УДК 530.12

СТАНДАРТНО-МЕРНЫЕ СИСТЕМЫ ПРЕОБРАЗОВАНИЙ ДЛЯ СПЕЦИАЛЬНОЙ ТЕОРИИ ОТНОСИТЕЛЬНОСТИ

Салума С.[a,1], Тауфик А. Н.[b,2]

[a] Университет Сохага, г. Сохаг, 82524, Египет.

[b] Университет будущего в Египте, г. Новый, 11835, Египет.

Мы представляем комплексную систему, включающую стандартные и размерные системы отсчета. Мы предлагаем теорию, состоящую из трех взаимосвязанных систем преобразований. Стандартно-мерная система преобразований сочетается с размерно-мерной системой преобразований, соответствующей типичному преобразованию Лоренца-Эйнштейна и стандартно-мерной системой. Скорость, с которой движется размерная рамка, играет решающую роль для того, чтобы уравнение сферических волн Максвелла оставалось инвариантным, а переход волновой природы света в природу частиц подчинялся системе преобразований.

Согласованность предложенных стандартно-мерных систем преобразований можно также проверить в следствиях. Мы привели уравнения массы и энергии свободной частицы и обнаружили, что скорость частицы и скорость движущейся рамки являются существенными. Мы также пришли к выводу, что уравнение Шредингера остается инвариантным при предложенных преобразованиях. Дальнейшие следствия для явлений, бросающих вызов специальной относительности, могут быть получены в другом месте.

Ключевые слова: Специальная относительность, системы отсчета, стандартные и размерные величины, основы квантовой механики.

STANDARD-DIMENSIONAL TRANSFORMATION SYSTEMS FOR SPECIAL RELATIVITY

Salouma S.[a,1], Tawfik A. N.[b,2]

[a] Sohag University, Sohag, 82524, Egypt.

[b] Future University in Egypt, Новый Каир, 11835, Egypt.

We introduce a comprehensive framework comprising standard and dimensional reference frames. We suggest a theory composed of three interconnected transformation systems. The standard-dimensional transformation system is combined with a dimensional-dimensional transformation system corresponding to the typical Lorentz–Einstein transformation and the standard-standard system. The velocity at which the dimensional frame moves plays a crucial role so that the Maxwell spherical wave equation remains invariant and the transition of the wave-nature to particle-nature of light becomes subject to the transformation system.

The consistency of the proposed standard-dimensional transformation systems can also be examined in implications. We drove the mass and energy equations of a free particle and found that the particle's velocity and that of the moving frame are essential. We also conclude that the Schrodinger equation remains invariant under the proposed transformation. Further implications to the phenomena challenging special relativity could be carried out elsewhere.

Keywords: Special Relativity; Reference Frames; Standard and Dimensional Values; Foundations of Quantum Mechanics.

PACS: 04.20.-q

DOI: 10.17238/issn2226-8812.2023.3-4.251-270

[1]E-mail: email@mail.ru

[2]E-mail: email2@inbox.ru

A. Introduction

The recently observed violations of some principles of the special theory of relativity urged theoretical interpretation and experimental confirmation to be proposed [1, 2, 3]. The Lorentz invariance violation [4, 5, 6, 7, 8, 9] and deformed special relativity (DSR) [10, 11] including doubly special relativity [12, 13] and modified dispersion relations [14, 15, 16] are a few examples to be recalled. The present script suggests an alternative i) to preserve the special theory of relativity but instead ii) to take into consideration Einstein's original ideas about *"time"* and *"space"* including the distinction between *"position"* and *"place"* [17, 18] and iii) to account for the situations where the observer would be incapable of tracking the trajectory of the motion [18, 19]. To resolve the observed violations of some special relativity principles, the restriction to the standard-standard transformation system is relaxed and a standard-dimensional transformation system shall be proposed.

The trio of event, reference frame, and observer can be categorized according to the observer's capability of monitoring the movement trajectory:

- Observer cannot monitor the movement trajectory: a set of values for time and space is perceived. These are the standard values.

- Observer can monitor the movement trajectory: a set of values for time and space known as dimensional values are then perceived. These are the dimensional values.

Let us assume that the stationary frame has i) the standard values (x', y', z', t') and ii) dimensional values (x, y, z, t). Accordingly, i) the standard values $(\xi', \eta', \zeta', T')$ and ii) dimensional values (ξ, η, ζ, T) can be assigned to the moving frame. From the definition of the inertial reference frame in special relativity [20], the spacetime transformation of an event from an inertial reference frame with velocity υ to an observer in a stationary inertial reference encompasses four types of spacetime transformations.

1. From the standard values of a stationary frame (rest) to the standard values of a moving frame (standard-standard system),

2. From standard values of a stationary frame (rest) to dimensional values of a moving frame (standard-dimensional system),

3. From the dimensional values of a stationary frame (rest) to the standard values of a moving frame (dimensional-standard system), and

4. From the dimensional values of a stationary frame (rest) to the dimensional values of a moving frame (dimensional-dimensional system).

With standard-dimensional, we mean that the frame of standard values is at rest while the frame of dimensional values is moving. Equivalently, with dimensional-standard, we refer to the frame of dimensional values which is at rest, while the frame of standard values moves. For the sake of simplicity, we suggest to combine both transformations into one category, standard-dimensional transformation system. Thus, the transformations in special relativity can be categorized into standard-standard, standard-dimensional, and dimensional-dimensional transformation systems.

In this regard, we emphasize that the standard values within a frame are unaffected by the frame's motion. But upon leaving the frame, the standard values become impacted. Therefore, the relationship between the standard values in a stationary frame and the ones in a moving frame relies on an unknown function $\delta(\upsilon)$.

The standard-dimensional transformation system entails transformation between the standard values conceived by an observer in a stationary frame and the corresponding dimensional values in a moving frame with velocity υ. Moreover, this system also represents the transformation between the dimensional values observed by an observer in a stationary frame and the corresponding standard values in a moving frame with velocity υ.

It should be noticed that the conclusions drawn from the three spacetime transformation systems are based on the following assumptions:

1. All laws of physics are subject to the standard-standard transformation system.

2. The physical laws which are subject to the dimensional-dimensional transformation system is also subject to the standard-dimensional transformation system with an assertion that the transition of the wave-nature to particle-nature of light under the standard-dimensional transformation system [20]. The physical laws which are not subject to the dimensional-dimensional transformation system will be subject to the standard-dimensional transformational system without any additional ingredients.

3. Under a standard-dimensional transformation, the speed of light in free space, c, remains constant across all inertial reference frames regardless their relative motion to the source or each another, and

4. The standard-dimensional transformation occurs in a homogeneous and isotropic spacetime [21].

When referring to subjecting laws to a specific transformation system, it implies that the laws remain consistent across all inertial reference frames within that transformation system.

This paper is structured as follows. The formalism is outlined in section B. The transformations under standard-dimensional system are introduced in section B.1. The velocity transformation and Maxwell spherical wave equation under standard-dimensional transformation system are derived in sections B.1.3 and B.1.4, respectively. The spacetime transformation under dimensional-dimensional system is given in section B.2. The results are discussed in section C. As examples, we first discuss the mass and energy under the standard-dimensional transformation system in section C.1. Then, in section C.2, the Schrödinger equation shall be derived under the standard-dimensional transformation system. Section D is devoted to the conclusions.

B. Formalism

We assume that the reference frame describing an event is denoted by k and has the dimensional values (ξ, η, ζ, T) and the standard values $(\xi', \eta', \zeta', T')$ of the spacetime. Also, let us assume that the reference frame describing an observer is denoted by $\mathbf{K}$ whose dimensional and standard values are (x, y, z, t) and (x', y', z', t'), respectively. In this regards, we emphasize that under the standard-dimensional transformation system, the reference frame k moves with velocity υ in the direction of increasing $\mathbf{x}$-axis relative to the observer's frame $\mathbf{K}$.

B.1. Standard-Dimensional Transformation Systems

In this section, we introduce comprehensive details about the proposed standard-dimensional transformation system. As introduced, such a transformation comprises two kinds. The first one is the translation from the observer's standard values in a stationary frame to the dimensional values in a frame which moves at velocity υ (standard-dimensional). The second one is the translation of the observer's dimensional values in a stationary frame to the standard values in a frame which moves at velocity υ (dimensional-standard).

B.1.1 Spacetime under Standard-Dimensional Transformation System

How the standard values observed by an observer in a stationary frame are to be transformed to the corresponding dimensional values in a moving frame with velocity υ? By using the specification of the standard-standard transformation system as a guide, we can suggest an answer to this question.

