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РАССЕЯНИЕ ЭЛЕКТРОНОВ НА ЧЕРВОТОЧИНЕ В БОРНОВСКОМ ПРИБЛИЖЕНИИ

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Одним из интригующих гипотетических объектов в физике гравитационного взаимодействия являются червоточины, которые соединяют либо две удаленные области одной и той же Вселенной, либо две разные вселенные. Для того, чтобы червоточины были проходимыми, т.е. позволяли путешественнику безопасно их пересекать, червоточины в общей теории относительности должны быть заполнены экзотической материей. В данной работе показано, что при определенных условиях количество упруго рассеянных на червоточине Эллиса-Бронникова электронов превосходит количество электронов падающего потока. Дополнительные электроны появляются в результате их перехода через червоточину с противоположной стороны катеноида. Иными словами, с помощью потока электронов, направленных на червоточину, в ней создается отрицательное давление. Это позволяет сделать вывод о том, что таким способом можно стабилизировать червоточину без экзотической материи.

Ключевые слова: Рассеяние электронов, функция Грина уравнения Дирака, червоточина, экзотическая материя.

ELECTRON SCATTERING ON A WORMHOLE IN THE BORN APPROXIMATION

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The wormholes which connect either two distant regions of the same Universe or two universes are one of the intriguing hypothetical objects in the physics of gravitational interaction. For wormholes to be traversable, i.e., to allow a traveler to cross them safely, the wormholes must be filled with exotic matter within the framework of general relativity. In this paper it is shown that under certain conditions the number of electrons elastically scattered on an Ellis-Bronnikov wormhole exceeds the number of electrons of the incident flux. The additional electrons appear as a result of their transition through the wormhole from the opposite side of the catenoid. In other words, a negative pressure is created in the wormhole by means of the flux of electrons directed to the wormhole. This allows us to conclude that the wormhole can be stabilized without exotic matter in this way.

Keywords: Electron scattering, Green's function of the Dirac equation, wormhole, exotic matter.

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Introduction

Since John Wheeler introduced the concept "wormhole" in 1957, wormholes have caused a huge interest among relativistic researchers, and till now this interest does not decrease. In particular, it is worth noting the message [1], which states that as of January 19, 2023 the word "wormhole" for all time occurs in the titles of 1614 articles on the resource ArXiv.org, and for the last 12 months 175 articles have this term in their titles. Over the past decades, scientists have done a lot of work to study the nature of wormholes, but there are still many unsolved problems. Nevertheless, scientists remain optimistic and, moreover, researchers are making various efforts to search for astrophysical wormholes

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in the Universe [2]. The main method of wormhole detection is currently gravitational lensing [3,4]. After the Event Horizon Telescope project obtained an image of the shadow of a black hole in the center of the galaxy M87 [5], the method of wormhole detection by its shadow became relevant [6-8]. Researchers are most interested in traversable wormholes. Such wormholes would allow to travel long distances without violating the velocity limit. In GR, for wormholes to be traversable, the presence of exotic matter is required [9-14]. Traversable wormholes that are not filled with an exotic type of matter are possible only in alternative theories of gravity [15-25]. However, unfortunately, one cannot be sure in the absolute correctness of any theory of gravitation for our Universe. Therefore, it is impossible to state definitively that the existence of traversable wormholes requires the presence or absence of exotic matter. Nevertheless, it is concluded in this work within the framework of GR that by means of a particle stream directed to the wormhole it is possible to create such a condition under which the wormhole could remain open without exotic matter. Thus, the following task is set: to find out within the framework of the Born approximation, what are the properties of the flux of electrons which are scattered on the Ellis-Bronnikov wormhole.

To solve the task, let us:

1. determine the Green's function for the Dirac equation in the gravitational field;
2. convert the Dirac equation to the integral form;
3. find the scattering amplitude in the Born approximation;
4. calculate the total cross sections for elastic and inelastic scattering;
5. describe the properties inherent in the character of electron scattering on a wormhole.

