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АНАЛИЗ СВЯЗАННЫХ ГРАВИТАЦИОННЫХ И ЭЛЕКТРОМАГНИТНЫХ ВОЛН*

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Рассматривается метод исследования связанных с электромагнитным полем гравитационных волн в резонаторах Фабри-Перо посредством детектирования свободных поперечных гравитационных волн в окружающем пространстве. Представлены внутренние решения уравнений гравитационного поля, описывающие связанные гравитационные волны и метод расчета характеристик свободных гравитационных волн. Также представлены оценки параметров источника и детектора для реализации экспериментов такого типа.

Ключевые слова: гравитационные волны, электромагнитные волны, резонатор Фабри-Перо.

THE ANALYSIS OF COUPLED GRAVITATIONAL AND ELECTROMAGNETIC WAVES

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A method for studying gravitational waves coupled with an electromagnetic field in the Fabry-Perot resonators by detecting free transverse gravitational waves in the surrounding space is considered. Internal solutions of the gravitational field equations describing bound gravitational waves and a method for calculating the characteristics of free gravitational waves are presented. Estimates of the source and detector parameters for implementing experiments of this type were also presented.

Keywords: gravitational waves, electromagnetic waves, the Fabry-Perot resonator.

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Introduction

Currently, various astrophysical processes and cosmological perturbations in the early universe are considered as sources of gravitational wave radiation [1]. Gravitational waves resulting from the merger of black holes and neutron stars were discovered in the LIGO and Virgo experiments [2, 3] based on the interference method proposed in [4]. Relic gravitational waves, which is one of the types of cosmological perturbations at the inflationary stage of the evolution of the early universe, are described both on the basis of Einstein gravity [5] and its modifications [6, 9] have not yet been discovered due to their small amplitude, which is less than the sensitivity of modern detectors [8].

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Various laboratory sources of gravitational waves are also considered, based both on the interaction of electromagnetic radiation and matter [9, 10], and considering the electromagnetic field as a source of gravitational waves [11, 12].

Thus, gravitational-wave fluctuations of the space-time metric are induced by both matter perturbations and an alternating electromagnetic fields $\delta T_{\mu\nu} = \delta T^{(M)}_{\mu\nu} + \delta T^{(EM)}_{\mu\nu}$, and the predominance of the first or second factor is associated with the specifics of the process of generating gravitational waves.

According to estimates given in [9, 10], the interaction of a short pulse of powerful laser radiation and matter $\delta T^{(M)}_{\mu\nu} \gg \delta T^{(EM)}_{\mu\nu}$ makes it possible to generate high-frequency gravitational waves with an amplitude on the order $h_0 \sim 10^{-40}$ forthe realistic parameters of this experiment, which is much less than the sensitivity limit of modern detectors $h_0 \sim 10^{-21} - 10^{-22}$ [2, 3].

In [13, 14, 15, 16] exact wave solutions of the equations of the gravitational field in linearized General Relativity were obtained for the case of gravitational waves induced by electromagnetic waves in vacuum, dielectric media and an external magnetic field. In these works, a Fabry-Perot resonator filled with electromagnetic radiation was considered as a source of gravitational waves, and the characteristics of external transverse gravitational waves were calculated based on the characteristics of gravitational waves coupled with the electromagnetic field inside the resonator.

Also, the analysis of the processes of generating gravitational wave radiation through electromagnetic fields of a special configuration in a vacuum, dielectric media and an external magnetic field, carried out in [15, 16], implies the possibility of obtaining gravitational waves with controlled characteristics in laboratory conditions.

The coupled states of gravitational and electromagnetic waves were considered as the basis for this analysis. These states were considered earlier both: within the framework of the analysis of gravitational and electromagnetic waves in the vicinity of astrophysical objects [17], and within the framework of the problem of generating gravitational waves in terrestrial conditions [18].

In this work, we consider the possibility of detecting free gravitational waves induced by a system of coupled gravitational-electromagnetic fields when separating a gravitational wave from such a system. Direct detection of free gravitational waves in the vicinity of a region filled with coupled gravitational-electromagnetic fields makes it possible to determine the characteristics of gravitational waves associated with the electromagnetic field.

1. Coupled gravitational and electromagnetic waves

Weak gravitational waves are considered based on the linearized Einstein gravity theory as small perturbations of Minkowski space-time [1]

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1, \tag{1.1}$$

where $\eta_{\mu\nu}$ is the metric tensor of Minkowski space-time with nonzero components $\eta_{00} = 1$, $\eta_{11} = \eta_{22} = \eta_{33} = -1$ and $\mu, \nu = 0, 1, 2, 3$.