First, we express the spacetime transformations under standard-standard system

$$\begin{aligned}
&(i)\;\; x' = \delta(\upsilon)\xi', \\
&(ii)\; y' = \delta(\upsilon)\eta', \\
&(iii) z' = \delta(\upsilon)\zeta', \\
&(iv)\; t' = \delta(\upsilon)T'.
\end{aligned} \tag{B.1}$$

This transformation system is valid regardless whether the frame moves in the direction of increasing x-axis or in the opposite direction. The set of transformations, Eq. (B.1), shall be utilized in deriving the standard-dimensional transformation system.

For an event in k, we assume that the origin points of k and $\mathbf{K}$ were identical, at a dimensional time T_0 and a corresponding standard time in t' in $\mathbf{K}$. We then suggest that a ray of light is emitted from the origin point at the time T_o along the ξ-axis and propagates in the direction of increasing ξ-axis. We also assume that the ray of light has to cover a standard distance ξ' over the time T_1 before it reflects back to its origin point in k. The ray of light returns in time T_2. Based on the assumptions outlined in section A, we can now express the relationships in k as

$$T_2 - T_1 = T_1 - T_0, \qquad \frac{1}{2}\left(T_o + T_2\right) = T_1. \tag{B.2}$$

From the definition of the dimensional values, we then conclude that

$$T = T\left(\xi', \eta', \zeta', T'\right), \quad T_0 = T\left(0,0,0,T_0'\right), \quad T_1 = T\left(\xi', 0, 0, T_1'\right), \quad T_2 = T\left(0,0,0,T_2'\right). \tag{B.3}$$

Also, when a ray of light is emitted along the ξ-axis, the concept of forward and backward directions is then applicable to the standard values of *"place"* in the ξ' dimension. However, this concept does not apply to (η', ζ') dimensions. Then, we obtain

$$T_0' = \frac{t'}{\delta(\upsilon)}, \qquad T_1' = T_0' + \frac{\xi'}{c} = \frac{t'}{\delta(\upsilon)} + \frac{\xi'}{c}, \qquad {T_2}' = T_0' + \frac{\xi'}{c} - \frac{\xi'}{c} = \frac{t'}{\delta(\upsilon)}. \tag{B.4}$$

When substituting these quantities into Eq. (B.2), we get

$$\frac{1}{2}\left[T\left(0,0,0,\frac{t'}{\delta(\upsilon)}\right) + T\left(0,0,0,\frac{t'}{\delta(\upsilon)}\right)\right] = T\left(\xi', 0, 0, \frac{t'}{\delta(\upsilon)} + \frac{\xi'}{c}\right). \tag{B.5}$$

By differentiating Eq. (B.5) with respect to t' and applying $t'/\delta(\upsilon) + \xi'/c = \rho$, we obtain

$$\frac{1}{2}\left[\frac{2}{\delta(\upsilon)}\frac{\partial T}{\partial t'}\right] = \frac{1}{\delta(\upsilon)}\frac{\partial T}{\partial \rho}, \qquad \frac{\partial T}{\partial \rho} = \frac{\partial T}{\partial t'}. \tag{B.6}$$

Now, by differentiating Eq. (B.5) with respect to ξ' and applying $\partial\rho/\partial\xi' = 1/c$, we find

$$\frac{\partial T}{\partial \xi'} + \left(\frac{1}{c}\frac{\partial T}{\partial \rho}\right) = 0, \tag{B.7}$$

in which Eq. (B.6) can be substituted,

$$\frac{\partial T}{\partial \xi'} + \frac{1}{c}\frac{\partial T}{\partial t'} = 0. \tag{B.8}$$

With this regard we emphasize that the origin point of a coordinate system would be any other point of the ray's starting position. As a result, Eq. (B.8) holds true for all possible values of (ξ', η', ζ'). The same conclusion could be drawn for η'- and ζ'-axes. It is important to realize that when observed from the stationary frame, the ray of light consistently travels along these axes, at the speed of light c, and therefore, the variation in both term of Eq. (B.8) vanish,

$$\frac{\partial T}{\partial \eta'} = 0, \qquad \frac{\partial T}{\partial \zeta'} = 0. \tag{B.9}$$

Now we recall expression (iv) in Eq. (B.1), that of the standard-standard transformation system, as well as (B.3), and (B.9). This allows to draw the conclusion that T represents a function, $T = T(t', \xi')$,

$$T = a \cdot t' + b \cdot \xi', \tag{B.10}$$

where a and b are constants. Here, the partial differentials of Eq. (B.10) with respect to t', ξ', separately, lead to

$$\frac{\partial T}{\partial t'} = a, \qquad \frac{\partial T}{\partial \xi'} = b. \tag{B.11}$$

From Eqs. (B.8) and (B.11), b can be related to a,

$$b \;\; = \;\; -\frac{1}{c}a. \tag{B.12}$$

whose substitution into Eq. (B.10) leads to

$$T \;\; = \;\; a\left(t' - \frac{1}{c}\xi'\right). \tag{B.13}$$

By substituting $t' = x'/c$ and $\xi' = x'/\delta(\upsilon)$, whose relationship reads $t' = \delta(\upsilon)\xi'/c$, into Eq. (B.13) and applying $\xi = cT$, we get

$$\xi = a\left[\delta(\upsilon) - 1\right]\xi'. \tag{B.14}$$

When taking into account that $\xi' = x' - \upsilon t'/\delta(\upsilon)$, which expresses the motion of the reference frame k, we conclude that

$$x' = \frac{\xi\delta(\upsilon)}{a\left[\delta(\upsilon) - 1\right]} + \upsilon t'. \tag{B.15}$$

Now, we consider that a ray of light is emitted along the η-axis in the direction of its increment and covers the standard distance η'. This leads to

$$T = a\left(t' - \frac{\eta'}{c}\right). \tag{B.16}$$

Given that $t' = y'/c$ and $\eta' = y'/\delta(\upsilon)$, Eq. (B.16) can be reexpressed as

$$T = \frac{a}{c}\left(1 - \frac{1}{\delta(\upsilon)}\right)y'. \tag{B.17}$$

By using $\eta = cT$, we can derive that

$$y' = \frac{\delta^2(\upsilon)}{a\left[\delta(\upsilon) - 1\right]}\frac{\eta}{\delta(\upsilon)}. \tag{B.18}$$

Similarly, for a ray of light which is emitted along ζ-axis in the direction of its increment and intersects the axis at the standard distance ζ', then

$$T = a\left(t' - \frac{\zeta'}{c}\right). \tag{B.19}$$

Given that $t' = z'/c$ and $\zeta' = z'/\delta(\upsilon)$ and by using $\zeta = cT$, we obtain

$$z' = \frac{\delta^2(\upsilon)}{a\left[\delta(\upsilon) - 1\right]}\frac{\zeta}{\delta(\upsilon)}. \tag{B.20}$$

We are now able to determine t'. Since $\xi' = x'/\delta(\upsilon)$ and $\xi' = cT'$, we find that

$$T' = \frac{x'}{c\delta(\upsilon)}. \tag{B.21}$$

Then from Eq. (B.15) and Eq. (B.21), we conclude that

$$T' = \frac{1}{\delta(\upsilon)} \left(\frac{\xi}{c} \frac{\delta(\upsilon)}{a\,[\delta(\upsilon)-1]} + \frac{\upsilon t'}{c} \right). \tag{B.22}$$

By substituting $\xi/c = T$ and $t'/\delta(\upsilon) = T'$ into Eq. (B.22), we obtain

$$\frac{T\delta(\upsilon)}{a\,[\delta(\upsilon)-1]} = t'\left(1 - \frac{\upsilon}{c}\right), \tag{B.23}$$

$$t' = \frac{\delta^2(\upsilon)}{a\,[\delta(\upsilon)-1]} \frac{T}{(1-\frac{\upsilon}{c})\delta(\upsilon)}. \tag{B.24}$$

Let $\varphi(\upsilon) = \delta^2(\upsilon)/a(\delta(\upsilon)-1)$. Then, we get

$$t' = \frac{\varphi(\upsilon)}{(1-\frac{\upsilon}{c})\delta(\upsilon)} T. \tag{B.25}$$

Now, we are ready to derive the spacetime transformations under standard-dimensional system. This comprises various cases as follows.