A. Green's function

It is known that the Green's function $G_o\left(x-x'\right)$ of the Dirac equation for a free particle

$$i\gamma^{(a)}\partial_a\psi - m\psi = 0$$

satisfies the equation

$$i\gamma^{(a)}\partial_a G_o\left(x-x'\right) - mG_o\left(x-x'\right) = \delta\left(x-x'\right)$$

and has the form

$$G_o\left(x-x'\right) = \frac{1}{(2\pi)^4}\int \frac{\widehat{p}+m}{\widehat{p}^2-m^2} e^{-ip\left(x-x'\right)} d^4p, \tag{1}$$

where $\widehat{p} = \gamma^{(a)}p_a$.

Similarly, we can define the Green's function $G\left(x-x'\right)$ for the Dirac equation in an external gravitational field

$$i\gamma^{\mu}\nabla_{\mu}\psi - m\psi = 0$$

as a solution of the equation

$$i\gamma^{\mu}\nabla_{\mu}G\left(x-x'\right) - mG\left(x-x'\right) = \delta\left(x-x'\right), \tag{2}$$

where $\gamma^{\mu} = \gamma^{(a)}e^{\mu}_{(a)}$, $e^{\mu}_{(a)}$ is an orthonormal tetrad.

To characterize the gravitational field let us define the operator $\widehat{\Gamma}$ by the equality

$$\widehat{\Gamma} = \gamma^{(a)}\left(e^{\mu}_{(a)}\nabla_{\mu} - e_{o}{}^{\mu}_{(a)}\nabla_{o\mu}\right),$$

where $\nabla_{\mu} = \partial_{\mu} + \Gamma_{\mu}$ is a covariant derivative in a gravitational field; $\Gamma_{\mu} = \frac{1}{4}\gamma^{\lambda}\gamma_{\lambda;\mu}$ is a spinor connection in a gravitational field; $\nabla_{o\mu} = \partial_{\mu} + \Gamma_{o\mu}$; $e_{o}{}^{\mu}_{(a)}$ and $\Gamma_{o\mu}$ are orthonormal tetrad and spinor connection in curvilinear coordinates in the absence of external gravitational field. In a Cartesian coordinate system $e_{o}{}^{\mu}_{(a)} = \delta^{\mu}_{a}$ and $\Gamma_{o\mu} = 0$.

Then equation (2) can be rewritten as

$$i\gamma^{(\mu)}\nabla_{o\mu}G\left(x-x^{'}\right)+i\widehat{\Gamma}G\left(x-x^{'}\right)-mG\left(x-x^{'}\right)=\delta\left(x-x^{'}\right).$$

This equation can be represented in the integral form

$$G\left(x-x^{'}\right)=G_o\left(x-x^{'}\right)+i\int G\left(x-x^{''}\right)\widehat{\Gamma}\left(x^{''}\right)G_o\left(x^{''}-x^{'}\right)\sqrt{-g\left(x^{''}\right)}d^4x^{''}, \tag{3}$$

where $g(x)$ is a determinant of the matrix $(g_{\mu\nu})$. It is possible to verify the validity of (3) by acting on the left and right parts of the equality (3) with the operator

$$i\gamma^{(\mu)}\nabla_{o\mu}+i\widehat{\Gamma}(x)-m.$$

In the case when the gravitational disturbance is small, it is possible to solve the equation (3) by applying the method of successive approximations and find the Green's function in the form of a series:

$$G\left(x-x^{'}\right)=G_o\left(x-x^{'}\right)+i\int G_o\left(x-x^{''}\right)\widehat{\Gamma}\left(x^{''}\right)G_o\left(x^{''}-x^{'}\right)\sqrt{-g\left(x^{''}\right)}d^4x^{''}+$$
$$+i^2\int G_o\left(x-x^{'''}\right)\widehat{\Gamma}\left(x^{'''}\right)G_o\left(x^{'''}-x^{''}\right)\widehat{\Gamma}\left(x^{''}\right)G_o\left(x^{''}-x^{'}\right)\sqrt{-g\left(x^{'''}\right)}\sqrt{-g\left(x^{''}\right)}d^4x^{'''}d^4x^{''}+\ldots. \tag{4}$$

