Bianchi IX there are no first integrals, so we analyze the behavior of anisotropy parameters numerically. However there are still freedom in choice of functions $K(\phi, X)$ and $G(\phi, X)$, the most interesting models that are being studied nowadays with $K(\phi, X) = c_K X$ and $G(\phi, X) = c_G X$. Therefore, on plots (1) represented the behavior of Hubble parameter and anisotropy $\sigma^2 = \dot{\beta}_+^2 + \dot{\beta}_-^2$ from cosmic time $H_0(t - t_0)$ for model $K(\phi, X) = c_K X$, $G(\phi, X) = c_G X$.

3.2. $G_5 = \xi\sqrt{2X}$ theory

This model can be obtained with the following choice of auxiliary functions $G_\alpha(\phi, X)$:

$$G_2 = X, \quad G_3 = 0, \quad G_4 = \frac{1}{16\pi}, \quad G_5 = \xi\sqrt{2X}, \tag{22}$$

this model was firstly analyzed in our work [12], and it gave interesting results, however in this work numerical analysis shows us, that behavior introduced in previous work cannot be repeated. With the choice of these auxiliary functions (22) yield following equations for mean Hubble parameter H:

$$\begin{aligned} 3\left(H^2 - \sigma^2 + \mathcal{K}^T\right) = 4\pi\dot{\phi}^2 &+ 32\pi\xi\dot{\phi}^2(H - 2\dot{\beta}_+)\left[(H + \dot{\beta}_+)^2 - 3\dot{\beta}_-^2\right] \\ &+ 8\pi\xi\dot{\phi}^2\left[3H\mathcal{K}^T + \dot{\beta}_+\mathcal{K}^T_+ + \sqrt{3}\dot{\beta}_-\mathcal{K}^T_-\right], \qquad (23) \\ 2\dot{H} + 3H^2 + 3\sigma^2 + (1 - 8\pi\xi\ddot{\phi}\dot{\phi})\mathcal{K}^T &= -4\pi\dot{\phi}^2 + 8\pi\frac{d}{dt}\left[\xi\dot{\phi}^2(H^2 - \sigma^2)\right] \\ &+ 16\pi\xi\dot{\phi}^2(H^3 + \dot{\beta}_+^3 - \dot{\beta}_+\dot{\beta}_-^2), \qquad (24) \end{aligned}$$

and equation for anisotropy parameters $\beta_\pm$:

$$\begin{aligned} \frac{1}{\mathrm{a}^3}\frac{d}{dt}\left[\mathrm{a}^3\dot{\beta}_+ + 8\pi\xi\mathrm{a}^3\dot{\phi}^2(\dot{\beta}_-^2 - \dot{\beta}_+^2 - H\dot{\beta}_+)\right] &= \frac{1}{6}\left(1 - 8\pi\xi\ddot{\phi}\dot{\phi}\right)\mathcal{K}^T_+, \qquad (25) \\ \frac{1}{\mathrm{a}^3}\frac{d}{dt}\left[\mathrm{a}^3\dot{\beta}_- + 8\pi\xi\mathrm{a}^3\dot{\phi}^2(2\dot{\beta}_+\dot{\beta}_- - H\dot{\beta}_-)\right] &= \frac{\sqrt{3}}{6}\left(1 - 8\pi\xi\ddot{\phi}\dot{\phi}\right)\mathcal{K}^T_-, \qquad (26) \end{aligned}$$

and in the same manner we can get background equation for scalar field:

$$\frac{d}{dt}\left[\mathrm{a}^3\dot{\phi}\left(1 - 2\xi(H - 2\dot{\beta}_+)\left[(H + \dot{\beta}_+)^2 - 3\dot{\beta}_-^2\right] + \xi\left(3H\mathcal{K}^T + \dot{\beta}_+\mathcal{K}^T_+ + \sqrt{3}\dot{\beta}_-\mathcal{K}^T_-\right)\right)\right] = 0. \tag{27}$$

Due to complexity of these equations, we use numerical methods to show solutions for mean Hubble parameter H and anisotropy σ from cosmic time $H_0(t - t_0)$ on the plots (2).

3.3. Non-minimal derivative coupling theory

Theory of gravity with non-minimal derivative coupling is defined by following auxiliary functions $G_\alpha(\phi, X)$:

$$G_2 = X, \quad G_3 = 0, \quad G_4 = \frac{1}{16\pi}, \quad G_5 = \frac{1}{2}\eta\phi, \tag{28}$$

isotropic case of this model, when $\eta = 0$, is analyzed in all details in our work [16], here we analyze the behavior of anisotropy on early and late stage of universe evolution. With auxiliary functions (28) we

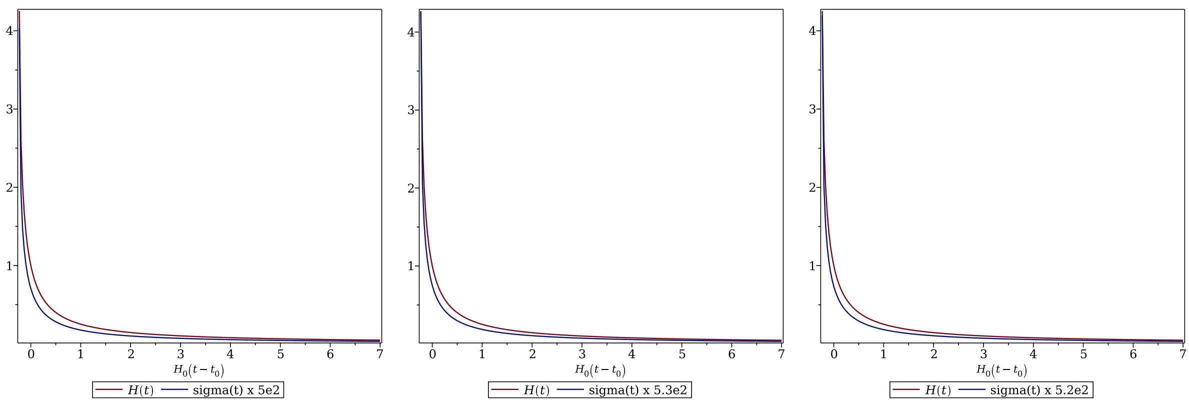

Fig. 2. Left panel: Bianchi type I model. Middle panel: Bianchi type V model. Right panel: Bianchi type IX model. Behaviour of Hubble parameter (red line) and anisotropy (blue line) with respect to cosmic time.

