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OSCILLATING SCALAR FIELDS AS DARK MATTER*

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The work considers oscillating scalar fields acting as cold dark matter in the expanding Universe. This behavior is inherent for both real and complex fields, and besides, the complex field reproduces the properties of cold dark matter even if the energy density per particle exceeds its mass.

Keywords: Axion, dark matter, oscillating scalar field.

ОСЦИЛЛИРУЮЩИЕ СКАЛЯРНЫЕ ПОЛЯ КАК ТЕМНАЯ МАТЕРИЯ

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В работе рассматриваются осциллирующие скалярные поля, действующие как холодная темная материя в расширяющейся Вселенной. Такое поведение имеет место как для действительного так и для комплексного поля, причем комплексное поле воспроизводит свойства холодной темной материи даже когда плотность энергии на одну частицу превышает ее массу.

Ключевые слова: Аксион, темная материя, осциллирующее скалярное поле.

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Introduction

Today, there is very little doubt that a considerable fraction of the matter in the Universe is hidden dark matter (DM). Its presence in galaxies is evidenced by observational data on the rotation curves [1] and gravitational lensing [2]. The CMB anisotropy data indicate a significant DM contribution to the evolution of the Universe [3]. The preferred DM model is a cold DM (CDM) formed by collisionless weakly interacting massive particles (WIMP) being nonrelativistic already from the moment of decoupling from a thermal bath [4]. CDM is in the best agreement with the most reliable observations, and despite some problems, it is a reference model in modern cosmology and astrophysics. Alternative versions of DM have to reproduce the main characteristics and properties of CDM.

The most significant CDM property for the Universe evolution is based on the fact that CDM consists of nonrelativistic particles and therefore is pressureless, i.e. its equation of state $p = 0$. Another important result is that CDM provides a rapid growth of spatial inhomogeneities at the matter dominated (MD) stage, which form modern large-scale structures. The density contrast δ for subhorizon perturbations satisfies the equation

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho\delta = 0\,, \qquad (1)$$

where H is the Hubble expansion rate, and ρ is the background energy density. Eq. (1) gives the solution growing as scale factor, $\delta \propto a$.

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Another DM model that is in good agreement with the observations is a massive real scalar field [5–7]. When its mass m satisfies the condition

$$H \ll m\,, \tag{2}$$

the scalar field behaves as CDM at all relevant cosmological scales. This model is primarily motivated by axions and axion-like particles (ALPs) as DM candidates. For cosmologically suitable axions with masses $m \sim 10^{-6} - 10^{-4}$ eV, relation (2) is satisfied from the moment of axion field creation whereas at the MD stage it is satisfied even for ultralight bosons with masses $m \sim 10^{-22}$ eV.

It is shown in this work that a complex scalar field produces similar results and also well simulates CDM under condition (2).

The paper is organized as follows. The evolution of the real scalar field in the expanding Universe is outlined in Sec. 1. This model is well-known [6] and it is presented here to compare it with results for the complex scalar field described in Sec. 2. Finally, the results obtained are discussed.

1. Real scalar field

Consider the scalar field with the Lagrange density

$$\mathcal{L} = \frac{1}{2}\partial_i\psi\partial^i\psi - \frac{1}{2}m^2\psi^2\,, \tag{3}$$

in the spatially flat Friedmann universe described by the metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right)\,. \tag{4}$$

The field equation reads as

$$\ddot{\psi} + 3\frac{\dot{a}}{a}\dot{\psi} + m^2\psi = 0\,. \tag{5}$$

To conveniently apply condition (2), one uses the substitution

$$\psi = a^{-3/2}\phi\,. \tag{6}$$

obtaining

$$\ddot{\phi} + \left(m^2 - \frac{3}{2}\frac{\ddot{a}}{a} - \frac{3}{4}\frac{\dot{a}^2}{a^2}\right)\phi = 0 \tag{7}$$

instead (5). The last two terms in the parenthesis are of order H^2 and can be ignored due to condition (2), so that the solution to Eq. (5) has the form

$$\psi = u(t)\cos mt\,, \tag{8}$$

where $u \propto a^{-3/2}$. Corrections to this solution are of the order H^2/m^2.

On the timescale of amplitude variations, the field makes a large number of oscillations, so that it is zero on the average. The same result is obtained when averaging over the period of oscillations.

The energy density and the pressure are determined by corresponding components of the stress-energy tensor. Neglecting the terms of order H^2/m^2 one obtains

$$T_0^0 = \frac{1}{2}\left(\dot{\psi}^2 + m^2\psi^2\right) = \frac{1}{2}m^2u^2\,, \tag{9}$$

$$T_\alpha^\beta = -\frac{1}{2}\left(\dot{\psi}^2 - m^2\psi^2\right)\delta_\alpha^\beta = \frac{1}{2}m^2u^2\cos 2mt\cdot\delta_\alpha^\beta\,. \tag{10}$$