B. Dirac Integral Equation

The following **theorem** is valid: Let a gravitational field be given in which the Green's function $G\left(x^{'}-x\right)$ can be expressed as a uniformly convergent series (4). Then at an arbitrary point $x^{'}$ of the space-time region bounded by the closed hypersurface Σ the bispinor describing the motion of an electron in this field can be represented as

$$\psi\left(x^{'}\right)=i\oint G\left(x^{'}-x\right)\gamma^{\mu}(x)\psi(x)\sqrt{-g(x)}dS_{\mu}, \tag{5}$$

where $\oint\ldots\sqrt{-g(x)}dS_{\mu}$ is the integral over the closed three-dimensional hypersurface Σ.

To prove the theorem let us first prove two auxiliary lemmas:

Lemma 1. The equality is valid:

$$\gamma^{\mu}_{;\mu}-\frac{1}{4}\gamma_{\lambda;\mu}\gamma^{\lambda}\gamma^{\mu}-\frac{1}{4}\gamma^{\mu}\gamma^{\lambda}\gamma_{\lambda;\mu}=0,$$

where $\gamma_{\lambda;\mu}=\nabla_{\mu}\gamma_{\lambda}$ is the covariant derivative of γ_{λ}.

Proof. It is known that

$$\gamma_{\lambda}\gamma^{\mu}\gamma^{\lambda}=-2\gamma^{\mu}.$$

Let us take the covariant derivative of both sides of this equality

$$\gamma_{\lambda;\mu}\gamma^{\mu}\gamma^{\lambda}+\gamma_{\lambda}\gamma^{\mu}_{;\mu}\gamma^{\lambda}+\gamma_{\lambda}\gamma^{\mu}\gamma^{\lambda}_{;\mu}=-2\gamma^{\mu}_{;\mu}. \tag{6}$$

Considering the equality

$$\gamma^{\mu}\gamma^{\lambda}+\gamma^{\lambda}\gamma^{\mu}=2g^{\mu\lambda}$$

let us rewrite (6) in the form

$$\gamma_{\lambda;\mu}\left(2g^{\mu\lambda}-\gamma^{\lambda}\gamma^{\mu}\right)+\gamma_{\lambda}\gamma^{\mu}_{;\mu}\gamma^{\lambda}+\left(2\delta^{\mu}_{\lambda}-\gamma^{\mu}\gamma_{\lambda}\right)\gamma^{\lambda}_{;\mu}=-2\gamma^{\mu}_{;\mu}.$$

From where

$$6\gamma^{\mu}_{;\mu}-\gamma_{\lambda;\mu}\gamma^{\lambda}\gamma^{\mu}-\gamma^{\mu}\gamma^{\lambda}\gamma_{\lambda;\mu}+\gamma_{\lambda}\gamma^{\mu}_{;\mu}\gamma^{\lambda}=0.$$

Since

$$\gamma_\lambda \gamma^\mu_{;\mu} \gamma^\lambda = \gamma_\lambda \left(e^\mu_{(a);\mu} \gamma^{(a)} \right) \gamma^\lambda = -2 e^\mu_{(a);\mu} \gamma^{(a)} = -2 \gamma^\mu_{;\mu},$$

we get

$$4\gamma^\mu_{;\mu} - \gamma_{\lambda;\mu} \gamma^\lambda \gamma^\mu - \gamma^\mu \gamma^\lambda \gamma_{\lambda;\mu} = 0.$$

Q.E.D.

