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QUASI-DE SITTER SOLUTION IN $F(T,(\nabla T)^2)$ TELEPARALLEL GRAVITY*

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This paper considers the relevance of the quasi-de Sitter solution for the modified $F(T,(\nabla T)^2)$ teleparallel theory of gravity. The Hubble parameter is represented in the form: $H = H_* + \epsilon e^{-Bt}$, where $H_*, B - const$. The cosmological equations of the model with a perfect fluid are written for the following choice of function F: $F = AT^n + \omega(T)\nabla_\mu T \nabla^\mu T$. The evolution of matter density perturbations in the absence of pressure perturbations is considered.

Keywords: Teleparallel gravity, quasi-de Sitter solution, cosmological perturbations.

РЕШЕНИЕ КВАЗИ-ДЕ СИТТЕРА В $F(T,(\nabla T)^2)$ ТЕЛЕПАРАЛЛЕЛЬНОЙ ГРАВИТАЦИИ

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В данной работе рассматривается актуальность решения квази-де Ситтера для модифицированной $F(T,(\nabla T)^2)$ телепаральнойной теории гравитации. Параметр Хаббла представлен в виде: $H = H_* + \epsilon e^{-Bt}$, где $H_*, B - const$. Записаны космологические уравнения модели с идеальной жидкостью при следующем выборе функции F: $F = AT^n + \omega(T)\nabla_\mu T \nabla^\mu T$. Рассмотрена эволюция возмущений плотности материи при отсутствии возмущений давления.

Ключевые слова: Телепараллельная гравитация, решение квази-де Ситтера, космологические возмущения.

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Introduction

In this work, we propose modified teleparallel gravity of $F(T,(\nabla T)^2)$ version and investigate cosmological quasi de Sitter solutions within it. Cosmological equations for general version $F(T,(\nabla T)^2,\Box T)$ have already been represented in the work of G. Otalora and E. Saridakis [1]. We study simplicated model using the result of [1].

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1. Cosmological equations

Modified teleparallel gravity with higher derivatives with respect to the torsion scalar of the form $F(T,(\nabla T)^2,\Box T)$ was constructed in [1]. The action of the model has the form:

$$S=\frac{1}{2}\int d^4x\, e\, F(T,(\nabla T)^2,\Box T)+S_m(e_\rho^A,\Psi_m), \tag{1}$$

where T is the torsion scalar, $e=det(e_\mu^A)=\sqrt{-g}$ tetrad determinant, $(\nabla T)^2=\eta^{AB}e_A^\mu e_B^\nu\nabla_\mu T\nabla_\nu T=g^{\mu\nu}\nabla_\mu T\nabla_\nu T$, $\Box T=\nabla_\mu\nabla^\mu T$, Ψ_m a general field of matter. We consider a truncated version $F(T,(\nabla T)^2)$ in the Friedmann-Robertson-Walker metric:

$$ds^2=dt^2-a^2(t)\delta_{ij}dx^idx^j, \tag{2}$$

which arises from the tetrad $e_\mu^A=diag(1,a(t),a(t),a(t))$, where $a=a(t)$ is the scale factor.

The dynamic equations for the cosmological model with a perfect fluid [1] for truncated $F(T,(\nabla T)^2)$ model are the following:

$$F_TH^2+24H^2F_X(3H\dot{H}+\ddot{H})H+24H^3\dot{H}\dot{F}_X+\frac{F}{12}=\frac{\rho_m}{6}, \tag{3}$$

$$F_T\dot{H}+H\dot{F}_T+24H[2H\ddot{H}+3(\dot{H}+H^2)\dot{H}]\dot{F}_X+24H^2F_XH^{(3)}+24H^2\dot{H}\ddot{F}_X+24F_X\dot{H}^2(12H^2+\dot{H})+$$
$$+24HF_X(4\dot{H}+3H^2)\ddot{H}=-\frac{p_m}{2}, \tag{4}$$

where $X=(\nabla T)^2$; $T=-6H^2$; $\dot{T}=\frac{dT}{dt}=-12H\dot{H}$; $X=\dot{T}^2=144H^2\dot{H}^2$, $F_X=\frac{dF}{dX}$. Hereinafter we omit the index m for density ρ_m and pressure p_m.

