UDC 53.1

EXACT COSMOLOGICAL SOLUTIONS OF SCALAR-TORSION GRAVITY WITH A SELF-INTERACTING FIELD OF THE GALILEAN TYPE*

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The equations of cosmological dynamics for inflation models based on scalar-torsion gravity with a non-minimal connection between torsion and scalar field are considered based on a power-law parameterization of the connection between the non-minimal interaction function and the Hubble parameter. Three classes of exact solutions to the equations of cosmological dynamics are obtained, which are the basis for further analysis of the evolution of cosmological perturbations and verification of cosmological models of this type using observational data.

Keywords: Torsion scalar gavity, Friedman's universe, precise cosmological solutions.

ТОЧНЫЕ КОСМОЛОГИЧЕСКИЕ РЕШЕНИЯ СКАЛЯРНО-ТОРСИОННОЙ ГРАВИТАЦИИ С ПОЛЕМ САМОДЕЙСТВИЯ ГАЛИЛЕОННОГО ТИПА

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Рассматриваются уравнения космологической динамики для инфляционных моделей на основе скалярно-торсионной гравитации с неминимальной связью кручения и скалярного поля на основе степенной параметризации связи функции неминимального взаимодействия и параметра Хаббла. Получены три класса точных решений уравнений космологической динамики, которые являются основой для дальнейшего анализа эволюции космологических возмущений и верификации космологических моделей данного типа по наблюдательным данным.

Ключевые слова: Торсионная скалярная гравитация, вселенная Фридмана, точные космологические решения.

PACS: 04.20, 04.50
DOI: 10.17238/issn2226-8812.2024.1.110-115

*Настоящая работа выполнена в рамках Дополнительного соглашения №073-03-2024-060/1 от 13.02.2024 к Соглашению о предоставлении субсидии из федерального бюджета на финансовое обеспечение выполнения государственного задания на оказание государственных услуг (выполнения работ) № 073-03-2024-060 от 18.01.2024, заключенным между ФГБОУ ВО «УлГПУ им. И.Н. Ульянова» и Министерством просвещения Российской Федерации.
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Introduction

The paper considers exact solutions of scalar-torsion gravity with a self-interacting scalar field of the Galilean type with a Lagrangian $L = \left[\frac{M_{pl}^2}{2}F(\phi)T + P(\phi, X) - G(\phi, X)\Box\phi\right]$. The functions are set here: $P = -\omega X + V$, $G = \gamma X$, $F = \left(\frac{H}{\lambda}\right)^n$, where γ, n - constant values, ω, V - functions of the scalar field ϕ, $X := -\frac{(\partial\phi)^2}{2}$, H is the Hubble parameter. We consider the model in Friedmann-Robertson-Walker metric:

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j, \tag{1}$$

which arises from the tetrad $e^A_\mu = diag(1, a(t), a(t), a(t))$, where $a = a(t)$ is the scale factor. Due to the complexity of the equations, the model's solutions are constructed with a different choice of the exponent n, scalar field ϕ and the Hubble parameter H. Thus, three classes of exact solutions are found: 1. When n is arbitrary and a scalar field is constant: $\phi = const$. 2. For $n = 0$, $\omega = \omega_* = const$ (which corresponds to the special case $F = const$). 3. For $n = -1$, $\omega = \omega_*$. Solutions were obtained for additional specified Hubble parameter H: de Sitter solution, power law evolution, exponential-power law evolution. The classes of presented solutions are not cover all possible solutions of the selected model, but this article is limited to studies of these three classes of exact solutions.

1. Exact Solutions of cosmological dynamic equations

The model of torsion-scalar gravity is considered in [1] and is based on the action:

$$S = \int d^4xe\left[\frac{M_{pl}^2}{2}F(\phi)T + P(\phi, X) - G(\phi, X)\Box\phi\right], \tag{2}$$

where $e = det[e^A_\mu] = \sqrt{-g}$. The article [1] is devoted to investigation of solutions in slow-roll regime for a certain selection of functions P, G, F. In our work we search for exact solutions with the following function selection: $P = -\omega X + V$, $G = \gamma X$, where $X := -\frac{(\partial\phi)^2}{2}$, T is a torsion scalar, $M_{pl}^2 = (8\pi G)^{-1}$, ω is a function of the scalar field ϕ, γ is a constant. The system of model equations can be reduced to [2, 3]:

$$V = \frac{3}{2}H\dot{\phi}^3\gamma + \frac{1}{2}\dot{\phi}^2\ddot{\phi}\gamma + \left(3H^2 + (1+n)\dot{H}\right)M_{pl}^2\left(\frac{H}{\lambda}\right)^n, \tag{3}$$

$$\dot{\phi}^2\left(\omega - 3H\dot{\phi}\gamma + \ddot{\phi}\gamma\right) + (1+n)\,2\dot{H}M_{pl}^2\left(\frac{H}{\lambda}\right)^n = 0. \tag{4}$$

The equations include more than two unknowns functions, what gives the right to fix some parameters. Thus, when constructing solutions, the parameter n and the function ϕ are initially selected, and if necessary, the Hubble parameter H can be selected too. Other classes of solutions are built based on the choice of parameters n and H, and if necessary, the function ϕ and ω can be set.

