UDC 53.1

INTERACTING SPINOR AND ELECTROMAGNETIC FIELDS IN COSMOLOGY

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Within the scope of a Bianchi type-I (BI) cosmological model we study the interacting system of spinor and electromagnetic fields. The corresponding system of Maxwell, Dirac and Einstein equations are obtained and solved. It was found that under such generalization nonlinear and massive spinor field can exist in a general BI space-time and give rise to different type of solutions depending on the choice of nonlinear and interacting terms.

Keywords: spinor field, BI cosmology, electromagnetic field, energy-momentum tensor.

ВЗАИМОДЕЙСТВУЮЩИЕ СПИНОРНОЕ И ЭЛЕКТРОМАГНИТНОЕ ПОЛЯ В КОСМОЛОГИИ

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В рамках космологической модели Бианки типа I (BI) изучается взаимодействующая система спинорных и электромагнитных полей. Получены и решены соответствующие системы уравнений Максвелла, Дирака и Эйнштейна. Было обнаружено, что при таком обобщении нелинейное и массивное спинорное поле может существовать в общем пространстве-времени типа BI и приводить к различным типам решений в зависимости от выбора нелинейных и взаимодействующих членов.

Ключевые слова: спинорное поле, космологическая модель типа Бианки I, электромагнитное поле, тензор энергии-импульса.

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Introduction

In some earlier studies it was found that due to the presence of non-diagonal components of energy-momentum tensor (EMT) the nonlinear spinor field in BI space-time automatically becomes massless and linear or leads to the isotropization of space-time [1], [2]. Whereas, in case pure electromagnetic field with induced nonlinearity there occurs severe restrictions on the components of vector potential. It was found that if the electromagnetic field is given by the 4-vector potential $\mathcal{A}_\mu = (0, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3)$, the corresponding non-diagonal components of EMT leads to $\mathcal{A}_1 = \mathcal{A}_2 = \mathcal{A}_3$ and space-time becomes isotropic [3]. The motivation for considering the interacting system was to see whether such generalization can remove the restrictions mentioned above.

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1. Basic Equations and their solutions

The interacting system of material fields we choose in the form

$$L = \frac{\imath}{2}\left[\bar\psi\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar\psi\gamma^\mu\psi\right] - m\bar\psi\psi - \lambda_1 Y(K) - \frac{1}{16\pi}F_{\tau\eta}F^{\tau\eta}X(K), \tag{1}$$

with $X(K) \equiv 1 + \lambda_2 Z(K)$. Here λ_1 is the self coupling constant, λ_2 is the coupling constant. In case of $\lambda_2 = 0$ we have spinor and electromagnetic fields those are minimally coupled to each other. Here $Y(K)$ and $Z(K)$ are some arbitrary functions of invariant K generated from the real bilinear forms of the spinor field, such that $K = \{I,\, J,\, I+J,\, I-J\}$, $I = S^2 = (\bar\psi\psi)^2$, $J = P^2 = (\imath\bar\psi\bar\gamma^5\psi)^2$. It can be shown that $K = C^2/V^2$, where $C = const$. This relation is true for $K = \{I,\, J,\, I+J,\, I-J\}$ for a massless spinor field, whereas for $K = I$ it is valid for massive spinor field as well.

The gravitational field is given by the metric

$$ds^2 = dt^2 - a_1^2 {dx_1}^2 - a_2^2 {dx_2}^2 - a_3^2 {dx_3}^2, \tag{2}$$

where the metric functions a_i are the functions of t only. We also consider the case when the spinor and electromagnetic fields depend on time only, i.e., $\psi = \psi(t)$, $\bar\psi = \bar\psi(t)$ and $\mathcal{A}_i = \mathcal{A}_i(t)$, $\quad i = 1, 2, 3$. On account of the non-diagonal components of the EMT in this case we find the following system of Einstein equations:

$$\frac{\ddot a_2}{a_2} + \frac{\ddot a_3}{a_3} + \frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3} = \kappa\left[P_1 + Q\left[q_1^2a_1^2 - q_2^2a_2^2 - q_3^2a_3^2\right] + \frac{4\lambda_2 K Z_K}{X}Q\bar Q\right], \tag{3}$$

$$\frac{\ddot a_3}{a_3} + \frac{\ddot a_1}{a_1} + \frac{\dot a_3}{a_3}\frac{\dot a_1}{a_1} = \kappa\left[P_1 + Q\left[q_2^2a_2^2 - q_3^2a_3^2 - q_1^2a_1^2\right] + \frac{4\lambda_2 K Z_K}{X}Q\bar Q\right], \tag{4}$$

