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BLACK HOLE IMAGE IN HEISENBERG-EULER ELECTRODYNAMICS*

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In this paper we use a geodesic method for constructing a black hole shadow. It's based on numerical integration of geodesic equations by the 5th order Dormand-Prince method. We construct an image of charged black hole shadow in the framework of Heisenberg-Euler electrodynamics and perform a comparative analysis of this image with a similar image in the classical case of Maxwell electrodynamics.

Keywords: Black hole image, Nonlinear electrodynamics, Gravitational lensing.

ИЗОБРАЖЕНИЯ ЧЕРНОЙ ДЫРЫ В ЭЛЕКТРОДИНАМИКЕ ГЕЙЗЕНБЕРГА-ЭЙЛЕРА

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В данной работе мы используем метод геодезических для построения изображения черной дыры. Он основан на численном интегрировании уравнений геодезических методом Дорманда-Принца 5-го порядка. В статье построено изображение заряженной черной дыры в рамках электродинамической модели Эйлера -Гейзенберга а также произведен сравнительный анализ этого изображения со подобным изображением в классическом максвелловском случае.

Ключевые слова: Изображение черной дыры, Нелинейная электродинамика, Гравитационное линзирование.

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Introduction

Nowadays, nonlinear electrodynamics is very useful in the different areas of theoretical physics: astrophysics and cosmology [1], quantum electrodynamics [2], condensed matter physics [3] and others. The most famous nonlinear electrodynamics models are Born-Infield [4], with based on symmetry of Lagrangian, and Euler-Heisenberg [5], witch based on one-loop QED corrections.

Also, the images of black holes [6] and other compact objects [7] are one of the most popular way of modern theoretical and observational investigations. Moreover, a nonlinear electrodynamics can produce a «fingerprints» in black hole shadows [8].

So, we have two goals: on the one hand, we are testing a new peculiarities in standard method of constructing of black hole shadow; on other hand, we are trying to find additional astrophysical manifestations of nonlinear electrodynamics. We use the «geometric» system with $k = c = 1$. Latin letters run from 0 to 3.

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1. The model

In this paper we consider static spherical symmetric model with standard ansatz about energy-momentum tensor: $T_0^0 = T_1^1$. This model is described by a following metric:

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,. \tag{1}$$

According to Einstein equations, metric function $f(r)$ is given by:

$$f(r) = 1 - \frac{2GM}{r} + \frac{8\pi G}{r}\int r^2 T_0^0(r)dr\,. \tag{2}$$

In the simplest case Maxwell Maxwell electrodynamics $f(r)$ reduce to Reisner-Nordstrom metric:

$$f(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}\,. \tag{3}$$

Here M, Q - mass and electric charge of black hole, respectively.

Let's consider a most famous nonlinear model - Heisenberg-Euler electrodynamics with effective Lagrangian:

$$L_{EH} = -\frac{1}{4}F_{ik}F^{ik} + \frac{\pi^2\alpha}{45E_0^2}((F_{ik}F^{ik})^2 + 7(F_{ik}F^{*ik})^2)\,, \tag{4}$$

where α - fine-structure constant, $F^*_{ik} = \frac{1}{2}E_{iklm}F^{lm}$ - Hodge dual of Maxwell tensor F_{ik}, E_0 -critical value of electric field. Using (2) we cam find corresponding metric function with first nonzero modification term:

$$f(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2} + \frac{7GhQ^4}{6r^6}\,, \quad h = \frac{\pi^2\alpha}{45E_0^2}\,. \tag{5}$$

2. Geodesics equations integration

Our approach is based on the of reversibility of light rays. If we know the structure of geodesic lines for photons near black holes, we may «reverse» the course of photons arriving at the observer, and find out the light intensity and red shift by calculating these quantities along geodesics in accordance with any accretion models of interest. We use accretion model $I \propto \frac{1}{r^3}$, where I - intensity of accretion [9].

The set of geodesic equations in standard form

$$\frac{d^2x^i}{ds^2} = \Gamma^i_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds} \tag{6}$$

is very easy transform to two subsets:

$$\frac{dx^i}{ds} = y^i\,, \qquad \frac{dx^i}{ds} = \Gamma^i_{kl}\frac{dy^k}{ds}\frac{dy^l}{ds}\,. \tag{7}$$

To integrate equations 7 we use the 5th order Dormand-Prince method [10] As initial conditions we choose a uniform mesh of photons which flying from a distant observers to black hole by trajectories, parallel to the observation axis, and having different impact parameters.

To execute the calculation program, we use Python in combined with a well known in machine learning libraries «pytorch» and «torchode». Using of this tools made possible to assemble a flexible structure program with parallelization and transfer functionality computing on graphics accelerators.

