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AETHERIC CONTROL OVER AXIONICALLY ACTIVE QUASI-ELECTROSTATIC SYSTEM AND LATENT STRUCTURE OF THE ROTATING GÖDEL UNIVERSE*

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Based on the Gödel spacetime platform we consider an example of exact solution to the master equations of the extended Einstein-Maxwell-aether-axion model. This exact solution describes a specific configuration of the unit vector field, associated with the velocity of the dynamic aether, of the axion field in the equilibrium state, of the electric field orthogonal to the axis of the Universe rotation, and of the dust cloud. The main feature of the solution is that the axion, vector and electric fields of this configuration turn out to be hidden from the point of view of the structure of the gravitational field of the rotating Universe.

Keywords: Axion electrodynamics, dynamic aether.

УПРАВЛЕНИЕ АКСИОННО-АКТИВНОЙ КВАЗИ-ЭЛЕКТРОСТАТИЧЕСКОЙ СИСТЕМОЙ, ОСУЩЕСТВЛЯЕМОЕ ДИНАМИЧЕСКИМ ЭФИРОМ, И СКРЫТАЯ СТРУКТУРА ВРАЩАЮЩЕЙСЯ ВСЕЛЕННОЙ ГЁДЕЛЯ

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На основе Гёделевской пространственно-временной платформы мы рассматриваем пример точного решения уравнений аксионного расширения эфирной версии модели Эйнштейна-Максвелла. Это точное решение описывает специфическую конфигурацию, состоящую из нормированного векторного поля, ассоциируемого со скоростью динамического эфира, аксионного поля в равновесном состоянии, электрического поля, ортогонального оси вращения Вселенной, и пылевого облака. Главной особенностью найденного решения является то, что аксионное, векторное и электрическое поля в такой конфигурации оказываются скрытыми с точки зрения структуры гравитационного поля вращающейся Вселенной.

Ключевые слова: Аксионная электродинамика, динамический эфир.

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Introduction

In 1949 Kurt Gödel has published the paper [1], in which a discussion about the cosmological solutions of a new type has been opened. Over the next seventy years various problems of the rotating Universe was studied (see, e.g., [2] for review and references). As a prologue for this short note we would like to recover some classical results.

The metric of the Gödel spacetime includes one parameter a:

$$ds^2 = a^2 \left[dt^2 - dx^2 + \frac{1}{2}e^{2x}dy^2 - dz^2 + 2e^x dtdy\right] . \qquad (1)$$

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The Ricci scalar is constant $R{=}\frac{1}{a^2}$, and five nontrivial Einstein's equations can be written as follows:

$$\kappa T_{00} = \frac{1}{2} - \Lambda a^2 = \kappa T_{02} e^{-x} \,, \quad \kappa T_{11} = \frac{1}{2} + \Lambda a^2 = \kappa T_{33} \,, \quad \frac{3}{2} - \Lambda a^2 = 2\kappa T_{22} e^{-2x} \,. \tag{2}$$

Λ is the cosmological constant, $\kappa{=}8\pi G$, $c{=}1$. In the work [1] Gödel assumed that the stress-energy tensor is of the form $T^{(\rm dust)}_{pq}{=}\rho U_p U_q$ and describes the dust with the velocity four-vector U^j and the mass density ρ; the mentioned quantities are found to be

$$U^j = \frac{1}{a}\, \delta^j_0 \,, \quad U_k = a\left(\delta^0_k + \delta^2_k\, e^x\right) \,, \quad \rho = \frac{1}{\kappa a^2} \,, \quad \Lambda = -\frac{1}{2a^2} \,. \tag{3}$$

The covariant derivative of the velocity four-vector in the spacetime with the metric (1) is the antisymmetric traceless tensor orthogonal to the velocity four-vector U^k

$$\nabla_k U_l = \frac{a}{2} e^x \left(\delta^1_k \delta^2_l - \delta^2_k \delta^1_l\right) = -\nabla_l U_k \,, \quad U^k \nabla_k U_l \equiv DU_l = 0 \,, \quad g^{mn} \nabla_m U_n \equiv \Theta = 0 \,. \tag{4}$$

This means that the flow of the dust in the Gödel Universe is characterized by vanishing acceleration, shear and expansion. Only the antisymmetric vorticity tensor $\omega_{kl} = \nabla_k U_l$ is nonvanishing; it has only one independent component $\omega_{12}{=}\frac{a}{2} e^x$, and its square $\omega^2 \equiv \omega_{kl}\omega^{kl}{=}\frac{1}{a^2}$ is constant. The angular velocity $\omega^m \equiv \omega^{*mn} U_n$, where ω^{*mn} is the pseudotensor dual to the ω_{kl}, happens to be of the form $\omega^m{=}{-}\delta^m_3 \frac{1}{\sqrt{2a^2}}$. In other words, the Universe rotates around the z-axis, and the angular velocity of this rotation is proportional to $\frac{1}{a}$. Also, it is clear that $\nabla_k \left(\rho U^k\right) = 0$ and $\nabla^q T^{(\rm dust)}_{pq} = 0$, i.e., the laws of conservation of the particle number and of the energy density hold.