$$\frac{\ddot a_1}{a_1} + \frac{\ddot a_2}{a_2} + \frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2} = \kappa\left[P_1 + Q\left[q_3^2a_3^2 - q_1^2a_1^2 - q_2^2a_2^2\right] + \frac{4\lambda_2 K Z_K}{X}Q\bar Q\right], \tag{5}$$

$$\frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2} + \frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3} + \frac{\dot a_3}{a_3}\frac{\dot a_1}{a_1} = \kappa\left[P_2 + Q\bar Q\right], \tag{6}$$

$$0 = \frac{a_2}{4a_1}\left(\frac{\dot a_1}{a_1} - \frac{\dot a_2}{a_2}\right)A^3 + 2q_1q_2a_2Q, \tag{7}$$

$$0 = \frac{a_1}{4a_3}\left(\frac{\dot a_3}{a_3} - \frac{\dot a_1}{a_1}\right)A^2 + 2q_3q_1a_1Q, \tag{8}$$

$$0 = \frac{a_3}{4a_2}\left(\frac{\dot a_2}{a_2} - \frac{\dot a_3}{a_3}\right)A^1 + 2q_2q_3a_3Q. \tag{9}$$

As one sees, the foregoing system is overridden. But one can use a few of them as constraints to define, for example, initial values for metric functions a_i and directional Hubble parameters H_i. In the foregoing system we denote, $P_1 = \lambda_1(Y - 2KY_K)$, $P_2 = m_{\rm sp}S + \lambda_1 Y$, $Q = 1/(8\pi V^2 X)$, $\bar Q = q_1^2a_1^2 + q_2^2a_2^2 + q_3^2a_3^2$, $Y_K = dY/dK$ and $Z_K = dZ/dK$. Here $A^\mu = \bar\psi\gamma^5\gamma^\mu\psi$ is the pseudo-vector constructed from Dirac spinor and q_i is integration constants: $\dot{\mathcal{A}}_i = -a_i^2q_i/(VX)$ with $V = a_1a_2a_3$ is the volume scale of BI- metric. Beside these, from the spinor field equations it is possible to show that

$$S^2 + P^2 + {A^0}^2 = \frac{c_0^2}{V^2}, \quad A^1 = \frac{c_1}{V}, \quad A^2 = \frac{c_2}{V}, \quad A^3 = \frac{c_3}{V}, \tag{10}$$

where c_0, c_1, c_2, c_3 are constants of integration. Introducing the directional Hubble parameters H_i we rewrite the Einstein equations (3) - (5) as follows:

$$\begin{aligned} \dot a_1 &= H_1 a_1, & (11)\\ \dot a_2 &= H_2 a_2, & (12)\\ \dot a_3 &= H_3 a_3, & (13)\\ \dot H_1 &= \frac{\kappa}{2}\left[P_1 + Q\left[q_3^2 a_3^2 + q_2^2 a_2^2 - 3 q_1^2 a_1^2 + \frac{2\lambda_2 K Z_K}{X}\bar Q\right]\right]\\ &- \frac{1}{2}\left[2H_1^2 + H_1 H_2 + H_3 H_1 - H_2 H_3\right], & (14)\\ \dot H_2 &= \frac{\kappa}{2}\left[P_1 + Q\left[q_3^2 a_3^2 + q_1^2 a_1^2 - 3 q_2^2 a_2^2\right] + \frac{2\lambda_2 K Z_K}{X}\bar Q\right]\\ &- \frac{1}{2}\left[2H_2^2 + H_1 H_2 + H_2 H_3 - H_3 H_1\right], & (15)\\ \dot H_3 &= \frac{\kappa}{2}\left[P_1 + Q\left[q_2^2 a_2^2 + q_1^2 a_1^2 - 3 q_3^2 a_3^2\right] + \frac{2\lambda_2 K Z_K}{X}\bar Q\right]\\ &- \frac{1}{2}\left[2H_3^2 + H_3 H_1 + H_2 H_3 - H_1 H_2\right]. & (16)\end{aligned}$$

The remaining equations we exploit to find the initial values of metric functions and Hubble parameters. From the non-diagonal equations (7) - (9) we have

$$c_1 c_2 q_1 q_2 a_1 a_2 + c_2 c_3 q_2 q_3 a_2 a_3 + c_3 c_1 q_3 q_1 a_3 a_1 = 0, \quad (17)$$

with the solution

$$a_1 = -\frac{c_2 c_3 q_2 q_3 a_2 a_3}{c_1 c_2 q_1 q_2 a_2 + c_3 c_1 q_3 q_1 a_3}, \quad (18)$$

whereas from (6) we find

$$H_1 = \frac{\kappa\left(P_2 + Q\bar Q\right) - H_2 H_3}{H_2 + H_3}. \quad (19)$$

