UDC 530.12, 531.51

COSMOLOGICAL MODELS BASED ON SCALAR-TORSION GRAVITY *

Fomin I. V.[a,1]

[a] Bauman Moscow State Technical University, Moscow, 105005, Russia.

Cosmological models based on teleparallel gravity with a non-minimal coupling between the scalar field and torsion (scalar-torsion gravity) are considered in the context of their comparison with the case of Einstein gravity (GR) and the teleparallel equivalent of general relativity (TEGR). A classification of cosmological inflationary models is proposed based on the type of dependence of the tensor-scalar ratio on the spectral index of scalar perturbations. The difference between inflationary models based on scalar-torsion gravity and models based on GR and TEGR is shown in the context of their verification using observational data based on this dependence.

Keywords: teleparallel gravity, general relativity, scalar fields.

КОСМОЛОГИЧЕСКИЕ МОДЕЛИ НА ОСНОВЕ СКАЛЯРНО-ТОРСИОННОЙ ГРАВИТАЦИИ

Фомин И. В.[a,1]

[a] МГТУ им. Н.Э. Баумана,, г. Москва, 105005, Россия.

Рассматриваются космологические модели на основе телепараллельной гравитации с неминимальной связью скалярного поля и кручения (скалярно-торсионная гравитация) в контексте их сравнения со случаем гравитации Эйнштейна (ОТО) и телепараллельным эквивалентом общей теориии относительности (TEGR). Предложена классификация моделей космологической инфляции по типу зависимости тензорно-скалярного отношения от спектрального индекса скалярных возмущений. Показано отличие инфляционных моделей на основе скалярно-торсионной гравитации от моделей на основе ОТО и TEGR в контексте их верификации по наблюдательным данным на основе этой зависисмости.

Ключевые слова: телепараллельная гравитация, общая теория относительности, скалярные поля.

PACS: 04.50.-h
DOI: 10.17238/issn2226-8812.2024.1.104-109

Introduction

Within the framework of constructing current cosmological models, the stage of cosmological inflation is important, since during this stage various physical processes occur that determine the further evolution of the universe [1, 2].

A different gravity theories are used to construct models of cosmological inflation [3], including Einstein gravity (GR) [1, 2] and the teleparallel equivalent of general relativity (TERG) [4] and teleparallel gravity with a non-minimal coupling between the scalar field and torsion (scalar-torsion gravity or STG) [5–7] as well.

In addition to the presence of the inflationary stage of accelerated expansion of the early universe, the exit from inflation and the stage of the second accelerated expansion of the universe at the present

*Настоящая работа выполнена в рамках Дополнительного соглашения №073-03-2024-060/1 от 13.02.2024 к Соглашению о предоставлении субсидии из федерального бюджета на финансовое обеспечение выполнения государственного задания на оказание государственных услуг (выполнения работ) № 073-03-2024-060 от 18.01.2024, заключенным между ФГБОУ ВО «УлГПУ им. И.Н. Ульянова» и Министерством просвещения Российской Федерации.

[1] E-mail: ingvor@inbox.ru

era, verifiable cosmological models must comply with observational constraints on the values of the parameters of cosmological perturbations [1, 2].

The main characteristics for verifying inflation models using observational constraints on the values of the parameters of cosmological perturbations based on measurements of anisotropy and polarization of CMB are the tensor-scalar ratio $r < 0.032$ and the spectral index of scalar perturbations $n_S = 0.9663 \pm 0.0041$ [8, 9].

Analysis of the evolution of cosmological perturbations for various inflationary models based on various theories of gravity leads to different dependencies $r = r(1 - n_S)$, the type of which may or may not correspond to observational constraints.

This paper examines the classification of cosmological models according to type $r = r(1 - n_S)$ dependence for arbitrary models. Cosmological models based on GR and TEGR are also compared with models based on the scalar-torsion gravity.