Averaging these quantities over the period one finds

$$\overline{T_0^0} = \rho = \frac{1}{2}m^2u^2 \propto a^{-3}\,, \tag{11}$$

$$\overline{T_\alpha^\beta} = -p\,\delta_\alpha^\beta \;= 0\,. \tag{12}$$

The oscillating scalar field acts as nonrelativistic matter. When the scalar field dominates in the Universe, it provides the same evolution as CDM. Note, that the field ψ can be interpreted as a collection of particles with a particle density

$$n = \frac{\rho}{m} \propto a^{-3}\,. \tag{13}$$

2. Complex scalar field

The approach developed in the previous section can be applied to the complex scalar field with a global U(1) symmetry. The theory is defined by the Lagrange density

$$\mathcal{L} = \partial_i \varphi^* \partial^i \varphi - m^2 |\varphi|^2\,, \tag{14}$$

so that the conserved current associated with the U(1) symmetry is

$$n_i = -i\left(\varphi^* \partial_i \varphi - \varphi \partial_i \varphi^*\right)\,, \tag{15}$$

and the corresponding conserved charge is a total particle number.

Using the amplitude-phase representation $\varphi = \psi e^{i\theta}$ one obtains from (14) the following equations of motion in the expanding Universe described by metric (4):

$$\partial_t\left(a^3 \psi^2 \dot\theta\right) = 0\,, \tag{16}$$

$$\ddot\psi + 3\frac{\dot a}{a}\dot\psi - \dot\theta^2 \psi + m^2 \psi = 0\,. \tag{17}$$

Eq. (16) represents the particle number density conservation arising from the global U(1) symmetry, so that the density evolves according to

$$2\psi^2 \dot\theta = n(t) \propto a^{-3}\,. \tag{18}$$

Taking into account (18) and neglecting terms of order H^2 when compared with m^2 the solution to Eq. (17) reads as

$$\psi = \frac{1}{2} u(t) \left(1 + \left(1 - \Delta^2\right)^{1/2} \cos 2mt\right)^{1/2}\,, \tag{19}$$

where $u \propto a^{-3/2}$, $\Delta = 2n/mu^2$ =const, and it is implied $\Delta < 1$ in this solution.

This solution describes the complex scalar field with oscillating amplitude so that its central value is nonzero. The phase θ of the complex field grows unevenly according to Eq. (18). Details of the field behavior are determined by parameter Δ.

The amplitude representation in Eq. (19) gives the same expression for nonzero components of the stress-energy tensor as for the real field:

$$T^0_0 = \dot\psi^2 + m^2\psi^2 + \dot\theta^2\psi^2 = \frac{1}{2} m^2 u^2\,, \tag{20}$$

$$T^\beta_\alpha = \dot\psi^2 - m^2\psi^2 + \dot\theta^2\psi^2 = -\frac{1}{2} m^2 u^2 \left(1 - \Delta^2\right)^{1/2} \cos 2mt \cdot \delta^\beta_\alpha\,. \tag{21}$$

The average values over the period yield the energy density and the pressure

$$\rho = \frac{1}{2} m^2 u^2 \propto a^{-3}\,, \qquad p = 0\,. \tag{22}$$

Taking into account (22) parameter Δ can be represented as $\Delta = mn/\rho$. In this form, it has a clear physical interpretation, an inverse value of the energy per particle, which remains constant while the field oscillates.

It is usually thought that nonrelativistic matter implies $\rho \approx mn$ as it takes place for the real scalar field. In solution (19) that corresponds $\Delta \approx 1$, and $\psi \approx u/2$. It follows from (18) that the phase function $\theta \approx mt$, and the field φ represents a coherent state of zero momentum scalar particles with density

$n = 2m\psi^2$. The field under this approximation was applied to study instabilities of the Affleck-Dine condensate [8] and to the problem of why the densities of baryons and dark matter in the Universe are similar [9].

The opposite case, $\Delta \ll 1$, can be considered as an ultrarelativistic one because of $\rho \gg mn$. However expressions (22) remain valid and the complex scalar field continues to mimic CDM.

The small value of Δ can emerge for light bosons decoupled after QCD phase transition, when decoupling temperature satisfies $T_{\rm d} \gg m \gg H$ [10]. In this case Δ can be estimated as $\Delta \sim m/T_{\rm d}$. Since the condition for oscillations is embodied, the quantity Δ holds this value throughout the further evolution.

Conclusions

We have demonstrated that the oscillating real and complex scalar fields in the expanding Universe behave as CDM, producing the apparent equation of state $p = 0$. For the real field, this means a usual expression for the energy density $\rho = mn$ while the complex field provides the nonrelativistic behavior even for $\rho > mn$. This property could provide additional opportunities to ALP models because of avoiding some constraints with respect to the particle density.

In conclusion, we present the equation for subhorizon density perturbations of the complex scalar field in the MD phase

$$\ddot{\delta} + 2H\dot{\delta} - \left(4\pi G\rho - \frac{k^4}{4m^2a^4}\right)\delta = 0\,. \tag{23}$$

Despite a more complicated background solution, the density contrast obeys the same equation as for the real field [7]. For long-wave perturbations (i. e. when $k \to 0$), which are the most important for large scale structure formation, Eq.(23) transforms to (1). Hence there is no distinction between oscillating scalar fields and CDM, on scales of observational interest.

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