In this case, the Einstein equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - g_{\mu\nu} R = \kappa T_{\mu\nu}, \tag{1.2}$$

taking into account the harmonic gauge, can be written as follows [1]

$$\Box h_{\mu\nu} = \Delta h_{\mu\nu} - \frac{1}{c^2}\frac{\partial^2 h_{\mu\nu}}{\partial t^2} = \left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T\right), \tag{1.3}$$

where G is the gravitational constant, c is the velocity of light in vacuum, $\kappa = \frac{16\pi G}{c^4}$ is the Einstein constant and $T_{\mu\nu}$ is the energy-momentum tensor.

For gravitational waves propagating in direction $x^1 = x$, non-zero components of tensor $h_{\mu\nu}$ determine three possible types [19]:

- h_{22}, h_{23}, h_{33} – transverse–transverse (TT),
- h_{12}, h_{13}, h_{20}, h_{30} – longitudinal–transverse (LT),
- h_{11}, h_{10}, h_{00} – longitudinal–longitudinal (LL).

However, in empty space ($T_{\mu\nu} = 0$), LT and LL–waves do not lead to a deviation from the flat Minkowski space-time and can be eliminated by additional coordinate transformations [19]. Also, one has three possible polarizations for TT-waves, namely h_{23}, $h_{22} - h_{33}$ and $h_{22} + h_{33}$. However, equations (1.3) in empty space lead to condition $h_{22} + h_{33} = 0$, and, thus, transverse-traceless gravitational waves with only two polarizations h_{23} and $h_{22} - h_{33}$ can propagate, which corresponds to so called transverse-traceless gauge [1].

Nevertheless, one can consider the possibility of the existence of coupled states of electromagnetic and gravitational waves for which LT and LL–types of the metric perturbations are non-zero and $h_{22} + h_{33} \neq 0$ on the basis of Einstein-Maxwell equations written in a general form.

The gravitational field equations (1.2) in the gauge-invariant form can be written as follow [20]

$$\nabla^2\Theta = -\kappa\rho, \tag{1.4}$$

$$\nabla^2\Phi = \frac{\kappa}{2}\left(\rho + 3P - 3\dot{S}\right), \tag{1.5}$$

$$\nabla^2\Xi_i = -2\kappa S_i, \tag{1.6}$$

$$\Box h_{ij}^{\mathrm{TT}} = -2\kappa\sigma_{ij}, \tag{1.7}$$

where h_{ij}^{TT} corresponds to the TT-waves, gauge-invariant potentials Θ, Φ, $\Xi_i i$ define the remaining possible modes h_{00}, h_{0j} and ρ, P, S, S_i, σ_{ij} are defined by the components of the energy-momentum tensor $T_{\mu\nu}$ of the source of gravitational field.

For the case of empty space these equations are reduced to the following form

$$\nabla^2\Theta^{\mathrm{vac}} = 0, \tag{1.8}$$

$$\nabla^2\Phi^{\mathrm{vac}} = 0, \tag{1.9}$$

$$\nabla^2\Xi_i^{\mathrm{vac}} = 0, \tag{1.10}$$

$$\Box h_{ij}^{\mathrm{TT,vac}} = 0. \tag{1.11}$$

As one can see, in empty space, only equation (1.11) has wave solutions corresponding to the free TT-waves.

Nevertheless, for a specific source, under conditions

$$\rho \sim \frac{\partial^2}{\partial t^2}\Theta, \tag{1.12}$$

$$\left(\rho + 3P - 3\dot{S}\right) \sim \frac{\partial^2}{\partial t^2}\Phi, \tag{1.13}$$

$$S_i \sim \frac{\partial^2}{\partial t^2}\Xi_i, \tag{1.14}$$

expressions (1.4)–(1.6) correspond to the Poisson type wave equations.

Solutions of these equations describe gravitational waves coupled with this specific source, which are not a product of the choice of coordinate system.

Electromagnetic waves in a Fabry-Perot resonator were considered as such a specific source in works [14, 15, 16]. Based on solutions to the gravitational field equations in these works, the characteristics of coupled gravitational waves inside the resonator were obtained. Also, in [14, 15, 16] a procedure for reconstructing characteristics of free gravitational TT-waves in the empty space was defined, that we will consider in more detail.