1. Classification of cosmological models based on dependence $r = r(1 - n_S)$

Since the value of the spectral index of scalar perturbations is $n_S \simeq 0.97$ and $1 - n_S \simeq 0.03 \ll 1$ [8,9], we can write the dependence $r = r(1 - n_S)$ as follows

$$r = \sum_{k=0}^{\infty} \beta_k (1 - n_S)^k = \beta_0 + \beta_1 (1 - n_S) + \beta_2 (1 - n_S)^2 + ..., \tag{1}$$

where $(1 - n_S)$ is the parameter of expansion and β_k are the constant coefficients, which depends on the type of inflationary model.

Also, the zeroth order term in this expansion $r(0) = \beta_0 = 0$ from condition $r(n_S = 1) = 0$ corresponding to the flat Harrison-Zel'dovich spectrum [1, 2], thus, we can rewrite expression (1) in the following form

$$r = \sum_{k=1}^{\infty} \beta_k (1 - n_S)^k = \beta_1 (1 - n_S) + \beta_2 (1 - n_S)^2 + ... \tag{2}$$

Thus, we can consider the classification of cosmological models according to non-zero orders of expansion of dependence $r = r(1 - n_S)$ as follows: first-order models for $\beta_1 \neq 0$, second-order models for $\beta_1 = 0$ and $\beta_2 \neq 0$ etc.

It should be noted that if the cosmological model satisfies observational constraints in a certain order of expansion (2), then the remaining orders of expansion can be neglected, since they make a smaller contribution to the value of the tensor-scalar ratio.

2. Cosmological models based on GR and TEGR

Cosmological models based on GR in the system of units $8\pi G = m_p^2 = c = 1$ correspond to the following action [1, 2]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right], \tag{3}$$

where R is the scalar curvature, ϕ is a scalar field, $V(\phi)$ is the potential of a scalar field and $g^{\mu\nu}$ is a metric tensor of a space-time.

The action for cosmological models based on the scalar-torsion gravity can be written as follows [5–7]

$$S = \int d^4x \, e \left[-\frac{1}{2} T - \frac{1}{2} \omega(\phi) \partial_\mu \phi \partial^\mu \phi - V(\phi) \right], \tag{4}$$

where T is the torsion scalar and $e = \det(e^A_{\ \mu}) = \sqrt{-g}$, $e^A_{\ \mu}$ is the tetrad field.

For the case of the flat Friedmann-Robertson-Walker (FRW) metric for GR

$$ds^2 = -dt^2 + a^2 \, \delta_{ij} dx^i dx^j, \tag{5}$$

where $a = a(t)$ is the scale factor, and t is the cosmic time, and for corresponding diagonal tetrad field $e^A_{\ \mu} = \mathrm{diag}(1, a, a, a)$ for TEGR we have the same cosmological dynamic equations [6, 7]

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{6}$$

$$-3H^2 - 2\dot{H} = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{7}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'_\phi = 0, \tag{8}$$

where $V'_\phi = dV/d\phi$, $H = H(t)$ is the Hubble parameter.

Also, we have the same parameters of cosmological perturbations for the case of GR and TEGR

$$n_S - 1 = -4\epsilon + 2\delta, \qquad r = 16\epsilon, \tag{9}$$

where the slow-roll parameters are defined as

$$\epsilon = -\frac{\dot{H}}{H^2} \ll 1, \qquad \delta = -\frac{\ddot{H}}{2H\dot{H}} \ll 1. \tag{10}$$

for any inflationary model [6, 7].

We also note that the definition of the Hubble parameter $H = H(t)$ corresponds to the definition of a specific dependence $\epsilon = \epsilon(\delta)$ in explicit or parametric form.

2.1. First-order cosmological models

Now, we consider the first-order models, which can be define by the linear dependence

$$\epsilon = m\delta + \mathcal{O}\left(\delta^2, \delta^3, ...\epsilon^2, \epsilon^3, ..., \dot{\epsilon}, \ddot{\epsilon}, ..., \dot{\delta}, \ddot{\delta}, ...\right) \simeq m\delta, \tag{11}$$

where m is a some constant.