Lemma 2. For the Green's function $G\left(x^{'} - x\right)$ the following equality is valid:

$$\gamma^{(0)} G^+ \left(x - x^{'}\right) \gamma^{(0)} = G\left(x^{'} - x\right).$$

Proof. From the equality

$$\gamma^{(0)} \gamma^{(\mu)+} \gamma^{(0)} = \gamma^{(\mu)},$$

for the operators of the form $\widehat{A} = \gamma^{(\mu)} A_\mu$ follows

$$\gamma^{(0)} \widehat{A}^+ \gamma^{(0)} = \widehat{A}, \tag{7}$$

and for the multiplication of operators follows

$$\gamma^{(0)} \left(\widehat{A}\widehat{B}\widehat{C} \right)^+ \gamma^{(0)} = \gamma^{(0)} \widehat{C}^+ \gamma^{(0)} \gamma^{(0)} \widehat{B}^+ \gamma^{(0)} \gamma^{(0)} \widehat{A}^+ \gamma^{(0)} = \widehat{C}\widehat{B}\widehat{A}. \tag{8}$$

Let us substitute the series (4) into the expression $\gamma^{(0)} G^+ \left(x - x^{'}\right) \gamma^{(0)}$. Given the uniform convergence of series (4) and using (7) and (8) we obtain

$$\gamma^{(0)} G^+ \left(x - x^{'}\right) \gamma^{(0)} = G\left(x^{'} - x\right). \tag{9}$$

Here the sign of the argument changes because the sign of the exponent changes at the complex conjugation (1).

Q.E.D.

Proof of the theorem. Consider 4-vector

$$F^\mu = \overline{G}\left(x - x^{'}\right) \gamma^\mu(x) \psi(x). \tag{10}$$

Let us calculate the covariant derivative of this 4-vector

$$\nabla_\mu F^\mu = \partial_\mu F^\mu + \Gamma^\mu_{\mu\nu} F^\nu = \frac{\partial \overline{G}\left(x - x^{'}\right)}{\partial x^\mu} \gamma^\mu(x)\psi(x) + \overline{G}\left(x - x^{'}\right) \frac{\partial \gamma^\mu(x)}{\partial x^\mu} \psi(x) +$$

$$+ \overline{G}\left(x - x^{'}\right) \gamma^\mu(x) \frac{\partial \psi(x)}{\partial x^\mu} + \overline{G}\left(x - x^{'}\right) \Gamma^\mu_{\mu\nu} \gamma^\nu(x) \psi(x) =$$

$$= -i \left(i \nabla_\mu \overline{G} \cdot \gamma^\mu \psi + \overline{G} \cdot i \gamma^\mu \nabla_\mu \psi \right) + \overline{G} \left(\nabla_\mu \gamma^\mu - \overline{\Gamma}_\mu \gamma^\mu - \gamma^\mu \Gamma_\mu \right) \psi.$$

Here spinor connections have the form

$$\Gamma_\mu = \frac{1}{4} \gamma^\lambda \gamma_{\lambda;\mu}, \quad \overline{\Gamma}_\mu = \frac{1}{4} \gamma_{\lambda;\mu} \gamma^\lambda.$$

Then given lemma 1 we obtain

$$\nabla_\mu F^\mu(x) = i \gamma^{(0)} \delta\left(x - x^{'}\right) \psi(x). \tag{11}$$

Substituting (10) and (11) into Gauss' formula

$$\int \nabla_\mu F^\mu \sqrt{-g} d^4x = \oint F^\mu \sqrt{-g} dS_\mu$$

we obtain

$$\psi\left(x^{'}\right)=-i\oint\gamma^{(0)}\overline{G}\left(x^{'}-x\right)\gamma^{\mu}(x)\psi(x)\sqrt{-g(x)}dS_{\mu}. \qquad (12)$$

By using lemma 2 it is possible to transform the equation (12) into the following form:

$$\psi\left(x^{'}\right)=-i\oint G\left(x^{'}-x\right)\gamma^{\mu}(x)\psi(x)\sqrt{-g(x)}dS_{\mu}.$$

Q.E.D.