2. Decoupling of gravitational and electromagnetic waves

Let us consider coupled gravitational waves induced by a system of the flat electromagnetic waves (or one wave) in a Fabry-Perot resonator along a some direction x as follows

$$h_{\mu\nu} = A_{\mu\nu}\left(U_s, E_k, \omega_k, L, x\right)\left\{\alpha \cos\left[\omega_g\left(t - \frac{x}{c}\right) + \varphi_{\mu\nu}\right] + \beta \cos\left[\omega_g\left(t + \frac{x}{c}\right) + \varphi_{\mu\nu}\right]\right\}, \quad (2.1)$$

where $A_{\mu\nu} = A_{\mu\nu}\left(E_k, \omega, L, x\right)$ are the amplitudes of the coupled gravitational waves, U_s $(s = 1, 2, 3, ...)$ are the characteristics of additional constant or variable fields (for example, magnetic fields or the medium with some dielectric and magnetic properties) inside resonator, E_k is the electric field strength and ω_k is the frequency for each electromagnetic wave $(k = 1, 2, 3, ...)$, L is the Fabry-Perot resonator length, the constants parameters α and β depend on the configuration of the resulting electromagnetic field, $\omega_g = \omega_g(\omega_k, U_s)$ is the frequency of the coupled gravitational waves and $\varphi_{\mu\nu}$ are a some additional phases.

Examples of such solutions of the gravitational field equations were considered in [14, 15, 16] for the case of a single electromagnetic wave [14], a system of electromagnetic waves and in the presence of a constant magnetic field and dielectric media as well [15, 16].

When electromagnetic part of the coupled gravitational-electromagnetic waves are reflected from the walls of the resonator (or absorbed by the walls of the resonator), gravitational and electromagnetic waves are decoupled, since gravitational waves interact extremely weakly with matter [1].

In empty space $T_{\mu\nu} = 0$, solutions of equations (1.3) can be written in the form of plane gravitational TT-waves [1]

$$h_{22} = -h_{33} = h_0 \cos\left[\omega_g\left(t \mp \frac{x}{c}\right)\right], \quad (2.2)$$

where h_0 and ω_g are the amplitude and frequency of gravitational waves in empty space, the upper sign corresponds to the case of wave propagation in the positive direction, for the lower sign wave propagates in the opposite direction.

Thus, internal solutions (2.1) for a given energy-momentum tensor $T_{\mu\nu}$ correspond to the gravitational field coupled with the source, and solution (2.2) in the form of plane transverse gravitational waves is external one. Consequently, the comparison of the characteristics of coupled and free gravitational waves presupposes the choice of a certain type of sources of the gravitational field.

The connection between internal and external solutions is determined from the condition of continuity of the energy density flux of gravitational waves [14, 15, 16]

$$ct^{01} = -\frac{c}{4\kappa}\left(\partial_0 h_{ij} \partial_1 h^{ij}\right), \quad i, j = 1, 2, 3, \quad (2.3)$$

at the boundary between the region filled with the source and empty space $\left(ct^{01}\right)_{in} = \left(ct^{01}\right)_{out}$.

When considering the process of generating gravitational waves by means of Fabry-Perot resonators filled with electromagnetic waves, the method of theoretical analysis of this process can be represented as follows:

1. Obtaining internal gravitational-wave solutions of equations (2.1) for a given source;
2. Calculation of the energy density flux ct^{01} of gravitational waves based on the obtained solutions;
3. Reconstruction of the characteristics of free gravitational waves in empty space based on the solutions (2.1) and (2.2) with condition $\left(ct^{01}\right)_{in} = \left(ct^{01}\right)_{out}$;
4. Assessment of the possibility of detecting external gravitational waves.

Thus, the analysis of gravitational waves coupled with the electromagnetic field is of interest both from the point of view of their difference from free gravitational waves in empty space, and for assessing the possibility of creating emitters of fundamentally observed gravitational waves.

In works [14, 15, 16], coupled gravitational waves induced by a system of standing electromagnetic waves in a Fabry-Perot resonator were considered

$$h_{00} = h_{11} = -\frac{4GE_1E_2}{c^3\tilde{\omega}}\left(\frac{L}{2}+x\right)\sin\left(\tilde{\omega}\left(t-\frac{x}{c}\right)\right) - \frac{4GE_1E_2}{c^3\tilde{\omega}}\left(\frac{L}{2}-x\right)\sin\left(\tilde{\omega}\left(t+\frac{x}{c}\right)\right), \tag{2.4}$$

$$h_{01} = h_{10} = \frac{4GE_1E_2}{c^3\tilde{\omega}}\left(\frac{L}{2}+x\right)\sin\left(\tilde{\omega}\left(t-\frac{x}{c}\right)\right) - \frac{4GE_1E_2}{c^3\tilde{\omega}}\left(\frac{L}{2}-x\right)\sin\left(\tilde{\omega}\left(t+\frac{x}{c}\right)\right), \tag{2.5}$$