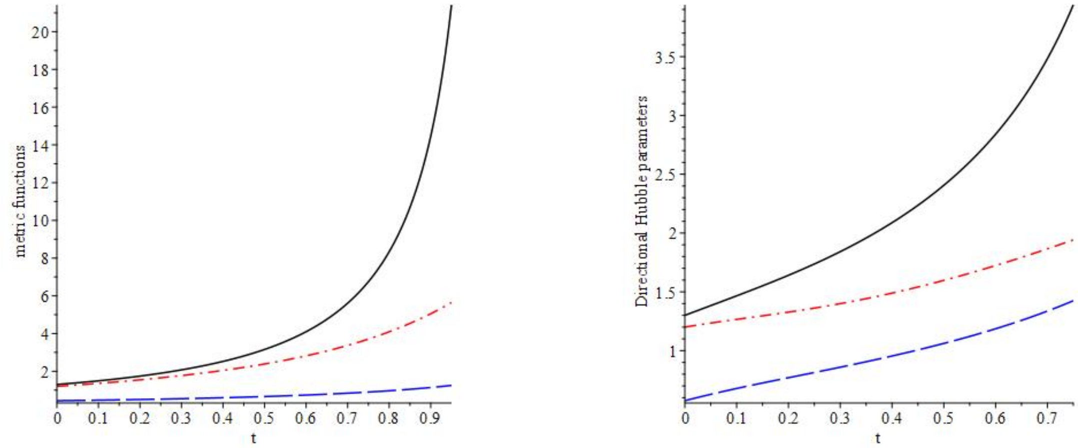
In what follows we solve the system (11) - (16) together with (18) and (19), numerically. The system, in question, is a multi-parametric one which can be exploited to find different types of solutions. For simplicity we choose $c_i : c_1 = c_2 = c_2 = 1$ and $q_1 = 0.2$, $q_2 = 0.5$, $q_3 = -0.03$. Note that we should choose q_i and c_i as well as $a_2(0)$ and $a_3(0)$ in such a way that (18) provides real value for $a_1(0)$. Further we choose the self-coupling constant $\lambda_1 = 0.4$ and the coupling constant $\lambda_2 = 0.5$. We choose $Z(K) = K^{n_2}$ and set $n_2 = 2$. Note that n_2 can be chosen both positive and negative. The initial values for the metric functions and Hubble parameters we set $a_2(0) = 1.2$, $a_3(0) = 1.3$, $H_2(0) = 1.2$ and $H_3(0) = 1.3$. For these values of problem parameters from (18) and (19) we find $a_1(0) = 0.439$ and $H_1(0) = 0.574$, respectively. The spinor field nonlinearity we choose in the form [4, 5]

$$Y(K) = \left(B + \lambda_1 K^{(1+\alpha)/2}\right)^{1/(1+\alpha)}, \quad B > 0, \quad 0 < \alpha \le 1, \quad (20)$$

that describes a Chapligin gas. Here we choose $B = 1$ and $\alpha = 0.5$. The corresponding solutions are given in Fig. 1. In Fig. 1 we plot the metric functions and Hubble parameters, respectively. In the figures blue long dash line, red dash dot line and black solid line stand for x, y and z-component of metric function and Hubble parameter, respectively. Note that the spinor field nonlinearity can simulate different types of sources including perfect fluid, quintessence, cosmological constant, phantom matter, Chaplygin gas, modified quintessence, modified Chaplygin gas, quintom matter etc. So exploiting this property of spinor field nonlinearity one can find different types of solutions.

Conclusion

Within the scope of BI cosmological model we have studied the interacting system of spinor and electromagnetic fields. The corresponding field equations are derived and solved numerically. Some severe restrictions, occurred in case of pure spinor or electromagnetic fields, were removed.



a) b)

Fig. 1. a) Evolution of the metric functions a_1 (blue long dash line), a_2 (red dash dot line) and a_3 (black solid line), when the spinor field nonlinearity describes a Chaplygin gas. and b)Evolution of the directional Hubble parameters H_1 (blue long dash line), H_2 (red dash dot line) and H_3 black solid line.

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Dedication: I would like to dedicate this report to the blessed memory of one of the best cosmologists of our time, Professor A.A. Starobinsky. Since 2007, when I first attended a conference organized by the Russian Gravitational Society, we have met several times at conferences. Whenever I asked his opinion about my work, he gave valuable advice. Although in earlier years he was very skeptical about using the spinor field in a classical manner, he eventually said that such an approach has a right to exist. I will definitely miss him at upcoming conferences.

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