This dependence can be determined up to higher order terms, which can be neglected due to the quasi-de Sitter character of the early universe's accelerated expansion [1, 2].

From definition of the slow-roll parameters (10) and dependence (11) we obtain

$$H(t) \simeq \left[\left(1 - \frac{2}{m}\right)(c_1 t + c_2)\right]^{\frac{m}{m-2}}, \tag{12}$$

where from the condition $\dot{H} < 0$ we get $0 < m < 2$.

From expressions (9) and (12) we obtain linear dependence

$$r = \frac{8m}{2m-1}(1 - n_S) > 0.16, \tag{13}$$

for the case $0 < m < 2$, and, therefore, the first-order cosmological models based on GR and TEGR don't correspond to the observational constraint $r < 0.032$ under condition $\dot{H} < 0$.

If condition $\dot{H} < 0$ is violated, for the case $m < -0.2$, condition $r < 0.032$ can be satisfied.

However, for $\dot{H} > 0$, the relative acceleration of the expansion of the universe

$$Q = \frac{\ddot{a}}{a} = H^2 + \dot{H}, \tag{14}$$

can only be positive $Q > 0$, that corresponds to absence of exit from inflation, which can be defined by condition $Q < 0$ after inflationary stage.

Thus, first-order cosmological models based on GR and TEGR cannot be verified by observational constraints, therefore, in expansion (2) for the case GR and TEGR we have $\beta_1 = 0$ for actual cosmological models.

2.2. Second-order cosmological models

For the second-order models we have the quadratic dependence

$$\epsilon \simeq m\delta^2, \tag{15}$$

with corresponding Hubble parameter

$$H(t) \simeq \lambda + \frac{m}{t}, \tag{16}$$

where condition $\dot{H} < 0$ is satisfied for $m > 0$.

From expressions (9) and (16) we obtain quadratic dependence

$$r = 4m(1 - n_S)^2 > 0, \tag{17}$$

and observational constraints $r < 0.032$, $n_S = 0.9663 \pm 0.0041$ are satisfied for $0 < m < 9$.

We also note that such a model can satisfy any future observational constraints on the value of the tensor-to-scalar ratio, which, in this case, are reduced to a refinement of the value of parameter m.

The cosmological model based on the Hubble parameter (16) or exponential power-law inflation based on Einstein gravity and beyond was discussed in detail in [10].

In this work it was shown that exponential power-law model implies different stages: $Q_1 > 0$ (inflation), $Q_{2,3} < 0$ (exit from inflation with transition to the radiation domination and matter domination eras) and $Q_4 > 0$ ($Q_4 \ll Q_1$) (second inflation).

Also, in [10] it was shown, that for the special case $m = 3/4$, exponential-power inflation is conformally equivalent to the Starobinsky model $f(R) = R + \alpha R^2$, which implies the following relation $r = 3(1 - n_S)^2$ [11, 12].

Thus, second-order models based on GR and TEGR, in contrast to first-order models, correspond to observational constraints and can be considered as relevant ones to describe the evolution of the universe.

3. Cosmological models based on the scalar-torsion gravity

The action for cosmological models based on the scalar-torsion gravity can be written as follows [13]

$$S = \int d^4x\, e\, \left[f(T,\phi) + \omega(\phi)X\right], \tag{18}$$

where $f = f(T,\phi)$ is an arbitrary differentiable function of a scalar field ϕ and torsion scalar T.

For the diagonal tetrad field $e^A_\mu = \mathrm{diag}(1, a, a, a)$ corresponding to the flat Friedmann-Robertson-Walker (FRW) metric one has following dynamic equations

$$f(T,\phi) - \frac{1}{2}\omega(\phi)\dot{\phi}^2 - 2Tf_{,T} = 0, \tag{19}$$

$$f(T,\phi) + \frac{1}{2}\omega(\phi)\dot{\phi}^2 - 2Tf_{,T} - 4\frac{d}{dt}\left(Hf_{,T}\right) = 0, \tag{20}$$

$$\omega(\phi)\ddot{\phi} + 3\omega(\phi)H\dot{\phi} + \frac{1}{2}\frac{d\omega(\phi)}{d\phi}\dot{\phi}^2 - f_{,\phi} = 0, \tag{21}$$

with additional relation $T = 6H^2$.