The closed 3-surface over which integration is performed (5) consists of infinitely distant timelike 3-surfaces and spacelike 3-surfaces $t=t_1$ and $t=t_1^{'}$ $\left(t_1<t_1^{'}\right)$. The Green's function in spacelike directions decreases to zero at infinity. Therefore, the integrals on timelike 3-surfaces will be equal to zero. Then given that

$$\gamma^{\mu}\sqrt{-g}dS_{\mu}=\gamma^{\mu}\sqrt{-g}n_{\mu}dS=\pm\gamma^{\mu}\sqrt{-g}e_{\mu}^{(a)}dS=\pm\gamma^{(a)}\sqrt{-g}dS,$$

we represent the equation (5) in the form

$$\psi\left(x_2\right)=i\int_{t_1}G\left(x_2-x_1\right)\gamma^{(0)}\psi\left(x_1\right)\sqrt{-g\left(x_1\right)}d^3x_1-i\int_{t_1^{'}}G\left(x_2-x_1^{'}\right)\gamma^{(0)}\psi\left(x_1^{'}\right)\sqrt{-g\left(x_1^{'}\right)}d^3x_1^{'}. \qquad (13)$$

The first summand corresponds to electrons with positive energy, and the second summand corresponds to electrons with negative energy. In this form the equation has a more universal character. It can be used not only for the Born approximation, but also in solving the particle scattering problem by the method of Feynman diagrams. In addition, the equation allows a visual interpretation. The Green's function determines the amplitude of the transition of the electron from the initial state with the wave function $\psi\left(x_1\right)$ to the state with the wave function $\psi\left(x_2\right)$ under the influence of the gravitational field.

C. Born Approximation

Let the gravitational field be central and stationary. Then we have

$$\psi(x)=\psi\left(\boldsymbol{r}\right)e^{-i\varepsilon t},$$

where ε is the energy of an electron. In this paper the case of interest is that the function $\psi\left(\boldsymbol{r}\right)$ at $r\to\infty$ has the form of a superposition of plane and spherical divergent waves

$$\psi\left(\boldsymbol{r}\right)\sim ue^{i\boldsymbol{p}_0\boldsymbol{r}}+A\left(\boldsymbol{n}\right)\frac{e^{ipr}}{r},$$

where $\boldsymbol{p}_0$ and $\boldsymbol{p}$ are electron momenta before and after scattering at infinity; u is a bispinor describing the state of an electron with momentum $\boldsymbol{p}_0$; $\boldsymbol{n}=\frac{\boldsymbol{r}}{r}$. Let the initial state of the electron be described by a plane wave

$$\psi\left(x_1\right)=ue^{-i\left(\varepsilon t-\boldsymbol{p}_0\boldsymbol{r}\right)}.$$

Let us substitute (4) into the first summand of equation (13). Let us restrict ourselves to the first approximation. Then after the transformations known from quantum mechanics [26, 27], we obtain the bispinor $A\left(\boldsymbol{n}\right)$ in the form

$$A\left(\boldsymbol{n}\right)=i\frac{1}{4\pi}\left(-\boldsymbol{\gamma}\cdot\boldsymbol{p}+\gamma^{(0)}\varepsilon+m\right)\int\Gamma\left(\boldsymbol{r}^{'}\right)ue^{i\boldsymbol{K}\boldsymbol{r}^{'}}d^3\boldsymbol{r}^{'}, \qquad (14)$$

where $\boldsymbol{K}=\boldsymbol{p}_0-\boldsymbol{p}$. Here the function $\Gamma\left(\boldsymbol{r}\right)$ is defined from the equality

$$\widehat{\Gamma}ue^{-i\left(\varepsilon t-\boldsymbol{p}_0\boldsymbol{r}\right)}=\Gamma\left(\boldsymbol{r}\right)ue^{-i\left(\varepsilon t-\boldsymbol{p}_0\boldsymbol{r}\right)}.$$

Choosing the normalization in the form $\overline{u}u=2m$, $\overline{u}\gamma^{\mu}u=2p^{\mu}$ [28], we find the scattering amplitude and differential cross section for scattering of electrons which possess polarization s in the final state in the form

$$f_s\left(\boldsymbol{n}\right)=\frac{1}{2m}\overline{u}_sA\left(\boldsymbol{n}\right), \qquad (15)$$

$$d\sigma_s = |f_s(\boldsymbol{n})|^2 \, d\Omega, \tag{16}$$

where u_s is a bispinor describing the state of elastically scattered electrons on a wormhole with momentum $\boldsymbol{p} = p_0\boldsymbol{n}$ and polarization s, $d\Omega$ is the solid angle element having direction $\boldsymbol{n}$.