1.1. Solutions for arbitrary n and $\phi = \phi_*$

Substituting the specified parameters in (3)-(4), we obtain solutions:

1. For $n = -1$, the potential takes the form $V = 3HM_{pl}^2\lambda$, and the Hubble parameter is arbitrary.
2. For $H = H_*$, $V = 3H_*^2M_{pl}^2\left(\frac{H_*}{\lambda}\right)^n$.
3. For $n = -1$, $H = H_*$, $V = 3H_*M_{pl}^2\lambda = V_*$.

Note that ω is arbitrary function. The down star means a constant.

1.2. Solutions for arbitrary n and $\phi = \phi_* t + \phi_0$

We insert H in (4) and find ω from resulting expression and then calculate V from (3).

1. For $H = H_*$, $\omega_* = 3H_*\phi_*\gamma$, $V_* = 3H_*\phi_*^3\gamma + 3H_*^{2+n}M_{pl}^2\lambda^{-n}$.
2. For $H = \frac{m}{t}$, $\omega = 2(1+n)M_{pl}^2 m^{1+n}\phi_*^n\lambda^{-n}(\phi-\phi_0)^{-2-n} + 3m\gamma\phi_*^2(\phi-\phi_0)^{-1}$,

$$V = \frac{3}{2}\frac{m\phi_*^4\gamma}{\phi-\phi_0} + (3m-n-1)\frac{m^{n+1}}{\lambda^n}M_{pl}^2\left(\frac{\phi_*}{(\phi-\phi_0)}\right)^{n+2}. \tag{5}$$

3. For $H = \frac{m}{t} + H_0$, $\omega = \frac{2m(1+n)M_{pl}^2}{\lambda^n(\phi-\phi_0)^2}\left(\frac{m\phi_*}{\phi-\phi_0} + H_0\right)^n + 3\phi_*\gamma\left(\frac{m\phi_*}{\phi-\phi_0} + H_0\right)$,

$$V = \frac{3}{2}\phi_*^3\gamma\left(\frac{m\phi_*}{\phi-\phi_0} + H_0\right) + \left(3\left(\frac{m\phi_*}{\phi-\phi_0} + H_0\right)^2 - (1+n)\frac{m\phi_*^2}{(\phi-\phi_0)^2}\right)\frac{M_{pl}^2}{\lambda^n}\left(\frac{m\phi_*}{\phi-\phi_0} + H_0\right)^n. \tag{6}$$

1.3. Solutions for arbitrary n and $\phi = \frac{\phi_*}{2}t^2 + \phi_0 t + \phi_1$

1. For $H = H_*$, $\omega = 3H_*\gamma\left(\pm\sqrt{\phi_0^2 - 2\phi_*(\phi_1-\phi)}\right) - \phi_*\gamma$,

$$V = \frac{3}{2}H_*\gamma\left(\pm\sqrt{\phi_0^2 - 2\phi_*(\phi_1-\phi)}\right)^3 + \frac{1}{2}\phi_*\gamma\left(\phi_0^2 - 2\phi_*(\phi_1-\phi)\right) + 3M_{pl}^2\frac{H_*^{n+2}}{\lambda^n}. \tag{7}$$

2. For $H = \frac{m}{t}$, $\omega = \frac{2(1+n)m^{n+1}M_{pl}^2}{\lambda^n t^{n+2}(\phi_* t+\phi_0)^2} + \frac{3m\gamma\phi_0}{t} + \phi_*\gamma(3m-1)$,

$$V = \frac{3}{2}\frac{m\gamma}{t}(\phi_* t+\phi_0)^3 + \frac{\phi_*\gamma}{2}(\phi_* t+\phi_0)^2 + \frac{M_{pl}^2 m^{n+1}}{\lambda^n t^{n+2}}(3m-n-1). \tag{8}$$