(i) When the frame k moves with velocity υ in the direction of increasing x-axis

$$\begin{aligned}
&(i)\ \ x' = \varphi(\upsilon)\left[\frac{\xi}{\delta(\upsilon)} + \frac{\upsilon}{(1-\frac{\upsilon}{c})\,\delta(\upsilon)}T\right],\\
&(ii)\ y' = \varphi(\upsilon)\frac{\eta}{\delta(\upsilon)},\\
&(iii) z' = \varphi(\upsilon)\frac{\zeta}{\delta(\upsilon)},\\
&(iv)\ t' = \frac{\varphi(\upsilon)}{(1-\frac{\upsilon}{c})\delta(\upsilon)}T.
\end{aligned} \tag{B.26}$$

Alternatively, when substituting Eq. (B.25) into Eqs. (B.15), (B.18), and (B.20), we obtain the same transformations as in Eq. (B.26).

(ii) When the frame k moves at velocity υ in the direction of increasing x-axis (inverse transformation), we find that

$$\begin{aligned}
&(i)\ \ \xi = \frac{\delta(\upsilon)}{\varphi(\upsilon)}\left[x' - (\upsilon t')\right],\\
&(ii)\ \eta = \frac{\delta(\upsilon)}{\varphi(\upsilon)}y',\\
&(iii)\zeta = \frac{\delta(\upsilon)}{\varphi(\upsilon)}z',\\
&(iv)\ T = \frac{\delta(\upsilon)(1-\frac{\upsilon}{c})}{\varphi(\upsilon)}t'.
\end{aligned} \tag{B.27}$$

(iii) In a specific scenario that the frame $\bar{k}$ moves at velocity υ in the opposite direction of increasing x-axis, an observation can be made but within the frame $\bar{\mathbf{K}}$. When assuming that the standard values of the frame $\bar{\mathbf{K}}$ read $(\bar{x}', \bar{y}', \bar{z}', \bar{t}')$ and the dimensional values of the frame $\bar{k}$ read $(\bar{\xi}, \bar{\eta}, \bar{\zeta}, \bar{T})$, then, we get

$$\begin{aligned}
&(i)\ \ \bar{x}' = \varphi(-\upsilon)\left[\frac{\bar{\xi}}{\delta(-\upsilon)} - \frac{\upsilon}{(1+\frac{\upsilon}{c})\,\delta(-\upsilon)}\bar{T}\right],\\
&(ii)\ \bar{y}' = \varphi(-\upsilon)\frac{\bar{\eta}}{\delta(-\upsilon)},\\
&(iii)\bar{z}' = \varphi(-\upsilon)\frac{\bar{\zeta}}{\delta(-\upsilon)},\\
&(iv)\ \bar{t}' = \varphi(-\upsilon)\frac{\bar{T}}{(1+\frac{\upsilon}{c})\delta(-\upsilon)},
\end{aligned} \tag{B.28}$$

where $\varphi(-\upsilon) = \delta^2(-\upsilon)/\{\bar{a}\,[\delta(-\upsilon) - 1]\}$.

(iv) The fourth case deals with the inverse transformation of the previous case. When the frame $\bar{k}$ moves at velocity υ in the opposite direction of increasing x-axis, we then find that

$$\begin{aligned}
&(i)\ \ \bar{\xi} = \frac{\delta(-\upsilon)}{\varphi(-\upsilon)}\left[\bar{x'} + \left(\upsilon\bar{t'}\right)\right],\\
&(ii)\ \bar{\eta} = \frac{\delta(-\upsilon)}{\varphi(-\upsilon)}\bar{y'},\\
&(iii)\bar{\zeta} = \frac{\delta(-\upsilon)}{\varphi(-\upsilon)}\bar{z'},\\
&(iv)\ \bar{T} = \frac{\delta(-\upsilon)}{\varphi(-\upsilon)}\left(1 + \frac{\upsilon}{c}\right)\bar{t'}.
\end{aligned} \tag{B.29}$$

Now, we are capable of describing different scenarios and even the wave front of the light pulse.

1. From the four sets of standard-dimensional transformation systems, Eqs. (B.26) - (B.29), we can define two scenarios as follows.

   (a) First scenario: if the dimensional values of the frame k are equivalent to the dimensional values of the frame $\bar{k}$, i.e., $(\xi, \eta, \zeta, T) = (\bar{\xi}, \bar{\eta}, \bar{\zeta}, \bar{T})$, then from Eqs. (B.27) and (B.29), the inverse transformations, we obtain that

$$\frac{\delta(\upsilon)}{\varphi(\upsilon)}\left[x' - (\upsilon t')\right] = \frac{\delta(-\upsilon)}{\varphi(-\upsilon)}\left[\bar{x'} + \left(\upsilon\bar{t'}\right)\right]. \tag{B.30}$$

   Since $(\xi, \eta, \zeta, T = (\bar{\xi}, \bar{\eta}, \bar{\zeta}, \bar{T}))$, it follows that $(x', y', z', t') = (\bar{x}', \bar{y}', \bar{z}', \bar{t}')$. Because the only possible condition which invalidates forward and backward directions should be $\upsilon = 0$. Then, the concept of direction along and opposite the frame of reference disappears, i.e., $(x', y', z', t')_{\upsilon=0} = (\bar{x}', \bar{y}', \bar{z}', \bar{t}')_{\upsilon=0}$, and we obtain

$$\frac{\delta(\upsilon)}{\varphi(\upsilon)}x' = \frac{\delta(-\upsilon)}{\varphi(-\upsilon)}x'. \tag{B.31}$$

   From $\delta(\upsilon)_{\upsilon=0} = \delta(-\upsilon)_{\upsilon=0} = 1$, it is obvious to conclude that,

$$\varphi(\upsilon) = \varphi(-\upsilon), \tag{B.32}$$

   i.e., equivalent forward and backward motion.

   (b) Second scenario: if the standard values of the frame **K** are equivalent to the dimensional values of the frame $\bar{k}$. Then from Eqs. (B.26) and (B.29), we obtain

$$\frac{\delta(-\upsilon)}{\varphi(-\upsilon)}\left(\bar{x'} + \left(\upsilon\bar{t'}\right)\right) = \varphi(\upsilon)\left[\frac{\xi}{\delta(\upsilon)} + \frac{\upsilon}{\left(1 - \frac{\upsilon}{c}\right)\delta(\upsilon)}T\right]. \tag{B.33}$$

   The condition $\left(\bar{x'}, \bar{y'}, \bar{z'}, \bar{t'}\right)_{\upsilon=0} = (\xi, \eta, \zeta, T)_{\upsilon=0}$ allows to rewrite Eqs. (B.33) as

$$\frac{\delta(-\upsilon)}{\varphi(-\upsilon)}\bar{x'} = \frac{\varphi(\upsilon)}{\delta(\upsilon)}\bar{x'}, \tag{B.34}$$

   which in turn leads to

$$\varphi(\upsilon)\varphi(-\upsilon) = \delta(\upsilon)\delta(-\upsilon). \tag{B.35}$$

   From $\delta(\upsilon)_{\upsilon=0} = \delta(-\upsilon)_{\upsilon=0} = 1$, Eq. (B.35) can be rewritten as

$$\varphi(\upsilon)\varphi(-\upsilon) = 1. \tag{B.36}$$

   Then, with Eq. (B.32), we obtain

$$\varphi(\upsilon) = \varphi(-\upsilon) = 1, \tag{B.37}$$

   i.e., a normalization condition.

