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ВОЗВРАЩЕНИЕ ПОЛЯ ПРОКА [*]

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Новая версия старой единой теории поля Вейля, основанная на строгом применении принципа калибровочной инвариантности, приводит к появлению массивного векторного поля, которое имеет полностью геометрическую природу и может быть интерпретировано, при некоторых условиях, как поле Прока. Мы исследуем некоторые возможные последствия присутствия этого поля в современном астрофизическом сценарии.

Ключевые слова: Единые теории поля, Поле Прока, Темная материя.

THE COMING BACK OF THE PROCA FIELD

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A new version of the old Weyl's unified field theory based on a strict application of the principle of gauge invariance leads to the appearance of a massive vector field, which has an entirely geometric nature and can be interpreted, under some conditions, as the Proca field. We investigate some possible consequences of the presence of this field in the modern astrophysical scenario.

Keywords: Unified field theories, Proca field, Dark matter.

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Introduction

Proca's theory appeared in 1936 in the context of the classical and quantum electrodynamics of a massive photon. It was proposed as a model to describe the weak interaction and the motion of spin-1 mesons [1]. Despite its interesting and original ideas, the model did not survive too long and soon gave away to other proposals, being subsequently almost forgotten. However, the quanta of the Proca field would reappear as the massive gauge bosons Z , W^{+} and W_{-}in the standard model of particle physics [2]. Recently, there has appeared new motivation to reconsider the role that Proca's field can play in other areas of physics. Mention to the Proca field mainly appears coming from two distinct contexts. Firstly, the idea of the presence of a massive vector field in the universe is motivated by current research in astrophysics and cosmology, namely, the dark matter problem [3]. Indeed, it has been argued that the massive vector field considered earlier by nuclear physicists can play a role in modelling what is called dark matter. The second motivation comes from the folowing fact: In standard gravitation theory, i.e., general relativity, the Proca field does not appear in a natural way, and has to be put in by hand as a matter field in much the same way as we do in the case of other physical (i.e, non-geometrical) fields. However, a recent proposed theory of gravity, deeply inspired in the original Weyl's unified field theory, seems to suggest the appearance of a massive vector field, which has an entirely geometrical nature.

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A. Weyl's theory

Let us recall that, in his attempt to unify gravity with electromagnetism, H. Weyl developed a new geometry, which constitutes one of the simplest generalizations of Riemannian geometry [4]. Recently Weyl's unified field theory was significantly reframed into a modified theory of gravity in order to allow matter to couple with the geometry in a gauge-invariant way [5]. This is done by strictly following a prescription of "minimum coupling which complies with the principle of gauge invariance postulated by Weyl. An interesting outcome of this procedure comes as an unexpected appearance of a vector field in the gravitation sector of the action. As it happens, for some choices of both the values of the cosmological constant Λ and ω (a free parameter of the theory) this vector field may be formally interpreted as a massive vector field satisfying an equation that is identical to Proca equation. Moreover, unlike the fact that the original Proca field, defined in Minkowski spacetime is not gauge-invariant, a fact that is characteristics of massive fields, here we should emphasize that the gauge invariance of the geometrical vector field is granted by first principles. Finally, let us remark that the investigation of massive vector fields within the framework of general relativity has been done by several authors [6]. A particularly interesting development of this line of research has recently shown the possibility of cosmic inflation being driven by a vector field [7].

B. The field equations

Let us start by recalling that the field equations in the Weyl's invariant theory are given by [5]

$$\frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}F^{\mu\nu}\right)=\frac{3\Lambda}{2\omega}\sigma^\mu, \tag{B.1}$$

$$\widetilde{R}_{\mu\nu}-\frac{1}{2}\widetilde{R}g_{\mu\nu}+\frac{\Lambda}{4}g_{\mu\nu}+\frac{3}{2}(\sigma_\mu\sigma_\nu-\frac{1}{2}g_{\mu\nu}\sigma^\alpha\sigma_\alpha)=\frac{\omega}{\Lambda}T_{\mu\nu}-\kappa T^{(m)}_{\mu\nu}, \tag{B.2}$$