3. For $H = \frac{m}{t} + H_*$, $\omega = \frac{2m(1+n)M_{pl}^2}{t^2(\phi_* t+\phi_0)^2}\left(\frac{m+tH_*}{t\lambda}\right)^n + 3\left(\frac{m}{t}+H_*\right)\gamma(\phi_* t+\phi_0) - \phi_*\gamma$,

$$V = \frac{3}{2}\left(\frac{m}{t}+H_*\right)(\phi_* t+\phi_0)^3\gamma + \frac{1}{2}(\phi_* t+\phi_0)^2\phi_*\gamma + \left(3\left(\frac{m}{t}+H_*\right)^2 - (1+n)\frac{m}{t^2}\right)M_{pl}^2\left(\frac{m+tH_*}{t\lambda}\right)^n. \tag{9}$$

1.4. Solutions for $n = 0$, $\omega = \omega_*$, $H = H_*$

1. $\phi = \phi_*$, $V = 3H_*^2 M_{pl}^2$.
2. $\phi = \frac{e^{3H_* y_*}}{9H_*^2}e^{3H_* t} + \frac{\omega_*}{3H_*\gamma}t + \phi_*$,

$$V = \frac{3}{2}H_*\left(\frac{e^{3H_* y_*}}{3H_*}e^{3H_* t} + \frac{\omega_*}{3H_*\gamma}\right)^3\gamma + 3H_*^2M_{pl}^2 + \frac{1}{2}\left(\frac{e^{3H_* y_*}}{3H_*}e^{3H_* t} + \frac{\omega_*}{3H_*\gamma}\right)^2 e^{3H_* y_*}e^{3H_* t}\gamma. \tag{10}$$

1.5. Solutions for $n = 0$, $\omega = \omega_*$, $H = \frac{m}{t}$

The expression for the potential takes the view

$$V = \frac{3}{2}\frac{m}{t}\dot{\phi}^3\gamma + 3\left(\frac{m}{t}\right)^2 M_{pl}^2 + \frac{1}{2}\dot{\phi}^2\ddot{\phi}\gamma - \frac{m}{t^2}M_{pl}^2. \tag{11}$$

The equation for the scalar field ϕ:

$$\omega_*\dot{\phi}^2 - \frac{3m}{t}\dot{\phi}^3\gamma + \dot{\phi}^2\ddot{\phi}\gamma - \frac{2m}{t^2}M_{pl}^2 = 0. \tag{12}$$

The study of this equation is beyond the scope of this article.

1.6. Solutions for $n = 0$, $\phi = \phi_* t + \phi_0$

$$H = Ce^{\frac{3\phi_*^3\gamma}{2M_{pl}^2}t} + \frac{\omega_*}{3\gamma}, \tag{13}$$

$$V = \frac{3}{2}\left(Ce^{\frac{3\phi_*^3\gamma}{2M_{pl}^2}\frac{\phi-\phi_0}{\phi_*}} + \frac{\omega_*}{3\gamma}\right)\phi_*^3\gamma + 3\left(Ce^{\frac{3\phi_*^3\gamma}{2M_{pl}^2}\frac{\phi-\phi_0}{\phi_*}} + \frac{\omega_*}{3\gamma}\right)^2 M_{pl}^2 + \frac{C3\phi_*^3\gamma}{2M_{pl}^2}e^{\frac{3\phi_*^3\gamma}{2M_{pl}^2}\frac{\phi-\phi_0}{\phi_*}}M_{pl}^2. \tag{14}$$

1.7. Solutions for $n = 0$, $\phi = \frac{\phi_*}{2}t^2 + \phi_0 t + \phi_1$

Equation (4) takes the form

$$\omega_* \left(\phi_* t + \phi_0\right)^2 - 3H\left(\phi_* t + \phi_0\right)^3\gamma + \left(\phi_* t + \phi_0\right)^2\phi_*\gamma + 2\dot{H}M_{pl}^2 = 0. \tag{15}$$

The solution of the equation in terms of Γ function is:

$$H = H_* e^{\frac{3\gamma}{8\phi_* M_{pl}^2}(\phi_* t+\phi_0)^4} + e^{\frac{3\gamma}{8\phi_* M_{pl}^2}(\phi_* t+\phi_0)^4}\frac{\Gamma\left(\frac{3}{4}, \frac{3\gamma}{8\phi_* M_{pl}^2}\left(\phi_* t + \phi_0\right)^4\right)}{4\phi\left(\frac{3\gamma}{8\phi_* M_{pl}^2}\right)^{\frac{3}{4}}}. \tag{16}$$

1.8. Solutions for $n = -1$, $\omega = \omega_*$, $H = H_*$

$$\phi = \frac{e^{3H_* y_*}}{9H_*^2}e^{3H_* t} + \frac{\omega_*}{3H_*\gamma}t + \phi_*, \tag{17}$$

$$V = \frac{3}{2}H_*\left(\frac{e^{3H_* y_*}}{3H_*}e^{3H_* t} + \frac{\omega_*}{3H_*\gamma}\right)^3\gamma + 3H_*\lambda M_{pl}^2 + \frac{1}{2}\left(\frac{e^{3H_* y_*}}{3H_*}e^{3H_* t} + \frac{\omega_*}{3H_*\gamma}\right)^2 e^{3H_* y_*}e^{3H_* t}\gamma. \tag{18}$$