2. To describe the wave front of the light pulse, an alternative set of the spacetime transformations must be formulated. Based on the information provided so far and the conditions outlined in section A, the wave front of the light pulse is then described as

$$\xi = cT, \tag{B.38}$$
$$x' = ct'. \tag{B.39}$$

By substituting (iv) of Eqs. (B.1) into Eq. (B.39), we get

$$x' = c\delta(v)T'. \tag{B.40}$$

Also, by substituting Eq. (B.38) and Eq. (B.40) into (i) of Eq. (B.26),

$$\delta^2(v)T' = T + \frac{\frac{v}{c}}{1-\frac{v}{c}}T. \tag{B.41}$$

From the assumption 3 which was introduced in section A and the propagation of ray of light in a straight line, we notice that $T = T'$. Therefore, with simple mathematical operations, we get

$$\delta(v) = \frac{1}{\sqrt{1-\frac{v}{c}}}. \tag{B.42}$$

Similarly, it is easy to find that

$$\delta(-v) = \frac{1}{\sqrt{1+\frac{v}{c}}}. \tag{B.43}$$

Now, we can apply this to the set of spacetime transformations under the standard-dimensional system of special relativity, Eq. (B.26),

$$\begin{aligned} &(i)\ x' = \frac{\xi}{\delta(v)} + v\delta(v)T,\\ &(ii)\ y' = \frac{\eta}{\delta(v)},\\ &(iii)\ z' = \frac{\zeta}{\delta(v)},\\ &(iv)\ t' = \delta(v)T \end{aligned} \quad . \tag{B.44}$$

B.1.2 Spacetime under Dimensional-Standard Transformation System

The second case derives a set of equations for the transformation of spacetime under dimensional-standard system. This translates the dimensional values observed by an observer in a stationary frame to the corresponding standard values in a moving frame with velocity v. According to Eq. (B.44), when the moving frame k is stationary within the standard-dimensional transformation system, the dimensional values (ξ, η, ζ, T) become equivalent to the standard values (x', y', z', t'). Consequently, when the moving frame k is at rest, the dimensional values (x, y, z, t) become equivalent to the standard values $(\xi', \eta', \zeta', T')$. Therefore, we consider the process of establishing a stationary state for the moving frame k by aligning it with respect to (x, y, z, t) and $(\xi', \eta', \zeta', T')$. To this end, another frame is utilized, This is a frame moving at velocity v in the opposite direction of increasing x-axis within the standard-dimensional system. The motion of this frame impacts both **K** and k. Consequently, to establish the values $(\xi', \eta', \zeta', T')$ and (x, y, z, t) at the rest of the standard-dimensional transformation system without impacting both frames, k and **K**, we suggest to follow a procedure as follows.

- The values $(\xi', \eta', \zeta', T')$ when transformed to the corresponding ones in the frame **K** become (x', y', z', t'). The latter values within the frame **K** have motion in the direction of increasing $x-$axes.

- Since the frame **K** is at rest, we can apply a frame with velocity υ in the opposite direction of increasing x-axes whose values are (x', y', z', t'). By transformation, this now frame get the values (x, y, z, t).

  In this regard, we draw two conclusions.

- The concept of opposite direction of velocity does appear in x and t dimensions only, because the movements of y and z are not influenced by the concept of opposite direction of increasing x-axis. Hence, the standard values x' and t' are affected by the value of $\delta(-\upsilon)$, while the standard values y' and z' are affected by the value of $\delta(\upsilon)$. In addition, the values of x' and t' represent the observed values outside the frame, while the standard values y' and z' represent the observed values within the frame.

- Based on the foregoing information and the spacetime transformations in a generalized form under the standard-standard system, it is possible to deduce that the standard values observed outside the frame are equal to the standard values observed inside the frame multiplied by the velocity function, υ-function.

  Accordingly, the dimensional values x and t can be expressed as

$$\begin{aligned} x &= \frac{x'}{\delta(-\upsilon)}, \\ t &= \frac{t'}{\delta(-\upsilon)}, \end{aligned} \tag{B.45}$$

while the values y and z read

$$\begin{aligned} y &= \delta(\upsilon)y', \\ z &= \delta(\upsilon)z'. \end{aligned} \tag{B.46}$$

From the standard-standard transformation system, Eqs. (B.45) and (B.46), we can summarize both kinds of spacetime transformation under standard-dimensional system,

1. the set of equations describing the spacetime transformation under standard-dimensional system when the frame k moves with velocity υ in the direction of increasing x-axis is given as

$$\begin{aligned} &(i)\ \ x = \frac{\delta(\upsilon)}{\delta(-\upsilon)}\xi', \\ &(ii)\ y = \delta^2(\upsilon)\eta', \\ &(iii) z = \delta^2(\upsilon)\zeta', \\ &(iv)\ t = \frac{\delta(\upsilon)}{\delta(-\upsilon)}T', \end{aligned} \tag{B.47}$$

2. we can now derive the spacetime transformations under standard-dimensional system by repeating the same steps when the frame k moves with velocity υ in the opposite direction of increasing x-axis

$$\begin{aligned} &(i)\ \ \bar{x} = \frac{\delta(-\upsilon)}{\delta(\upsilon)}\bar{\xi}', \\ &(ii)\ \bar{y} = \delta^2(-\upsilon)\bar{\eta}', \\ &(iii) \bar{z} = \delta^2(-\upsilon)\bar{\zeta}', \\ &(iv)\ \bar{t} = \frac{\delta(-\upsilon)}{\delta(\upsilon)}\bar{T}'. \end{aligned} \tag{B.48}$$

Obviously, we find that the velocity plays a crucial role in the proposed theory of the standard-dimensional transformation system. In the section that follows, we derive the velocity transformations under the same system.

B.1.3 Velocity under Standard-Dimensional Transformation System

To derive the velocity under standard-dimensional transformation system in frame k which moves with velocity υ in the direction of increasing x-axis, we have two scenarios.

1. **The first scenario** deals with the transformation between the observer's standard values in a stationary frame and the corresponding dimensional values in a moving frame. By differentiating the expression (i) of Eqs. (B.44) with respect to t', we get

$$\frac{dx'}{dt'} = \frac{1}{\delta(\upsilon)}\left(\frac{d\xi}{dT}\frac{dT}{dt'}\right) + \delta(\upsilon)\upsilon\frac{dT}{dt'}, \tag{B.49}$$

and by differentiating the expression (iv) of Eqs. (B.44) with respect to t', we obtain

$$1 = \delta(\upsilon)\frac{dT}{dt'} \Rightarrow \frac{dT}{dt'} = \frac{1}{\delta(\upsilon)}. \tag{B.50}$$

The substitution of Eq. (B.50) into Eq. (B.49) leads to

$$\frac{dx'}{dt'} = \frac{1}{\delta^2(\upsilon)}\frac{d\xi}{dT} + \upsilon. \tag{B.51}$$

Hence, the velocity in the x-axis reads

$$u_{x'} = \frac{u_\xi}{\delta^2(\upsilon)} + \upsilon, \tag{B.52}$$

where $u_{x'} = dx'/dt'$. Likewise, the velocity transformations in y- and z-axis can be expressed, respectively, as

$$\begin{aligned} u_{y'} &= \frac{u_\eta}{\delta^2(\upsilon)}, \\ u_{z'} &= \frac{u_\zeta}{\delta^2(\upsilon)}. \end{aligned} \tag{B.53}$$

2. **Second scenario** elaborates the transformation between the observer's dimensional values in a stationary frame and the corresponding standard values in a moving frame.

By differentiating the expression (i) of Eqs. (B.47) with respect to t, we find that

$$\frac{dx}{dt} = \frac{\delta(\upsilon)}{\delta(-\upsilon)}\left(\frac{d\xi'}{dT'}\frac{dT'}{dt}\right). \tag{B.54}$$

Also, by differentiating the expression (iv) of Eqs. (B.47) with respect to t, we obtain

$$1 = \frac{\delta(\upsilon)}{\delta(-\upsilon)}\frac{dT'}{dt} \rightarrow \frac{dT'}{dt} = \frac{\delta(-\upsilon)}{\delta(\upsilon)}. \tag{B.55}$$

Then, the substitution of Eq. (B.55) into Eq. (B.54) leads to

$$\frac{dx}{dt} = \frac{d\xi'}{dT'}. \tag{B.56}$$

Hence, the velocity transformation in the x-axis reads

$$u_x = \frac{dx}{dt} = u_{\xi'}. \tag{B.57}$$

Likewise, the velocity transformations in y- and z-axis can be respectively expressed as

$$u_y = \frac{dy}{dt} = \delta(\upsilon)\delta(-\upsilon)u_{\eta'}, \tag{B.58}$$

$$u_z = \frac{dy}{dt} = \delta(\upsilon)\delta(-\upsilon)u_{\zeta'}. \tag{B.59}$$

Now, we conclude this section by studying the Maxwell spherical wave equation in spacetime transformations under standard-dimensional system, section B.1.4.

B.1.4 Maxwell Spherical Wave Equation under Standard-Dimensional Transformation System

To study the Maxwell spherical wave equation under standard-dimensional transformation system [22, 23], we assume that the frame k moves at velocity υ in a specific direction relative to an observer in the frame $\mathbf{K}$. Furthermore, we suggest that at time $t = T = 0$, the origins and axes of both frames, k and $\mathbf{K}$, coincide. Also, we assume that a light pulse, which was emitted at time $t = T = 0$ in the frame $\mathbf{K}$ has a spherical wave front which is characterized by

$$x^2 + y^2 + z^2 = c^2 t^2, \qquad (x')^2 + (y')^2 + (z')^2 = c^2 (t')^2. \tag{B.60}$$

Based on the assumptions outlined in section A, the wave front of light pulse when observed from the perspective of the frame k has two scenarios.