In this work we consider the spacetime platform of the Gödel type and add three new players to the known model. First, we introduce the unit timelike vector field, which describes the velocity four-vector of the dynamic aether [3]; we use the same symbol U^j for this quantity. Second, we consider the pseudoscalar field, associated with the axionic dark matter (see, e.g., the review [4] for basic ideas and references). Third, we work with the electromagnetic field coupled both to the aether and to the axions. In other words, we add elements of the aetheric extension of the axion electrodynamics. But the principally new feature of our work is that the dynamic aether carries out the control over the evolution of the axion electrodynamic system. What does this mean? As it was established in the works [5], the dynamic aether guides the evolution of the axionically active electrodynamic system, first, via the aetheric effective metric $G^{mn}{=}g^{mn}{+}\mathcal{H} U^m U^n$ with the guiding function of the first type $\mathcal{H}$, incorporated into the kinetic terms of the axions and photons; second, via the guiding function of the second type Φ_* inserted into the potential of the axion field [6]. Generally, the guiding functions $\mathcal{H}$ and Φ_* depend on four differential invariants of the aether velocity four-vector. When we deal with the Gödel spacetime platform, only the square of the vorticity tensor is nonvanishing, $\omega^2{=}\frac{1}{a^2}$. In this context, we take the general model elaborated in [5], reduce it to the Gödel case and solve the obtained master equations.

1. The formalism and its application to the Gödel model

As it was advocated in [5] the aetherically extended action functional of the Einstein-Maxwell-axion theory contains two guiding functions $\mathcal{H}$ and Φ_*. Both guiding functions enter the master equations for the electromagnetic field

$$\nabla_k \left[F^{ik} + \mathcal{H}\left(F^{iq}U^k U_q - F^{kq}U^i U_q\right) + F^{*ik} \frac{\Phi_*}{2\pi} \sin\left(\frac{2\pi\phi}{\Phi_*}\right)\right] = 0 \,. \tag{5}$$

The second subset of the electrodynamic equations remains standard, $\nabla_k F^{*ik}{=}0$. We assume that there exists the electromagnetic field with the potential $A_j{=}\delta^0_j A_0(x){+}\delta^2_j A_2(x)$, for which there are only two nonvanishing components of the Maxwell tensor, F_{10} and F_{12}, and thus $F^{*m1}{=}\frac{1}{2\sqrt{-g}} E^{m1pq} F_{pq}{=}0$. We obtain two nontrivial master equations

$$\frac{d}{dx}\left[2F_{21} + (1-\mathcal{H})\, F_{10} e^x\right] = 0 \,, \quad \frac{d}{dx}\left[F_{10} + e^{-x} F_{21}\right] = 0 \,, \tag{6}$$

the solutions to which depend on the assumption about the value of the guiding function of the first type $\mathcal{H}$. The functions $\mathcal{H}(\omega^2)$ and $\Phi_*(\omega^2)$ happen to be constant since $\omega^2{=}\frac{1}{a^2}{=}$const. When $\mathcal{H} \neq -1$, the appropriate solution to the second equation (6) is $F_{10}{=}e^{-x}F_{12}$; the solution to the first equation is $F_{12} = \mathcal{E}$, where $\mathcal{E}$ is constant. When $\mathcal{H} = -1$, the solutions for F_{10} and F_{12} have the same form, but now $\mathcal{E}$ is not constant obligatory. The four-vector of the electric field $E_p \equiv F_{pk}U^k$ is equal to $E_p = \delta^1_p \frac{\mathcal{E}}{a} e^{-x}$; the four-vector of the magnetic field is vanishing, $B^p \equiv F^{*pq}U_q{=}0$. The first invariant of the electromagnetic field is $F_{mn}F^{mn}{=}2E_pE^p = -\frac{2\mathcal{E}^2}{a^4}e^{-2x}$, and the second (pseudo)invariant vanishes, $F^*_{mn}F^{mn}{=}0$. The stress-energy tensor of the electromagnetic field coupled to the dynamic aether (see [5]) has now five non-zero components, which happen to be proportional to the multiplier $(1{+}\mathcal{H})$:

$$T^{(\mathrm{EM})}_{00} = -T^{(\mathrm{EM})}_{11} = T^{(\mathrm{EM})}_{33} = \frac{(1{+}\mathcal{H})\,\mathcal{E}^2}{2a^2}e^{-2x}\,, \quad T^{(\mathrm{EM})}_{22} = \frac{3\,(1{+}\mathcal{H})\,\mathcal{E}^2}{4a^2}\,, \quad T^{(\mathrm{EM})}_{02} = \frac{(1{+}\mathcal{H})\,\mathcal{E}^2}{2a^2}e^{-x}\,. \tag{7}$$