Exact solutions of the system of equations (19)–(21) for an arbitrary Hubble parameter were considered in [14, 15], and can be written as follows

$$f(T,\phi) = -F(\phi)\sqrt{T} - V(\phi) = f(T,\phi)_{STG} - V(\phi), \tag{22}$$

$$\omega(\phi) = -\frac{1}{3}\frac{F_{,\phi}^2}{V(\phi)}, \qquad \dot{\phi} = \frac{\sqrt{6}V(\phi)}{F_{,\phi}}, \tag{23}$$

where $F = F(\phi)$ is the coupling function, and the scalar-torsion gravity is defined by the function

$$f(T,\phi)_{STG} = -F(\phi)\sqrt{T}. \quad (24)$$

Based on the general analysis of cosmological perturbations for the models based on the scalar-torsion gravity [13] we can define the spectral index of scalar perturbations and tensor-to-scalar ratio corresponding to solutions (22)–(23) as follows

$$n_S - 1 = 2\frac{\dot{H}}{H^2} + 2\frac{\dot{F}}{HF} - \frac{\ddot{F}}{H\dot{F}}, \quad (25)$$

$$r = -16\frac{\dot{F}}{HF}. \quad (26)$$

Thus, for the following relation between the Hubble parameter and coupling function

$$H(t) \sim \dot{F}^{1/2}F^{m-1}, \quad (27)$$

from expressions (25)–(26) we obtain the linear dependence

$$r = \frac{8}{m}(1 - n_S) > 0, \quad (28)$$

for an arbitrary cosmological dynamics, where $\beta_1 = 8/m$ and $m > 0$ is the positive constant.

Thus, for $m > 7.5$ these models satisfy observational constraints $r < 0.032$ and $n_S = 0.9663 \pm 0.0041$.

It should be noted, that these models can satisfy any future observational constraints on the value of the tensor-to-scalar ratio due to a refinement of the value of parameter m similar as for the case second-order models based on GR and TEGR.

Also, due to the fact that this result is valid for an arbitrary type of cosmological dynamics, such models can correctly describe inflation, exit from inflation, and the second accelerated expansion of the universe.

Consequently, in the case of scalar-torsion gravity, in contrast to GR and TEGR, first-order cosmological models comply with observational constraints.

Conclusion

In this paper, we considered a method for classifying cosmological models according to the order of expansion of dependence $r = r(1 - n_S)$ for arbitrary models built on the basis of arbitrary theories of gravity.

It was shown that for the case of Einstein gravity and its teleparallel equivalent, first-order models $r \sim (1 - n_S)$ don't satisfy observational constraints, however second-order models $r \sim (1 - n_S)^2$ are verifiable ones.

This means that if, when analyzing arbitrary cosmological models based on GR or TEGR, the expansion of dependence $r = r(1 - n_S)$ for this model contains a non-zero linear term $r \sim (1 - n_S)$, this cosmological model is unverifiable due to the observational constraints.

In contrast to cosmological models based on GR and TEGR, models based on scalar-torsion gravity correspond to observational constraints already at the first order $r \sim (1 - n_S)$.

Thus, scalar-torsional gravity corresponds to a wider class of verifiable cosmological models than Einstein gravity or its teleparallel equivalent.

References

1. Baumann D., McAllister L. *Inflation and String Theory*. Cambridge Monographs on Mathematical Physics (Cambridge University Press, 2015). https://doi.org/10.1017/CBO9781316105733

2. Chervon S., Fomin I., Yurov V., Yurov A. *Scalar Field Cosmology*. Series on the Foundations of Natural Science and Technology, Volume 13 (WSP, Singapur, 2019). https:https://doi.org/10.1142/11405

3. Odintsov S.D., Oikonomou V.K., Giannakoudi I.,Fronimos F.P., Lymperiadou E.C. Recent Advances in Inflation. *Symmetry*, 2023, vol. 15, no. 9, p. 1701.