First scenario considers the difference between the observer's standard values in a stationary frame and the corresponding dimensional values in a moving frame at velocity υ along the direction of increasing x-axis. In view of Eqs. (B.44) and the second part of Eq. (B.60), we obtain

$$\left[\frac{\xi}{\delta(\upsilon)} + (\delta(\upsilon)\upsilon T)\right]^2 + \left[\frac{\eta}{\delta(\upsilon)}\right]^2 + \left[\frac{\zeta}{\delta(\upsilon)}\right]^2 = c^2 \left[\delta(\upsilon) T\right]^2. \tag{B.61}$$

Therefore,

$$\xi^2 + \eta^2 + \zeta^2 + \upsilon^2 \delta^4(\upsilon) T^2 + 2\delta^2(\upsilon)\xi\upsilon T - c^2\delta^4(\upsilon)T^2 = 0. \tag{B.62}$$

Let us assume $\xi^2 + \eta^2 + \zeta^2 = c^2 T^2$. Then, we find that

$$\left[\frac{c^2}{\delta^4(\upsilon)} + \upsilon^2 - c^2\right]\delta^2(\upsilon) T = -2\xi\upsilon. \tag{B.63}$$

With some substitutions, we reach at the wave front

$$\xi \quad = \quad cT. \tag{B.64}$$

Second scenario considers the difference between the observer's dimensional values in a stationary frame and the corresponding standard values in a moving frame at velocity υ along the direction of increasing x-axis. In view of Eqs. (B.47) and the first part of Eq. (B.60), we obtain

$$\left[\frac{\delta(\upsilon)}{\delta(-\upsilon)}\right]^2 (\xi')^2 + \delta^4(\upsilon)\,(\eta')^2 + \delta^4(\upsilon)\,(\zeta')^2 = \left[\frac{\delta(\upsilon)}{\delta(-\upsilon)}\right]^2 c^2\,(T')^2. \tag{B.65}$$

Therefore,

$$\frac{\left(1+\frac{\upsilon}{c}\right)^2 (\xi')^2}{\left(1+\frac{\upsilon}{c}\right)\left(1-\frac{\upsilon}{c}\right)} + \frac{(\eta')^2}{\left(1+\frac{\upsilon}{c}\right)\left(1-\frac{\upsilon}{c}\right)} + \frac{(\zeta')^2}{\left(1+\frac{\upsilon}{c}\right)\left(1-\frac{\upsilon}{c}\right)} \quad = \quad \frac{\left(1+\frac{\upsilon}{c}\right)^2 c^2\,(T')^2}{\left(1+\frac{\upsilon}{c}\right)\left(1-\frac{\upsilon}{c}\right)}. \tag{B.66}$$

This implies that

$$(\xi')^2 + \frac{\upsilon^2}{c^2}(\xi')^2 + 2\frac{\upsilon}{c}(\xi')^2 + (\eta')^2 + (\zeta')^2 = c^2 (T')^2 + \upsilon^2 (T')^2 + 2\upsilon c\,(T')^2. \tag{B.67}$$

With the assumption that $(\xi')^2 + (\eta')^2 + (\zeta')^2 = c^2(T')^2$, the wave front becomes

$$\xi' = cT'. \tag{B.68}$$

To summarize the findings of this section, we recall the analytical observation that the Maxwell spherical wave equation which was determined under spacetime transformations of the standard-dimensional-type is found invariant, i.e., the Maxwell spherical wave equation remains unchanged although this type of spacetime transformations. Furthermore, this constancy can be interpreted as being conditioned to the motion within the frame k which follows a straight line parallel to the x-axis with velocity equal to the speed of light, c. Also, according to principles of optics, which assert that the phenomena resulting from the propagation of light in straight lines support the hypothesis that light has particle-nature, this type of motion seems to be consistent with the particle's behavior. As a result of these observations and by consulting refs. [24, 25], we conclude that the spacetime transformations under standard-dimensional system adheres to the second assumption outlined in section A.

B.2. Spacetime under Dimensional-Dimensional Transformation System

In this section, we delve into the dimensional-dimensional transformation system. As the dimensional values are in the moving frame, we divide the moving observer's frame into two possible directions. The first case considers that the frame k moves at velocity υ *in the same direction* of increasing x-axis. The second case counts for frame k which moves at velocity υ *in the opposite direction* of increasing x-axis.

Case 1: Spacetime transformations under dimensional-dimensional system in the frame k which moves at velocity υ in the same direction of increasing x-axis. In view of Eqs. (B.44), Eqs. (B.45) and Eqs. (B.46), we obtain,

$$\begin{aligned} &(i)\ \ x = \frac{\xi}{\delta(\upsilon)\delta(-\upsilon)} + \frac{\delta(\upsilon)}{\delta(-\upsilon)}\upsilon T,\\ &(ii)\ y = \eta,\\ &(iii) z = \zeta,\\ &(iv)\ t = \frac{\delta(\upsilon)}{\delta(-\upsilon)}T. \end{aligned} \tag{B.69}$$

From the identities $\delta(\upsilon) = 1/\sqrt{1-\frac{\upsilon}{c}}$ and $\delta(-\upsilon) = 1/\sqrt{1+\frac{\upsilon}{c}}$, the corresponding set of dimensional-dimensional transformation equations becomes

$$\begin{aligned} &(i)\ x = \frac{\xi}{\sqrt{1-\frac{\upsilon^2}{c^2}}} + \frac{\delta(\upsilon)}{\delta(-\upsilon)}\upsilon T,\\ &(ii)\ y = \eta,\\ &(iii)\ z = \zeta,\\ &(iv)\ t = \frac{T}{\left(1-\frac{\upsilon}{c}\right)/\sqrt{1-\frac{\upsilon^2}{c^2}}}. \end{aligned} \tag{B.70}$$

From the third assumption introduced in section A, we notice that the wave front of the light pulse can be described by

$$\xi = cT, \qquad\qquad t = \frac{x}{c}. \tag{B.71}$$

Let $\gamma = 1/\sqrt{1-\frac{\upsilon^2}{c^2}}$, then we get

$$T = \gamma\left[t - t\frac{\upsilon}{c}\right]. \tag{B.72}$$

From $t = \frac{x}{c}$ for the light pulse [22], we find that

$$T = \gamma\left[t - \frac{\upsilon x}{c^2}\right]. \tag{B.73}$$

Then, in view of expression (i) in Eq. (B.70), we obtain

$$x = \frac{\xi}{\gamma} + \frac{\delta(\upsilon)}{\delta(-\upsilon)}\upsilon T. \tag{B.74}$$

By substituting Eq. (B.73) into (**??**), we get

$$x - \frac{xv^2}{c^2} = \frac{1}{\gamma}(\xi + vT). \tag{B.75}$$

Therefore, we suggest that

$$x = \gamma(\xi + vT). \tag{B.76}$$

Also, by substituting Eq. (**??**) into (B.73), we arrive at

$$t = \frac{T}{\gamma} + \frac{xv}{c^2}. \tag{B.77}$$

Hence, we derive t

$$\begin{aligned} t &= \frac{T}{\gamma} + \frac{v}{c^2}\left(\frac{\xi}{\gamma} + vt\right), \\ t - \frac{tv^2}{c^2} &= \frac{T}{\gamma} + \frac{v}{c^2}\frac{\xi}{\gamma}, \\ t &= \gamma\left(T + \frac{v\xi}{c^2}\right). \end{aligned} \tag{B.78}$$

This allows to summarize the dimensional-dimensional transformation system as

$$\begin{aligned} &(i) \;\; x = \gamma(\xi + vT), \\ &(ii) \; y = \eta, \\ &(iii) z = \zeta, \\ &(iv) \; t = \gamma\left(T + \frac{v\xi}{c^2}\right). \end{aligned} \tag{B.79}$$

It is obvious that Eq. (B.79) represents the inverse Lorentz–Einstein spacetime transformations in special relativity [18, 22].

Case 2: The spacetime transformations under dimensional-dimensional system in the case that the frame k moves at velocity v in the opposite direction to increasing x-axis. Based in Eq. (B.79), we obtain

$$\begin{aligned} &(i) \;\; \bar{x} = \gamma\left(\bar{\xi} - v\bar{T}\right), \\ &(ii) \; \bar{y} = \bar{\eta}, \\ &(iii) \bar{z} = \bar{\zeta}, \\ &(iv) \; \bar{t} = \gamma\left(\bar{T} - \frac{v\bar{\xi}}{c^2}\right). \end{aligned} \tag{B.80}$$

Again, these transformations are the Lorentz–Einstein spacetime transformations in special relativity [18, 22].