In the context of the Gödel model the master equation for the axion field transforms into

$$-\frac{1}{a^2}\left(\phi'' + \phi'\right) + \frac{m^2_A \Phi_*}{2\pi}\sin\left(\frac{2\pi\phi}{\Phi_*}\right) = 0\,. \tag{8}$$

For our example we choose the exact solution to this equation in the form $\phi{=}n\Phi_*{=}$const, which corresponds to the localization of the axion field in the minimum of the potential with the serial number n. For this solution all the components of the stress-energy tensor of the axion field take zero values:

$$T^{(\mathrm{A})}_{00} = T^{(\mathrm{A})}_{11} = -T^{(\mathrm{A})}_{33} = \Pi\,, \quad T^{(\mathrm{A})}_{22} = \frac{1}{2}\Pi e^{2x}\,, \quad T^{(\mathrm{A})}_{02} = \Pi e^{x}\,, \quad \Pi \equiv \frac{1}{2}\Psi^2_0 n^2 {\Phi'_*}^2{=}0\,. \tag{9}$$

Master equations for the unit vector field in the context of the Gödel model with $\phi{=}n\Phi_*$, $U^j{=}\frac{1}{a}\delta^j_0$, and the electric field presented above, can be written as follows (see [5]):

$$\nabla_a \mathcal{J}^{aj} = \lambda U^j {+} \kappa\mathcal{H}U^j E_m E^m {+} \kappa\nabla_n\left(\frac{\partial\mathcal{H}}{\partial\omega^2}E_m E^m \omega^{jn}\right)\,. \tag{10}$$

These equations are compatible with the assumption that $U^j{=}\frac{1}{a}\delta^j_0$, if $\frac{\partial\mathcal{H}}{\partial\omega^2}{=}0$, and we obtain that

$$\mathcal{J}^{aj} = (C_1{-}C_3)\omega^{aj}\,, \quad \nabla_a\mathcal{J}^{aj} = -\delta^j_0\frac{(C_1{-}C_3)}{a^3}\,, \quad \lambda = U_j\nabla_a\mathcal{J}^{aj}{-}\kappa\mathcal{H}E_mE^m\,. \tag{11}$$

The stress-energy tensor, associated with the aether velocity field (see [5]) has now the form

$$T^{(\mathrm{U})}_{00}{=}\frac{3}{2}\left(C_1{-}C_3\right)\,, \quad T^{(\mathrm{U})}_{11}{=}\frac{1}{2}\left(C_1{-}C_3\right){=}{-}T^{(\mathrm{U})}_{33}\,, T^{(\mathrm{U})}_{22}{=}\frac{7}{4}\left(C_1{-}C_3\right)e^{2x}\,, \quad T^{(\mathrm{U})}_{02}{=}\frac{3}{2}\left(C_1{-}C_3\right)e^{x}\,. \tag{12}$$

Conclusions

Master equations for the gravitational field (2) contain now the total stress-energy tensor, which has the form

$$\kappa T_{ik} = T^{(\mathrm{U})}_{ik} + \kappa T^{(\mathrm{A})}_{ik} + \kappa T^{(\mathrm{EM})}_{ik} + T^{(\mathrm{INT})}_{ik} + \kappa T^{(\mathrm{dust})}_{ik}\,. \tag{13}$$

Clearly, the contribution of the dynamic aether (12) into the total stress-energy tensor vanishes, i.e., $T^{(\mathrm{U})}_{ik}{=}0$, when two Jacobson constants coincide, $C_1{=}C_3$. Keeping in mind that the sum of these parameters has been estimated after the event GRB 170817A, one could fix the constraint $|C_1| = |C_3| < 10^{-15}$. Since the square of the vorticity tensor is constant and thus the second guiding function also is constant, we obtain that the contribution of the axion field, which is in one of minima of the axion potential, $\phi{=}n\Phi_*$, also vanishes (see (9)). When the guiding function of the first type takes the critical value $\mathcal{H}{=}{-}1$ the contribution of the electromagnetic field (7) also disappears, $\kappa T^{(\mathrm{EM})}_{ik}{=}\,0$. Finally, the part of the total stress-energy tensor indicated in [5] as $T^{(\mathrm{INT})}_{ik}$ also vanishes, when $\frac{d\mathcal{H}}{d\omega^2}{=}0$ at $\mathcal{H}{=}{-}1$. Formally speaking, the condition $\mathcal{H}{=}{-}1$ means that the dielectric permittivity of the aether is equal to zero (see [5]). In other words, if we accept all the mentioned conditions, we obtain that $T_{ik}{=}T^{(\rm dust)}_{ik}$; this means that the solution obtained by Gödel in [1] describes not only the simple dust, but also the specific hidden configuration of the aetheric, axion and electric fields presented above as an example of exact solutions to the master equations of the Einstein-Maxwell-aether-axion model.

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