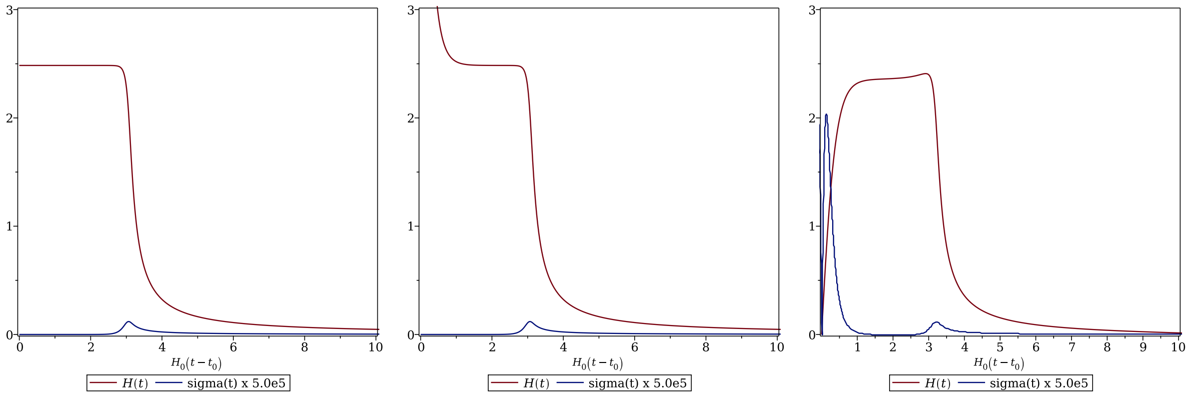

Fig. 3. Left panel: Bianchi type I model. Middle panel: Bianchi type V model. Right panel: Bianchi type IX model. Behaviour of Hubble parameter (blue line) and anisotropy (orange line) with respect to cosmic time.

get following background equations for gravitational field:

$$
\begin{aligned}
3\left(H^2-\sigma^2+\mathcal{K}^T\right)+12\pi\eta\dot\phi^2\left(3H^2-3\sigma^2+\mathcal{K}^T\right) &= 4\pi\dot\phi^2, && (29)\\
\left(1+4\pi\eta\dot\phi^2\right)(2\dot H+3H^2+3\sigma^2)+2H\eta\dot\phi\ddot\phi+(1-4\pi\eta\dot\phi^2)\mathcal{K}^T &= -4\pi\dot\phi^2, && (30)\\
\frac{1}{\mathrm{a}^3}\frac{d}{dt}\left[\mathrm{a}^3\dot\beta_\pm\left(1+4\pi\eta\dot\phi^2\right)\right] &= \frac{1}{6}\left(1-4\pi\eta\dot\phi^2\right)\mathcal{K}^T_+, && (31)\\
\frac{1}{\mathrm{a}^3}\frac{d}{dt}\left[\mathrm{a}^3\dot\beta_\pm\left(1+4\pi\eta\dot\phi^2\right)\right] &= \frac{\sqrt{3}}{6}\left(1-4\pi\eta\dot\phi^2\right)\mathcal{K}^T_-, && (32)
\end{aligned}
$$

and background equation for scalar field:

$$
\frac{d}{dt}\left[\mathrm{a}^3\dot\phi\left(1-3\eta(H^2-\sigma^2+\mathcal{K}^T)\right)\right]=0. \tag{33}
$$

As in theory of Kinetic Gravity Braiding for model of Bianchi type I and V, equations for anisotropy parameters $\dot\beta_\pm$ (31), (32) have zero right-hand side, thus the behavior of anisotropy parameters $\dot\beta_\pm$ are the following:

$$
\dot\beta_\pm=\frac{C_\pm}{\mathrm{a}^3(1+4\pi\eta\dot\phi^2)}. \tag{34}
$$

However for the model of Bianchi type IX right-hand side of anisotropy parameters $\dot\beta_\pm$ is a bit complex, thus we use numerical methods to show the behavior of mean Hubble parameter H and anisotropy σ.

Conclusion

We analyzed diverse anisotropic cosmological models and built numeric solutions for different theories. In the gravity theories of Kinetic Gravity Braiding and $G_{5X} = \sqrt{2X}$ as it is shown on plots the anisotropy function σ increases with a $\to 0$, while on the other side with a $\to \infty$ anisotropy function goes to zero, $\sigma \to 0$. It is important to mention here, that in Bianchi type IX models anisotropy behaves as periodic one, however it is being damped on early and late stages of universe evolution. In the theory of Non-minimal Derivative Coupling anisotropy function behaves in a different way. On early and late stages we can see that anisotropy function σ goes to zero, $\sigma \to 0$. And we can see that in the theory of Non-minimal Derivative Coupling we obtain three scenarios of universe evolution on early stage: for Bianchi type I eternal inflation, for Bianchi type V primordial singularity, for Bianchi type IX bounce.

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