We then conclude that in both directions of the moving frame k, the resulting spacetime transformations under dimensional-dimensional system are the Lorentz–Einstein spacetime transformations in special relativity.

C. Consistency Results

C.1. Mass and Energy Equations under Standard-Dimensional Transformation System

In this section, we refer to the standard-dimensional transformation system as the equations which govern the transformation of mass and energy in the spacetime. This relates the observed standard values measured by an observer in the rest frame to the corresponding dimensional values in the moving frame at velocity v.

Case I: Moving frame k at velocity υ in the direction of increasing x-axis. Assuming a particle has a mass m_g in the frame $\mathbf{K}$. According to Newton's second law motion in the frame $\mathbf{K}$, this particle is affected by a force f [26, 27]

$$f = m_g \frac{d^2 x'}{d(t')^2}. \tag{C.1}$$

By differentiating Eq. (B.52) with respect to t', we get

$$\frac{d^2 x'}{d(t')^2} = \frac{d}{dt'}\left(\frac{\frac{d\xi}{dT}}{\delta^2(\upsilon)} + \upsilon\right) = \frac{dT}{dt'}\frac{d}{dT}\left(\frac{\frac{d\xi}{dT}}{\delta^2(\upsilon)} + \upsilon\right). \tag{C.2}$$

By using $t' = \delta(\upsilon)\,T$, we then obtain

$$\frac{d^2 x'}{d(t')^2} = \frac{1}{\delta(\upsilon)}\frac{d}{dT}\left(\frac{\frac{d\xi}{dT}}{\delta^2(\upsilon)} + \upsilon\right) = \frac{1}{\delta^3(\upsilon)}\frac{d^2\xi}{dT^2}. \tag{C.3}$$

Accordingly, we obtain that

$$\frac{d^2 x'}{d(t')^2} = \left[1 - \left(\frac{\upsilon}{c}\right)\right]^{\frac{3}{2}} \frac{d^2\xi}{dT^2}. \tag{C.4}$$

If we assume that the particle was designated through that moment as being momentarily at rest from an observer's point of view in frame $\mathbf{K}$ and by using Eq. (B.52), then, the particle's velocity in relation to frame k at that time becomes

$$\upsilon = \frac{\frac{d\xi}{dT}}{\frac{d\xi}{dT}\frac{1}{c} - 1}. \tag{C.5}$$

By substituting Eq. (C.5) into Eq. (C.4), we obtain

$$\frac{d^2 x'}{dt'^2} = \left[1 - \left(\frac{d\xi}{dT}\frac{1}{c}\right)\right]^{\frac{-3}{2}} \frac{d^2\xi}{dT^2} \tag{C.6}$$

Also by substituting Eq. (C.6) into Eq. (**??**), the equation of motion in the frame k reads

$$f = m_g\left[1 - \left(\frac{d\xi}{dT}\frac{1}{c}\right)\right]^{-\frac{3}{2}} \frac{du_\xi}{dT}. \tag{C.7}$$

The velocity of the particle in the frame k, is given as $d\xi/dT = u_\xi$. Then, Eq. (C.7) can be rewritten as

$$f = m_g\left(1 - \frac{u_\xi}{c}\right)^{-\frac{3}{2}} \frac{du_\xi}{dT}. \tag{C.8}$$

In the special case that $u_\xi = \upsilon$, we arrive as

$$f = m_g\left(1 - \frac{\upsilon}{c}\right)^{\frac{-3}{2}} \frac{du_\xi}{dT}. \tag{C.9}$$

Back to the general situation, we suggest that

$$\frac{d}{dT}\frac{2m_g c}{\left(1 - \frac{u_\xi}{c}\right)^{\frac{1}{2}}} = \frac{m_g}{\left(1 - \frac{u_\xi}{c}\right)^{\frac{3}{2}}}\frac{du_\xi}{dT}. \tag{C.10}$$

From Eqs. (C.8) and (C.10), we can reformulate the force f under standard-dimensional transformation system as

$$f = \frac{d}{dT}\left(2m_g \frac{\left(\frac{c}{u_\xi}\right)u_\xi}{\sqrt{1 - \frac{u_\xi}{c}}}\right). \tag{C.11}$$

Consequently, we can derive the rate of the change in the momentum, i.e., the force. Now, the mass of a particle in the frame k can be determined,

$$M_k = \left(\frac{c}{u_\xi}\right)\frac{2m_g}{\sqrt{1 - \frac{u_\xi}{c}}}. \tag{C.12}$$

Case II: the frame k moves at velocity υ in the opposite direction to the direction of increasing x-axis. Similarly, by using the same method as in the previous case, but with transformations in the opposite direction of the increasing x-axis, we determine the mass of a particle in the frame $\bar{k}$, whose velocity becomes $u_{\bar{\xi}}$,

$$M_{\bar{k}} = \left(\frac{c}{u_{\bar{\xi}}}\right) \frac{2m_g}{\sqrt{1 - \frac{u_{\bar{\xi}}}{c}}}. \tag{C.13}$$

Consequently, the generalization of the mass transformation can be suggested as

$$M_{gd} = \left(\frac{c}{u}\right) \frac{2m_{gd}}{\sqrt{1 - \frac{u}{c}}}, \tag{C.14}$$

where u is the particle's velocity, m_{gd} is the mass of the particle in the rest frame, while M_{gd} is the mass of the particle in the moving frame. This specific transformation of mass should not be mixed with the relativistic mass as dictated by special relativity.

Now, we can derive a relationship between mass and energy.

$$\frac{d}{dx'} \left(2m_{gd} \frac{\left(2c^2 - cu\right)}{\sqrt{1 - \frac{u}{c}}} \right) = m_{gd} \frac{\frac{du}{dt}}{\left(1 - \frac{u}{c}\right)^{\frac{3}{2}}}. \tag{C.15}$$

Since $f = (m_{gd} \cdot du/dt) / \left(1 - \frac{u}{c}\right)^{\frac{3}{2}}$, hence

$$d \left(2m_{gd} \frac{\left(2c^2 - cu\right)}{\sqrt{1 - \frac{u}{c}}} \right) = f dx'. \tag{C.16}$$

By integrating both sides, we derive the work

$$2m_{gd} \frac{\left(2c^2 - cu\right)}{\sqrt{1 - \frac{u}{c}}} = \int f dx' \equiv \texttt{Work}. \tag{C.17}$$

From the work-energy theorem [27], the energy can be obtained

$$\texttt{Energy} = \frac{2m_{gd} \left(\frac{c}{u}\right) \left(2cu - u^2\right)}{\sqrt{1 - \frac{u}{c}}}. \tag{C.18}$$

Since $M_{gd} = 2m_{gd} \left(\frac{c}{u}\right) / \sqrt{1 - \frac{u}{c}}$, the energy can be expressed as

$$\texttt{Energy} = M_{gd} \left(2cu - u^2\right). \tag{C.19}$$

We conclude that the energy in the moving frame is proportional to u. Its positiveness is conditioned to $2c > u$, which is obviously fulfilled in special relativity.