By analogy with investigation of $f(R,(\nabla R)^2)$ [2] we choose the function $F(T,(\nabla T)^2)$ as

$$F=AT^n+\omega(T)X,\ A>0,\ A=const. \tag{5}$$

An exponent of torsion scalar n is a natural number.

The derivatives of the function $F(T,(\nabla T)^2)$ with respect to T,X and time t are

$$F_T=AnT^{n-1}+\omega_TX,\quad \dot{F}_T=An(n-1)T^{n-2}\dot{T}+\omega_{TT}\dot{T}X+\omega_T\dot{X},$$
$$F_X=\omega(T),\quad \dot{F}_X=\omega_T\dot{T}=\dot{\omega}_T,\quad \ddot{F}_X=\omega_{TT}\dot{T}^2+\omega_T\ddot{T}. \tag{6}$$

Here we are using abbreviated notation $\omega_T=\frac{d\omega}{dT}$, $\omega_{TT}=\frac{d^2\omega}{dT^2}$ and so on.

Substituting the relations above into equations of cosmological dynamics (3) and (4) we get:

$$(AnT^{n-1}+\omega_TX)H^2+24H^3\omega(3H\dot{H}+\ddot{H})+24H^3\dot{H}\omega_T\dot{T}+\frac{AT^n+\omega X}{12}=\frac{\rho_m}{6}, \tag{7}$$

$$(AnT^{n-1}+\omega_TX)\dot{H}+H(An(n-1)T^{n-2}\dot{T}+\omega_{TT}X\dot{T}+\omega_T\dot{X})+24H(2H\ddot{H}+3(\dot{H}+H^2)\dot{H})\omega_T\dot{T}+$$
$$+24H^2\dot{H}(\omega_{TT}\dot{T}^2+\omega_T\ddot{T})+24H^2H^{(3)}\omega+24\omega\dot{H}^2(12H^2+\dot{H})+24H\omega(4\dot{H}+3H^2)\ddot{H}=-\frac{p_m}{2}. \tag{8}$$

To solve the field equations (7) and (8), it is necessary to choose the values of the Hubble parameter. In present work, as an example, let us consider the quasi-de Sitter model.

Let us represent quasi-de Sitter model in the following form:

$$H=H_*+\epsilon e^{-Bt},\quad a=a_0\exp\left(H_*t-\frac{\epsilon}{B}e^{-Bt}\right), \tag{9}$$

where $\epsilon\ll 1$, H_*, $B-const.$ and H_*, $B>0$.

In our case, the Hubble parameter contains zero and first order parts with respect to infinitesimal parameter ϵ. Therefore we represent the density and pressure in the same manner:

$$\rho=\rho_0+\epsilon\delta\rho,\quad p=p_0+\epsilon\delta p. \tag{10}$$

2. Solutions for zeroth and first order perturbations in ϵ

Taking into account quasi-de Sitter model (9) with $H = H_* + \epsilon e^{-Bt}$ and its derivatives $\dot{H} = -\epsilon B e^{-Bt}$, $\ddot{H} = \epsilon B^2 e^{-Bt}$, we can derive equations for zeroth and first order perturbations in ϵ. By substituting $H(t)$ from (9) into equations (7) - (8), and taking into account $\dot{T} = 12H_*\epsilon B e^{-Bt}$, $X = (\dot{T})^2 = O(\epsilon^2)$, $\ddot{T}^2 = O(\epsilon)$ we obtain the equations on the density and pressure up to first order in ϵ:

$$12AnH_*^2\xi^{n-1} + 36AnH_*\epsilon e^{-Bt}\xi^{n-1} + 288\omega H_*^3\epsilon B e^{-Bt}(B - 3H_*) + A\xi^n = 2(\rho_0 + \epsilon\delta\rho_m), \tag{11}$$

$$2AnB\epsilon e^{-Bt}\xi^{n-1} + 4An(n-1)\epsilon e^{-Bt}\xi^{n-1} - 48\omega H_*^2 B^3\epsilon e^{-Bt} - 144H_*^3 B^2\epsilon e^{-Bt} = p_0 + \epsilon\delta p, \tag{12}$$

where $\xi = -6H_*^2$, $\xi < 0$.

Next, extracting zero-order term from obtained equations we find the following relations for the density ρ_0 and pressure p_0 of the matter:

$$A\xi^n(1-2n) = 2\rho_0, \quad p_0 = 0. \tag{13}$$

Note, there is no vacuum solution due to absence of possibility to set $\rho_0 = 0$ without setting $A = 0$. Also, let us mention that for odd n we have phantom type model $\rho_0 < 0$, or in scalar field representation $\frac{1}{2}\dot{\phi}^2 < -V(\phi)$. So if $V(\phi) > 0$ kinetic term should be negative.