Given that $\overline{u}_s\left(-\boldsymbol{\gamma}\cdot\boldsymbol{p} + \gamma^{(0)}\varepsilon - m\right) = 0$,, from (14) and (15) we obtain

$$f_s(\boldsymbol{n}) = \frac{i}{4\pi}\overline{u}_s \int \Gamma\left(\boldsymbol{r}'\right) u e^{i\boldsymbol{K}\boldsymbol{r}'} d^3\boldsymbol{r}'.$$

By substituting this expression into the (16) and averaging the differential scattering cross section over the polarization states, we obtain it in the form

$$d\sigma = \overline{B}(\boldsymbol{n})\,\overline{u_s\overline{u}_s} B(\boldsymbol{n})\, d\Omega,$$

where $B(\boldsymbol{n}) = \frac{i}{4\pi}\int \Gamma\left(\boldsymbol{r}'\right) u e^{i\boldsymbol{K}\boldsymbol{r}'} d^3\boldsymbol{r}'$; the line above $\overline{u_s\overline{u}_s}$ means averaging along the direction of vector $\boldsymbol{n}$. It can be shown that the density matrix of electrons which are scattered by the solid angle element $d\Omega$,

$$\rho(\boldsymbol{n}) = \overline{u_s(\boldsymbol{n})\,\overline{u}_s(\boldsymbol{n})}$$

and the density matrix of electrons for the incident flux

$$\rho_0 = u\overline{u}$$

are equal. This means that the fraction of electrons of a certain polarization which are scattered in the direction of the vector $\boldsymbol{n}$ is equal to the fraction of electrons of the same polarization in the incident flux. Then for the differential cross section for scattering we have

$$d\sigma = \overline{B}(\boldsymbol{n})\, u\overline{u} B(\boldsymbol{n})\, d\Omega.$$

From where we find that the scattering amplitude of the electron is equal to

$$f(\boldsymbol{n}) = \overline{u}B(\boldsymbol{n}) = \frac{i}{4\pi}\overline{u}\int \Gamma\left(\boldsymbol{r}'\right) u e^{i\boldsymbol{K}\boldsymbol{r}'} d^3\boldsymbol{r}'. \tag{17}$$

D. Electron Scattering on an Ellis-Bronnikov Wormhole

As a scattering center we choose the Ellis-Bronnikov wormhole, the metric of which has the following form

$$ds^2 = dt^2 - \frac{dr^2}{1 - \frac{R^2}{r^2}} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{18}$$

where R is the radius of the throat. In spite of the fact that in spacetime with metric (18) there is no gravitation, the scattering on such a wormhole can be described by the above formulas. To find the operator $\widehat{\Gamma}$ let us write out the nonzero components of the Christoffel symbols for the metric (17):

$$\Gamma^1_{11} = -\frac{R^2}{r^3 - rR^2}, \quad \Gamma^1_{22} = -r\left(1 - \frac{R^2}{r^2}\right), \quad \Gamma^1_{33} = -r\sin^2\theta\left(1 - \frac{R^2}{r^2}\right),$$
$$\Gamma^2_{12} = \frac{1}{r}, \quad \Gamma^2_{33} = -sin\theta\; cos\theta, \quad \Gamma^3_{13} = \frac{1}{r}, \quad \Gamma^3_{23} = ctg\theta, \tag{19}$$

and also choose the orthonormal tetrad in the form

$$e^{\mu}_{(a)} = diag\left(1; \sqrt{1 - \frac{R^2}{r^2}}; \frac{1}{r}; \frac{1}{rsin\theta}\right). \tag{20}$$