C.2. Schrödinger Equation under Standard-Dimensional Transformation System

The Schrödinger equation [28, 29, 30] can be expressed in the scenario where the spacetime transformation relates the observer's standard values in the stationary frame **K** to the corresponding dimensional values in the frame k which moves at velocity υ in the direction of increasing x-axis. We assume that the potential V can be solely determined by the *"position"*.

$$i\hbar \frac{\partial}{\partial t'} \psi\left(x', t'\right) = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial \left(x'\right)^2} \psi\left(x', t'\right) + V\left(x'\right) \psi\left(x', t'\right). \tag{C.20}$$

The solutions $\psi\left(x', t'\right) = A \exp[i(\kappa \cdot x' - \omega \cdot t')]$ satisfy Schrödinger equation, where A is a constant, $\kappa = \omega / c$ is wave number and ω is angular frequency. Then, Eqs. (B.44) leads to

$$\psi\left(x', t'\right) = \psi\left(\frac{\xi}{\delta(\upsilon)} + \delta(\upsilon)\upsilon T, \delta(\upsilon) T\right) = A \exp\left\{ i \left[\left(\frac{\kappa}{\delta(\upsilon)}\right) \xi - \left[\omega\delta(\upsilon) - \kappa\delta(\upsilon)\upsilon\right] T \right] \right\}. \tag{C.21}$$

When assuming $a = \kappa/\delta(\upsilon)$ and $b = \omega\delta(\upsilon) - \kappa\delta(\upsilon)\upsilon$, we obtain

$$\psi\left(x', t'\right) = A \exp\left[i\left(a\xi - bT\right)\right]. \tag{C.22}$$

Now b can be reexpressed as

$$\begin{aligned} b &= \omega\delta(\upsilon) - \kappa\delta(\upsilon)\upsilon = \kappa\delta(\upsilon)(c-\upsilon) = \frac{\kappa}{\delta(\upsilon)}\delta(\upsilon)(c-\upsilon)\delta(\upsilon) \\ &= ac\left(1 - \frac{\upsilon}{c}\right)\frac{1}{1-\frac{\upsilon}{c}}. \end{aligned}$$

Hence, we find that b is scaled by the speed of light, c,

$$b = c\,a. \tag{C.23}$$

From Eq. (C.23), we realize that $A\exp[i(a\xi - bT)]$ solves the Schrödinger equation in ξ and T,

$$A \cdot \exp\left[i\left(a\xi - b \cdot T\right)\right] = \psi\left(\xi, T\right), \tag{C.24}$$

i.e., the solution represents an optical wave function of ξ and T. From Eq. (C.22) and Eq. (C.24).

$$\psi\left(x', t'\right) = \psi\left(\xi, T\right). \tag{C.25}$$

By substituting Eq. (C.25) and (iv) in Eq. (B.44), we obtain

$$i\hbar\frac{\partial}{\partial t'}\psi\left(x', t'\right) = \frac{1}{\delta(\upsilon)}i\hbar\frac{\partial}{\partial T}\psi\left(\xi, T\right). \tag{C.26}$$

By differentiating (i) in Eq. (B.44) with respect to ξ, we find that

$$\frac{\partial x\prime}{\partial\xi} = \frac{1}{\delta(\upsilon)}. \tag{C.27}$$

Hence,

$$\frac{\partial\omega}{\partial\xi} = \frac{1}{\delta(\upsilon)}\frac{\partial\omega}{\partial x'}, \tag{C.28}$$

$$\frac{\partial^2\omega}{\partial\xi^2} = \frac{1}{\delta^2(\upsilon)}\frac{\partial^2\omega}{\partial(x')^2}. \tag{C.29}$$

According to the transformations introduced in section C.1, we conclude that the Newton's second law, $f = m_g[d^2x'/d(t')^2]$, in the frame $\mathbf{K}$ becomes

$$f = m_g\left[1 - \left(\frac{d\xi}{dT}\frac{1}{c}\right)\right]^{-\frac{3}{2}}\frac{du_\xi}{dT}. \tag{C.30}$$

Consequently, we express the transformation of the mass m from the frame $\mathbf{K}$ to the frame k, i.e.,

$$m|_{\texttt{frame}\,\mathbf{K}} \to m\left(1 - \frac{\upsilon}{c}\right)^{-\frac{3}{2}}\Bigg|_{\texttt{frame}\,k}. \tag{C.31}$$

From Eqs. (C.26), (**??**), and (C.31), we get

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial\left(x'\right)^2}\psi\left(x', t'\right) = \frac{-\hbar^2}{2m}\delta(\upsilon)^{-3}\frac{\partial^2}{\partial\xi^2}\psi\left(\xi, T\right)\delta^2(\upsilon). \tag{C.32}$$

According to the relation $x' = \xi/\delta(\upsilon) + \delta(\upsilon)\upsilon T$, we note that x' can be represented as a summation of two parts. The first part is $\xi/\delta(\upsilon)$, which is the value assigned to x' in the frame k. The second part is

$\delta(v)vT$, which is the value that results from the movement of frame k. Therefore, the potential energy $V(x')$ in the frame $\mathbf{K}$ can be related to the potential energy $V(\xi/\delta(v))$ in the frame k,

$$V\left(x'\right) = \frac{1}{\delta(v)} V(\xi). \tag{C.33}$$

From Eqs. (C.26), (C.32), and (C.33), we get

$$i\hbar \frac{\partial}{\partial T} \psi(\xi, T) = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial \xi^2} \psi(\xi, T) + V(\xi) \psi(\xi, T). \tag{C.34}$$

We conclude that the Schrödinger equation under standard-dimensional transformation system is invariant. Also, this finding obviously demonstrates that the second assumption in section A is upheld by the Schrödinger equation.

D. Conclusions and Outlook

The recently observed violations of some principles of special relativity such as Lorentz invariance violation and modified dispersion relations urged theoretical interpretations. We suggest alternative transformation systems preserving the current version of special theory but taking into consideration that Einstein's original ideas about *"time"* and *"space"* and also his distinction between *"position"* and *"place"*. The proposed theory extends the standard-standard transformation system. The standard-dimensional transformation system suggested combines the dimensional-dimensional transformation system which corresponds to the typical Lorentz–Einstein transformation and the standard-standard transformation system.

The key ingredient is whether the observer able to monitor the movement trajectory (standard values time and space are perceived) or not (dimensional values time and space are then perceived). Accordingly, standard-standard transformation system from the standard values of a stationary frame to the standard values of a moving frame, standard-dimensional transformation system from standard values of a stationary frame to dimensional values of a moving frame or vice verse, i.e., dimensional-standard transformation system and finally dimensional-dimensional transformation system from the dimensional values of a stationary frame to the dimensional values of a moving frame can be defined. We conclude that the standard-dimensional transformation system combines both dimensional-dimensional system, which is typical to the Lorentz–Einstein transformation and the standard-standard transformation system. In this regard, we find that the relationship between the standard values in a stationary frame and the ones in a moving frame seems to rely on a velocity function $\delta(v)$. This means that the velocity at which the dimensional frame moves plays a crucial role. Therefore, even the velocity transformations of this velocity under the standard-dimensional transformation system are found $\delta(v)$-dependent.

Under standard-dimensional transformation system, we conclude that the Maxwell spherical wave equation remains unchanged although this type of spacetime transformations. This observed invariance is conditioned to the motion within the moving frame in the rays are parallel to the x-axis and moving at speed of light. We conclude that straight rays manifest the particle-nature of light. We found that the dimensional-dimensional transformation system is identical to the typical Lorentz–Einstein spacetime transformation. Also, we conclude that the spacetime transformations under standard-dimensional system adheres the assumption that the physical laws are straightforwardly subject to the standard-dimensional transformational system.

For the mass and energy equations of a free particle under the standard-dimensional transformation system, we conclude that both quantities in both standard and dimensional frames depend on the velocity of the free particle and that of the moving frame. Another implication, we discussed, is the Schrödinger equation under standard-dimensional transformation system. We conclude that the Schrödinger equation remains invariant, which means that assumption of speed of light is upheld by the Schrödinger equation. Further implications, especially where special relativity is challenged, shall be carried out elsewhere

Acknowledgment

AT acknowledges the generous support by the Egyptian Center for Theoretical Physics (ECTP) and World Laboratory for Cosmology And Particle Physics (WLCAPP) at the Future University in Egypt (FUE)!

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this published article!

Dataset Availability

All data generated or analyzed during this study are included in this published article. The data used to support the findings of this study are included within the published article and properly cited! All of the material is owned by the authors.

Funding

The authors declare that this research received no specific grants from any funding agency in the public, commercial, or not-for-profit sectors.

Competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Список литературы/References

1. A. K. A. Maciel and J. Tiomno. On Experiments To Detect Possible Weak Violations of Special Relativity. *Phys. Rev. Lett.*, 55:143, 1985. 10.17238/issn2226-8812.2023.3-4.251-27010.1103/PhysRevLett.55.143. [Erratum: Phys.Rev.Lett. 57, 1382 (1986)].
2. Gian Luigi Fogli, E. Lisi, A. Marrone, and G. Scioscia. Testing violations of special and general relativity through the energy dependence of muon-neutrino <—> tau-neutrino oscillations in the Super-Kamiokande atmospheric neutrino experiment. *Phys. Rev. D*, 60:053006, 1999. 10.17238/issn2226-8812.2023.3-4.251-27010.1103/PhysRevD.60.053006.
3. T. Jacobson, Stefano Liberati, and D. Mattingly. A Strong astrophysical constraint on the violation of special relativity by quantum gravity. *Nature*, 424:1019–1021, 2003. 10.17238/issn2226-8812.2023.3-4.251-27010.1038/nature01882.
4. S. L. Dubovsky and P. G. Tinyakov. Violation of Lorentz invariance and neutral component of UHECR. *Astropart. Phys.*, 18:89–96, 2002. 10.17238/issn2226-8812.2023.3-4.251-27010.1016/S0927-6505(02)00114-7.
5. O. W. Greenberg. CPT violation implies violation of Lorentz invariance. *Phys. Rev. Lett.*, 89: 231602, 2002. 10.17238/issn2226-8812.2023.3-4.251-27010.1103/PhysRevLett.89.231602.
6. G. Amelino-Camelia, John R. Ellis, N. E. Mavromatos, and Dimitri V. Nanopoulos. Distance measurement and wave dispersion in a Liouville string approach to quantum gravity. *Int. J. Mod. Phys. A*, 12:607–624, 1997. 10.17238/issn2226-8812.2023.3-4.251-27010.1142/S0217751X97000566.
7. Giovanni Amelino-Camelia. Testable scenario for relativity with minimum length. *Phys. Lett. B*, 510:255–263, 2001. 10.17238/issn2226-8812.2023.3-4.251-27010.1016/S0370-2693(01)00506-8.
8. P. Adamson et al. A Search for Lorentz Invariance and CPT Violation with the MINOS

Far Detector. *Phys. Rev. Lett.*, 105:151601, 2010. 10.17238/issn2226-8812.2023.3-4.251-27010.1103/PhysRevLett.105.151601.

9. A. Tawfik, H. Magdy, and A. Farag Ali. Lorentz Invariance Violation and Generalized Uncertainty Principle. *Phys. Part. Nucl. Lett.*, 13:59–68, 2016. 10.17238/issn2226-8812.2023.3-4.251-27010.1134/S1547477116010179.
10. Florian Girelli, Tomasz Konopka, Jerzy Kowalski-Glikman, and Etera R. Livine. The Free particle in deformed special relativity. *Phys. Rev. D*, 73:045009, 2006. 10.17238/issn2226-8812.2023.3-4.251-27010.1103/PhysRevD.73.045009.
11. Sergio Doplicher, Klaus Fredenhagen, and John E. Roberts. Spacetime quantization induced by classical gravity. *Physics Letters B*, 331(1):39–44, 1994. 10.17238/issn2226-8812.2023.3-4.251-270https://doi.org/10.1016/0370-2693(94)90940-7. URL `https://www.sciencedirect.com/science/article/pii/0370269394909407`.
12. J. Kowalski-Glikman and S. Nowak. Doubly special relativity theories as different bases of kappa Poincare algebra. *Phys. Lett. B*, 539:126–132, 2002. 10.17238/issn2226-8812.2023.3-4.251-27010.1016/S0370-2693(02)02063-4.
13. Giovanni Amelino-Camelia. Doubly special relativity. *Nature*, 418:34–35, 2002. 10.17238/issn2226-8812.2023.3-4.251-27010.1038/418034a.
14. A. Tawfik. Impacts of Generalized Uncertainty Principle on Black Hole Thermodynamics and Salecker-Wigner Inequalities. *JCAP*, 07:040, 2013. 10.17238/issn2226-8812.2023.3-4.251-27010.1088/1475-7516/2013/07/040.
15. Abdel Nasser Tawfik and Eiman Abou El Dahab. Corrections to entropy and thermodynamics of charged black hole using generalized uncertainty principle. *Int. J. Mod. Phys. A*, 30(09):1550030, 2015. 10.17238/issn2226-8812.2023.3-4.251-27010.1142/S0217751X1550030X.
16. Abdel Nasser Tawfik and Abdel Magied Diab. Black Hole Corrections due to Minimal Length and Modified Dispersion Relation. *Int. J. Mod. Phys. A*, 30(12):1550059, 2015. 10.17238/issn2226-8812.2023.3-4.251-27010.1142/S0217751X15500591.
17. J.B. Marion. *Classical Dynamics of Particles and Systems*. Academic Press International Edition. Academic Press, 1965. URL `https://books.google.com.eg/books?id=-FjvAAAAMAAJ`.
18. A. Einstein. *The Special and General Theory: The Special and General Theory by Albert Einstein: The Special and General Theory - Einstein's Groundbreaking Work*. Prabhat Prakashan, 1948. ISBN 9788184302288. URL `https://books.google.com.eg/books?id=CbwwBQAAQBAJ`.
19. Rachel E Scherr, Peter S Shaffer, and Stamatis Vokos. Student understanding of time in special relativity: Simultaneity and reference frames. *American Journal of Physics*, 69(S1):S24–S35, 2001.
20. Youshan Dai and Liang Dai. New derivation of space-time linear transformation between inertial frames based on the principle of relativity. *Results in Physics*, 19:103552, 2020. ISSN 2211-3797. 10.17238/issn2226-8812.2023.3-4.251-270https://doi.org/10.1016/j.rinp.2020.103552. URL `https://www.sciencedirect.com/science/article/pii/S2211379720319975`.
21. Frank Verheest. On the linearity of the generalized Lorentz transformation. *American Journal of Physics*, 90(6):425–429, 06 2022. ISSN 0002-9505. 10.17238/issn2226-8812.2023.3-4.251-27010.1119/10.0010234. URL `https://doi.org/10.1119/10.0010234`.
22. S.T. Thornton and A. Rex. *Modern Physics for Scientists and Engineers*. Cengage Learning, 2012. ISBN 9781133712237. URL `https://books.google.com.eg/books?id=pWAKAAAAQBAJ`.
23. R.H. Dicke and J.P. Wittke. *Introduction to Quantum Mechanics, by Robert H. Dicke and James P. Wittke*. Addison-Wesley Publishing Company, 1961. URL `https://books.google.com.eg/books?id=tbHCswEACAAJ`.
24. E. Huggins. *Physics 2000*. Moose Mountain Digital Press, 1999. ISBN 9780970783639. URL `https://books.google.com.eg/books?id=03HvwQEACAAJ`.

25. M. Katz. *Introduction to Geometrical Optics*. G - Reference,Information and Interdisciplinary Subjects Series. World Scientific, 2002. ISBN 9789812382245. URL https://books.google.com.eg/books?id=agsOmxYsyCIC.
26. I. Newton, A. Motte, and N.W. Chittenden. *Newton's Principia: The Mathematical Principles of Natural Philosophy*. Nineteenth Century Collections Online (NCCO): Science, Technology, and Medicine: 1780-1925. Geo. P. Putnam, 1850. URL https://books.google.com.eg/books?id=N-hHAQAAMAAJ.
27. H. Goldstein, C.P. Poole, and J.L. Safko. *Classical Mechanics*. Addison-Wesley series in physics. Addison Wesley, 2002. ISBN 9780321188977. URL https://books.google.com.eg/books?id=EE-wQgAACAAJ.
28. D. McMahon. *Quantum Mechanics Demystified*. Demystified series. McGraw-Hill Education, 2005. ISBN 9780071486705. URL https://books.google.com.eg/books?id=VAq1Z4GClUoC.
29. D.J. Griffiths and D.F. Schroeter. *Introduction to Quantum Mechanics*. Cambridge University Press, 2018. ISBN 9781107189638. URL https://books.google.com.eg/books?id=82FjDwAAQBAJ.
30. N. Ashby and S.C. Miller. *Principles of Modern Physics*. Holden-Day series in physics. Holden-Day, 1970. URL https://books.google.com.eg/books?id=my4bAQAAIAAJ.

Авторы

Салем Салума,Математическое отделение, факультет естественных наук, Университет Сохаг, Сохаг 82524, Египет
E-mail: email@mail.ru

Абдель Насер Тауфик, Университет будущего в Египте (FUE), Пятое поселение, 11835 Новый Каир, Египет
E-mail: email2@inbox.ru

Просьба ссылаться на эту статью следующим образом:

Салума С., Тауфик А. Н. Стандартно-мерные системы преобразований для специальной теории относительности. *Пространство, время и фундаментальные взаимодействия.* 2023. № 3-4. С. 251–270.

Authors

Salem Salouma,Mathematics Department, Faculty of Science, Sohag University, Sohag 82524, Egypt
E-mail: email@mail.ru

Tawfik Abdel Nasser, Future University in Egypt (FUE), Fifth Settlement, 11835 New Cairo, Egypt.
E-mail: email2@inbox.ru

Please cite this article in English as:

Salouma S., Tawfik A. N. Standard-Dimensional Transformation Systems for Special Relativity. *Space, Time and Fundamental Interactions*, 2023, no. 3-4, pp. 251–270.