4. Yi-Fu Cai, Capozziello S., Mariafelicia De Laurentis, Saridakis E. f(T) teleparallel gravity and cosmology. *Rep. Prog. Phys.*, 2016, vol. 2, no. 1, p. 106901.

5. Gonzalez-Espinoza M., Otalora G. Cosmological dynamics of dark energy in scalar-torsion $f(T, \phi)$ gravity. *Eur. Phys. J. C*, 2021, vol. 81, no.5, p. 480.

6. Fomin I.V. Methods for constructing and verifying inflationary models of the early universe. *Space, Time and Fundamental Interactions*, 2022, no. 40, pp. 50–63.

7. Chervon S.V., Fomin I.V. Reconstruction of Scalar-Torsion Gravity Theories from the Physical Potential of a Scalar Field. *Symmetry*, 2023, vol. 15, no. 2, p. 291.

8. Aghanim N., et al. [Planck] Planck 2018 results. VI. Cosmological parameters. *Astron. Astrophys.*, 2020, vol. 641, A6.

9. Tristram M., Banday A.J., Górski K.M., Keskitalo R., Lawrence C.R., Andersen K.J., Barreiro R.B., Borrill J., Colombo L.P.L., Eriksen H.K., et al. Improved limits on the tensor-to-scalar ratio using BICEP and Planck data. *Phys. Rev. D*, 2022, vol. 105, no. 8, p. 08.

10. Fomin I., Chervon S. Exact and Slow-Roll Solutions for Exponential Power-Law Inflation Connected with Modified Gravity and Observational Constraints. *Universe*, 2020, vol. 6, no. 11, p. 199.

11. Starobinsky A.A. A New Type of Isotropic Cosmological Models Without Singularity. *Phys. Lett. B*, 1980, vol. 91, pp. 99–102.

12. Mishra S.S., Sahni V., Toporensky A.V. Initial conditions for Inflation in an FRW Universe. *Phys. Rev. D*, 2018, vol. 98 no. 8, p. 083538.

13. Gonzalez-Espinoza M., Otalora G. Generating primordial fluctuations from modified teleparallel gravity with local Lorentz-symmetry breaking. *Phys. Rev. B*, 2020, vol. 809, p. 135696.

14. Dentsel E.S., Fomin I.V. Generalized exact solutions in cosmological models with non-minimal coupling of scalar field and torsion, *Space, Time and Fundamental Interactions*, 2022, no. 40, pp. 110–118.

15. Dentsel E.S., Fomin I.V. Exact solutions in cosmological models with non-minimal coupling of scalar field and torsion. *Space, Time and Fundamental Interactions*, 2023, no. 1, pp. 46–50.

Authors

Fomin Igor Vladimirovich, Doctor of Physical and Mathematical Sciences, Professor of Physics Department of Bauman Moscow State Technical University, 2-nd Baumanskaya street, 5, Moscow, 105005, Russia.
E-mail: ingvor@inbox.ru

Please cite this article in English as:

Fomin I. V. Cosmological models based on scalar-torsion gravity. *Space, Time and Fundamental Interactions*, 2024, no. 1, pp. 104–109.

Авторы

Фомин Игорь Владимирович, д. ф.-м. н., профессор кафедры физики, Московский государственный технический университет имени Н.Э. Баумана, 2-я Бауманская ул., д. 5, г. Москва, 105005, Россия.
E-mail: ingvor@inbox.ru

Просьба ссылаться на эту статью следующим образом:

Фомин И. В. Космологические модели на основе скалярно-торсионной гравитации. *Пространство, время и фундаментальные взаимодействия.* 2024. № 1. С. 104–109.