Substituting (19) and (20) into the expression for spinor connection

$$\Gamma_\mu = \frac{1}{4} e^{\lambda}_{(b)} e_{(c)\lambda;\mu} \gamma^{(b)} \gamma^{(c)},$$

we find that all its components are equal to zero. Then assuming R is small, for the operator $\widehat{\Gamma}$ we get

$$\widehat{\Gamma} = -\frac{1}{2}\gamma^{(1)} \frac{R^2}{r^2} \frac{\partial}{\partial r}.$$

Then from (17) we obtain the scattering amplitude in the form

$$f\left(\boldsymbol{n}\right) = \frac{1}{4}\pi R^2 p_0^2 \int \frac{\cos\alpha}{r'^2} e^{i\boldsymbol{K}\boldsymbol{r}'} d^3\boldsymbol{r}',$$

where $\alpha = \left(\widehat{\boldsymbol{r}', \boldsymbol{p}_0}\right)$, $\boldsymbol{r}'$ is the radius-vector of an arbitrary point. The integration is performed over such points. The angle α is equal to

$$\alpha = \frac{\pi}{2} - \frac{\theta}{2} - \theta', \text{where } \theta' = \left(\widehat{\boldsymbol{r}', \boldsymbol{K}}\right).$$

Writing down the result of the integration as a series and leaving only the terms that give the summands containing the sixth degree R^6 and below for the total cross section for elastic scattering, we obtain

$$f\left(\boldsymbol{n}\right) = -\frac{\pi}{4} R^3 p_0^2 \cos\frac{\theta}{2} + \frac{i}{2} R^2 p_0 \left(1 - \frac{2}{3} R^2 p_0^2 \sin^2\frac{\theta}{2}\right).$$

As a result, we obtain the total cross section for elastic scattering in the form

$$\sigma_{\text{el}} = \pi R^4 p_0^2 + 2\pi\left(\frac{\pi^2}{16} - \frac{1}{3}\right) R^6 p_0^4.$$

From the optical theorem

$$\sigma_{\text{tot}} = \frac{4\pi}{p_0} \cdot Im f(0).$$

we find that the total cross section for scattering is equal to

$$\sigma_{\text{tot}} = 2\pi R^2.$$

Then the total cross section for inelastic scattering has the form

$$\sigma_{\text{inel}} = 2\pi R^2 - \pi R^4 p_0^2 - 2\pi\left(\frac{\pi^2}{16} - \frac{1}{3}\right) R^6 p_0^4,$$

and the fraction of inelastically scattered electrons is equal to

$$\frac{\sigma_{\text{inel}}}{\sigma_{\text{tot}}} = 1 - \frac{1}{2} R^2 p_0^2 - \left(\frac{\pi^2}{16} - \frac{1}{3}\right) R^4 p_0^4.$$

It follows that at $Rp_0 \approx 1,0923$ electron scattering will be completely elastic, and when the condition $Rp_0 > 1,0923$ is satisfied, the total cross section for inelastic scattering becomes negative.

Conclusion

Within the framework of the approximations used, the following statements can be made:

1. There is no total absorption of electrons by the wormhole. The existence of inelastic reaction channels necessarily entails the existence of elastic scattering.

2. There is a condition on the size of the throat R and on the value of the initial electron momentum p_0, at which the flux of electrons will be completely scattered elastically.

3. There is a condition on the size of the throat R and on the value of the initial electron momentum p_0, at which the total cross section for inelastic scattering becomes negative. The Born approximation used in this study restricts the inelastic scattering channels. The electron creation processes in this problem can be excluded. Therefore, there remains only one possibility to increase the number of elastically scattered electrons which exceeds the number of electrons in the initial flux. Additional electrons can appear only as a result of transition of electrons through the wormhole from the region of space located on the opposite side of the wormhole. Thus, it is possible to create such a flow directed to the wormhole which will create a "negative pressure" in the region containing the wormhole. Perhaps wormholes can be stabilized without exotic matter in this way.

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