where $\widetilde{R}_{\mu\nu}$ and $\widetilde{R}$ denote, respectively, the Ricci tensor and the scalar curvature defined with respect to the Riemannian connection, σ is a 1-form field, $T_{\mu\nu}=F_{\mu\alpha}F^{\alpha}_{\ \nu}+\frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}$ and $T^{(m)}_{\mu\nu}$ represents the energy-momentum tensor of matter, κ being a coupling constant. Let us make a short comment on the role of σ. In Weyl's original approach, σ led naturally to a new notion of curvature, a sort of "length curvature"represented by the 2-form $F=d\sigma$ (*Streckenkrummung*) in addition to the "direction curvature"(*Richtungkrummung*), the latter given by the Riemann tensor [4]. To his amazement, Weyl found that the length curvature $F=d\sigma$ presents striking similarities with the electromagnetic tensor, and it was this discovery, together with the invariance of his modified compatibiliy condition (between the metric and the affine connection), that led him to the attempt to geometrize the electromagnetic field. It is worth of mention here that the discovery of this new symmetry, which Weyl called *gauge symmetry,* is now celebrated as one of the most significant facts in the history of modern physics: it represents the birth of modern gauge theories [8].

Let us remark that the above equations may be obtained from varying the action given by

$$S=\int d^4x\sqrt{-g}[\widetilde{R}+\frac{\omega}{2\Lambda}F_{\mu\nu}F^{\mu\nu}+6\sigma_\mu\sigma^\mu-\frac{\Lambda}{2}+\kappa L_m],$$

which is identical to the action of the Proca's neutral spin-1 field in curved space-time with the cosmological constant coupled to gravity.

It is not difficult to verify that the equation B.2 can be rewritten as

$$\tilde{R}_{\mu\nu}-\frac{1}{2}\tilde{R}g_{\mu\nu}+\frac{\Lambda}{4}g_{\mu\nu}=\frac{\omega}{\Lambda}T^{(P)}_{\mu\nu}-\kappa T^{(m)}_{\mu\nu}, \tag{B.3}$$

where L_m denotes the Lagrangian density of matter, and

$$T^{(P)}_{\mu\nu}=F_{\mu\alpha}F^{\alpha}_{\ \nu}+\frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\ -\frac{3\Lambda}{2\,\omega}\left(\sigma_\mu\sigma_\nu-\frac{1}{2}g_{\mu\nu}\sigma_\alpha\sigma^\alpha\right). \tag{B.4}$$

C. The original Proca's theory

Let us start by writing the Lagrangian assumed by Proca in his theory. As is well known, the 4-potential A^α may be thought as a massive vector boson field that governs the weak interaction and the motion of spin-1 mesons. Proca's equation describe this kind of fields and is frequently used in quantum field theory [1]. The mentioned Lagrangian has the form

$$L = -\frac{1}{16\pi c}F_{\alpha\beta}F^{\alpha\beta} + \frac{m^2}{8\pi c}A^\alpha A_\alpha, \tag{C.1}$$

where m is the mass of the field and c is the speed of light. The energy-momentum tensor of the Proca field obtained from the above Lagrangian will be given by

$$T_{\mu\nu} = F_{\mu\alpha}F^{\alpha}_{\ \nu} + \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + m^2\left(A_\mu A_\nu - \frac{1}{2}g_{\mu\nu}A_\alpha A^\alpha\right). \tag{C.2}$$

By comparing (C.2) with (B.4), we see that $T^{(P)}_{\mu\nu}$ may be formally considered as the energy-momentum tensor of the Proca field provided that we define $m = \sqrt{-\frac{3\Lambda}{2\,\omega}}$ as its mass. (If $\Lambda > 0$ is interpreted as the cosmological constant, then we must set $\omega < 0$). In virtue of the analogy with Proca theory, it seems plausible to reinterpret the former Weyl field σ not as the electromagnetic field, as Weyl did, but as a sort of massive vector field, which enters the theory through a purely geometrical reasoning.

Possible applications of the Proca geometrical field in modern astrophysics

One of the main open questions in modern astrophysics, still unsolved, is to know what is the nature of the so-called *dark matter* [3]. As is well known, it is a component of the universe whose presence is inferred from its gravitation attraction rather than its luminosity. Dark matter does not interact with ordinary matter and is believed to be composed of not yet discovered subatomic particles. In the Lambda-CDM model it is thought to be responsible for approximately 26,8% of the total energy of the cosmos. Some proposed candidates for explaining dark matter are *wimps*, *primordial black holes* and *axions*. Recently, however, there have appeared new proposals considering the possibility of the Proca field being a component of dark matter [9].

Conclusion

The reinterpretation of Weyl's theory considered here allows us to build models to treat the origin of dark matter in a different way. Indeed, there is no longer need to explain why we cannot detect the nature of the supposed non-interacting particles and fields which constitute dark matter. This is because, in this approach, their nature is entirely geometric. We only need to consider a modified form of the theory of general relativity [10].

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