1.9. Solutions for $n = -1$, $\omega = \omega_*$, $H = \frac{m}{t}$

$$\phi = \frac{C_1 t^{3m+1}}{3m+1} - \frac{\gamma}{2\omega_*}t^2 + C_2, \tag{19}$$

$$V = \frac{3}{2}\frac{m}{t}\left(C_1 t^{3m} - \frac{\gamma}{\omega_*}t\right)^3\gamma + \frac{1}{2}\left(C_1 t^{3m} - \frac{\gamma}{\omega_*}t\right)^2\left(3mC_1 t^{3m-1} - \frac{\gamma}{\omega_*}\right)\gamma + 3\frac{m}{t}M_{pl}^2\lambda. \tag{20}$$

For further construction of solutions, it is necessary to choose m. For example, choosing $m = 1$, we get solutions:

$$\phi = \frac{C_1 t^4}{4} - \frac{\gamma}{2\omega_*}t^2 + C_2,\ t = \sqrt{-\frac{\gamma}{\omega_* C_1} \pm \sqrt{\frac{\gamma^2 - 4\left(C_2 - \phi\right)\omega_*}{\omega_* C_1}}}. \tag{21}$$

1.10. Solutions for $n = -1$, $\omega = \omega_*$, $H = \frac{m}{t} + H_*$

Substituting parameters into the original equation (4) and making simple algebra we get

$$\ddot{\phi} = 3\left(\frac{m}{t} + H_*\right)\dot{\phi} - \frac{\omega_*}{\gamma}. \tag{22}$$

The solution with respect to $\dot{\phi}$ in terms of Γ function is:

$$\dot{\phi} = Ct^{3m}e^{3H_* t} + t^{3m}e^{3H_* t}\frac{\omega_*}{\gamma}\frac{\Gamma\left(1-3m, 3H_* t\right)3^{3m-1}\left(H_* t\right)^{3m}}{H_* t^{3m}}. \tag{23}$$

The study of this equation we do not consider in this article.

1.11. Solutions for $n = -1$, $\omega = \omega_*$ and different values ϕ

1. $\phi = \phi_*$, $V = 3HM_{pl}^2\lambda$, where H - any function.
2. $\phi = \phi_* t + \phi_0$, $H = \frac{\omega_*}{3\phi_*\gamma} = H_*$, $V = \frac{\omega_*\gamma\phi_*^2}{2} + \frac{\omega_* M_{pl}^2\lambda}{\gamma\phi_*}$.
3. $\phi = \frac{\phi_*}{2}t^2 + \phi_0 t + \phi_1$, $V = \frac{(\omega_*+\phi_*\gamma)\left(\phi_0^2-2\phi_*(\phi_1-\phi)\right)}{2} + \frac{M_{pl}^2\lambda(\omega_*+\phi_*\gamma)}{\gamma\sqrt{\phi_0^2-2\phi_*(\phi_1-\phi)}}$.

The obtained exact solutions of the equations of cosmological dynamics are the basis for further analysis of the evolution of cosmological disturbances and verification of cosmological models of this type based on observational data.

Conclusion

The article [1] investigates two kind of generalized scalar-torsion gravity in slow-roll regime. The authors of [1] show good correspondence with observation data for chaotic inflation. Our work is devoted to search of exact solutions for scalar-torsion gravity of the Galilean type, given by the action (2). Three classes of exact solutions are found based on the selection of model parameters. Solutions were obtained for additional specified Hubble parameter H: de Sitter solution, power law evolution, exponential-power law evolution. We note that listed Hubble parameters are in good agreement with observation data for Friedmann cosmology. Presented solutions are not cover all possible solutions of the considered model. The search of new solutions and verification of them on observational data is the following task for future investigations.

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Please cite this article in English as:
Chaadaeva T. I., Chervon S. V., Fomin I. V. Exact cosmological solutions of scalar-torsion gravity with a self-interacting field of the Galilean type. *Space, Time and Fundamental Interactions*, 2024, no. 1, pp. 110–115.

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Просьба ссылаться на эту статью следующим образом:
Чаадаева Т. И., Червон С. В., Фомин И. В. Точные космологические решения скалярно-торсионной гравитации с полем самодействия галилеонного типа. *Пространство, время и фундаментальные взаимодействия.* 2024. № 1. С. 110–115.