As a result, the program processes about 10^3 trajectories per minute on the nVidia Tesla T4 accelerator, available for free on Google Colaboratory [11], which is significantly faster than methods without using parallelization of calculations. This allows to build images in good resolution in a reasonable time and test different metric functions. The initial conditions grid is chosen in the form evenly filled circle of 12 thousand points, maximum impact parameter - 8 in mass units. This program can be downloaded from following link: [12].

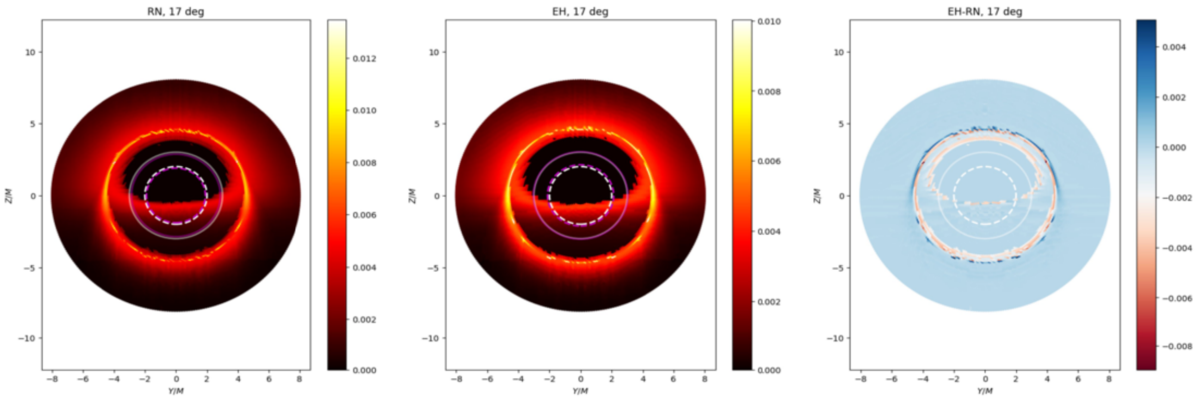

Fig. 1. Comparison between image of Reissner-Nordstrom (left panel) and Heisenberg - Euler (central panel) black holes at viewing angle of 17°. Scale demonstrate an integration intensity of light.

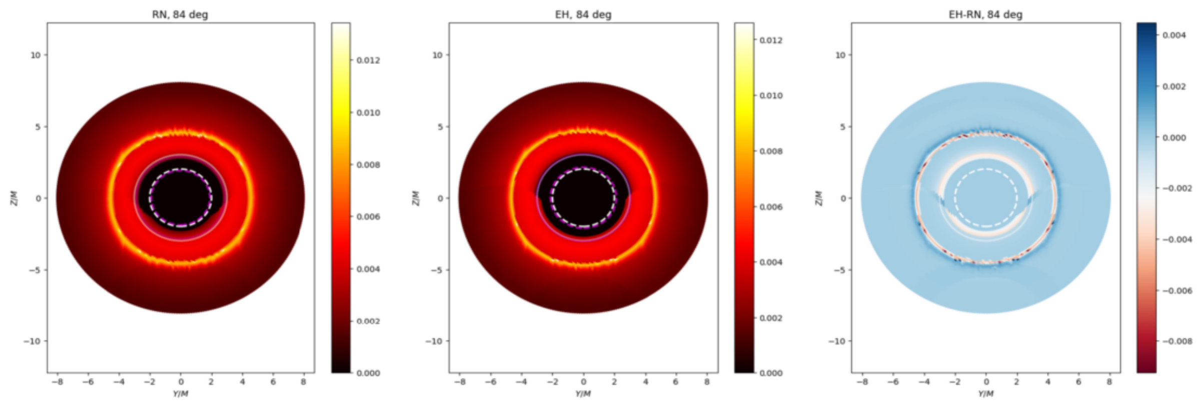

Fig. 2. Comparison between image of Reissner-Nordstrom (left panel) and Heisenberg - Euler (central panel) black holes at viewing angle of 84°. Scale demonstrate an integration intensity of light.

We are simulate images of a Reissner-Nordstrom black hole with a non-physically large charge ($C = 0.45$ in geometric units) at viewing angles of 17° and 84° degrees to the plane of the accretion disk. The angles are chosen as follows: for the purpose of further comparison of results with images compact objects in the center of the galaxy M87 and SgrA*, which obtained at the same viewing angles. Incredible underestimation of the critical field in the Euler-Heisenberg model justified by the desire to test the fundamental possibility detect differences in images.

In 1 and 2 we build a black hole images in Heisenberg - Euler electrodynamics and show the difference between that images and images in Reissner-Nordstrom electrodynamics. Obviously, that this differences are minimal and hardly observable.

Conclusion

In this work we construct a black hole image by geodesic method. We build a black hole images of Reissner-Nordstrom and Heisenberg - Euler models. Essentially, in principle it is assumed the extremely large monopole electric charge in black hole. Differences between images, which obtain in this method, are extremely small. We hope that modernization of geodesic method, based on Hamilton - Jacobi equation, will build more explicit images.

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