Considering the first order of smallness with respect to ϵ from equations (11) and (12), we obtain expressions for the density perturbations $\delta\rho$ and pressure δp:

$$\delta\rho = 18AnH_* e^{-Bt}\xi^{n-1} + 144\omega H_*^3 B e^{-Bt}(B - 3H_*), \tag{14}$$

$$\delta p = 2Ane^{-Bt}\xi^{n-1}(B + 2n - 2) - 48H_*^2B^2e^{-Bt}(\omega B - 3H_*). \tag{15}$$

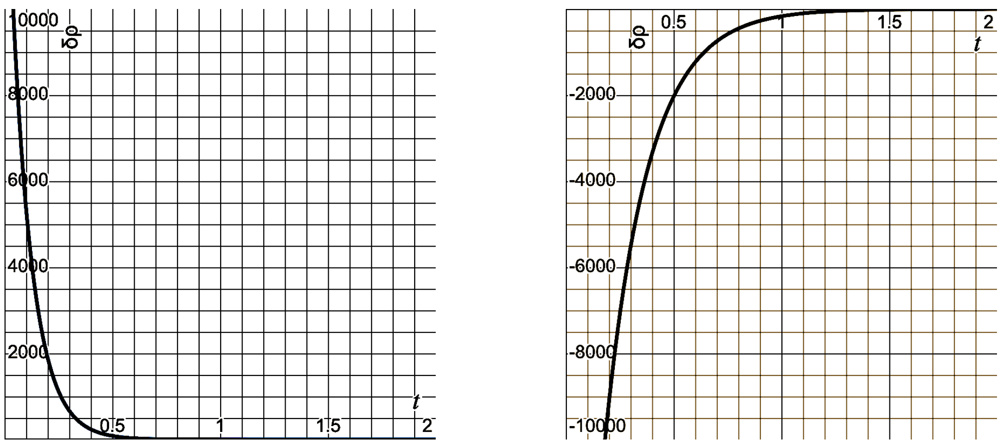
From equation (15), for the case of zero pressure perturbation $\delta p = 0$ we express ω:

$$\omega = \frac{An}{4B^3}\left[B + 2(n-1)\right]\xi^{n-2} + \frac{3H_*}{B}. \tag{16}$$

Then we substitute this expression into the density perturbation equation (14) and obtain the dependence of the density perturbation on time in the absence of pressure perturbation:

$$\delta\rho = e^{-Bt}\left(12\xi^2(B - 3H_*) + AnH_*\xi^{n-1}\left[19 + \frac{2(n-1) - 3\xi}{B} - \frac{6(n-1)\xi}{B^2}\right]\right). \tag{17}$$

Now we can construct graphs of the dependence the density perturbation on time in the absence of pressure perturbation.



a) b)

Fig. 1. Graph of the dependence $\delta\rho(t)$ for a) odd n values and b) even n values

As we can see, the perturbation of density exponentially decreases over time, which can be clearly observed in the graphs. In both cases, the perturbation of density decreases and always tends towards zero. The construction of these graphs is only possible when $n \in N$, and depending on the parity of its value. The behavior of the perturbation of density will tend from positive or negative values towards zero. Regardless of the parameters introduced for the following positive constants H_*,A, and B, their influence on the dependency is insignificant.

Conclusion

Thus, we considered a truncated model of the form $F(T,(\nabla T)^2) = AT^n + \omega(T)X$ and showed that in the presence of cosmic dust ($p_0 = 0,\ \delta p = 0$) there is a stable quasi-de Sitter solution of the form $H = H_* + \epsilon e^{-Bt}$. Pure de Sitter solution (13) is allowed also. Investigated model belongs to the class of eternal inflation.

References

1. Otalora G., Saridakis E. *Modified teleparallel gravity with higher-derivative torsion terms.* arXiv, 2016. https://arxiv.org/abs/1605.04599
2. Chervon S., Nikolaev A., Mayorova T., Odintsov S., Oikonomou V. *Kinetic Scalar Curvature Extended $f(R)$ Gravity.* arXiv, 2018. https://arxiv.org/abs/1810.01900

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