$$h_{22} = -\frac{4\alpha GE_1E_2}{c^2\tilde{\omega}^2}\left[\cos\left(\tilde{\omega}\left(t-\frac{x}{c}\right)\right) + \cos\left(\tilde{\omega}\left(t+\frac{x}{c}\right)\right)\right], \tag{2.6}$$

$$h_{33} = -\frac{4\beta GE_1E_2}{c^2\tilde{\omega}^2}\left[\cos\left(\tilde{\omega}\left(t-\frac{x}{c}\right)\right) + \cos\left(\tilde{\omega}\left(t+\frac{x}{c}\right)\right)\right], \tag{2.7}$$

and corresponding transverse gravitational waves in the surrounding space were defined as follows

$$h_{22} = -h_{33} = \pm\frac{2GE_1E_2}{c^2\tilde{\omega}^2}\cos\left(\tilde{\omega}\left(t-\frac{x}{c}\right)\right), \tag{2.8}$$

where $\tilde{\omega} = |\omega_2 - \omega_1| = \omega_g$ is the difference frequency of electromagnetic waves.

Thus, the space-time metric in the vicinity of an electromagnetic resonator filled with two standing electromagnetic waves can be defined as

$$ds^2 = c^2dt^2 - dx^2 - \left[1 - h_0\cos\left(\tilde{\omega}\left(t \mp \frac{x}{c}\right)\right)\right]dy^2 - \left[1 + h_0\cos\left(\tilde{\omega}\left(t \mp \frac{x}{c}\right)\right)\right]dz^2, \tag{2.9}$$

i.e. it contains only components of one type of polarization

$$h_+\left(x,t\right) = h_0\cos\left(\tilde{\omega}\left(t \mp \frac{x}{c}\right)\right). \tag{2.10}$$

Within the framework of the problem of generating and detecting gravitational waves, the controlled parameters of the source are the electric field strengths $E_1 = E_2 = E_0$ in the resonator and their difference frequency $\tilde{\omega}$, and the controlled parameter of the detector is the amplitude of detectable gravitational waves h_0, which is restricted by detector sensitivity.

After substitution of expression for electric field strengths

$$E_0^2 = \frac{8\pi}{c}I_L \times Q = \frac{8\pi}{c}\left(\frac{P_S}{S}\right)\times Q, \tag{2.11}$$

and $\omega_g = 2\pi f_{gw}$ into (2.8) we obtain the amplitude of gravitational waves

$$h_0 = \left(\frac{4G}{\pi c^3}\right)\times\left(\frac{I_L}{f_{gw}^2}\right)\times Q, \tag{2.12}$$

where Q is the quality factor of the resonator, I_L is the laser radiation intensity, P_L is the laser radiation power and S is the cross-sectional area of the beam.

For the quality factor of the electromagnetic resonator $Q \simeq 10^6$, the continuous laser radiation power $P_L \simeq 10^3\ W$ and the cross-sectional area of the beam $S \simeq 10^{-4}\ m$ we obtain following amplitude of decoupled gravitational waves in empty space

$$h_0 \simeq 3\times 10^{-23}\times\left(\frac{\mathrm{Hz}}{f_{gw}}\right)^2. \tag{2.13}$$

Thus, indirect analysis of the states of coupled gravitational and electromagnetic waves in the Fabri-Perot resonator can be performed by detecting decoupled gravitational waves in the surrounding space.

As one can see, the frequency of detected gravitational waves or the difference frequency of standing electromagnetic waves in the resonator for the successful implementation of such an experiment must be determined by the sensitivity of the detector.

3. The detection of decoupled gravitational waves

As a possible method for detecting decoupled gravitational waves we consider their influence on an external magnetic field.

The equations of electrodynamics for arbitrary space-time geometry can be written as follows

$$\partial_\mu F^{\mu\nu} + \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\right) F^{\mu\nu} = \mu_0 j^\nu, \tag{3.1}$$

$$\frac{\partial F_{\beta\gamma}}{\partial x^\alpha} + \frac{\partial F_{\gamma\alpha}}{\partial x^\beta} + \frac{\partial F_{\alpha\beta}}{\partial x^\gamma} = 0, \tag{3.2}$$

where components of the tensor of electromagnetic field is

$$F_{0k} = \frac{1}{c}\partial_t A_k - \partial_k A_0 \equiv \frac{E_k}{c}, \tag{3.3}$$

$$F_{jk} = \partial_j A_k - \partial_k A_j \equiv -\epsilon_{ijk} B^i. \tag{3.4}$$

Considering the magnetic field $\vec{B} = (0, B^y, B^z)$ for the case of the unperturbed Minkowski space-time, in the gravitational wave field for metric (2.9) its components are determined from equations (3.1)–(3.2) as follows [21]

$$\tilde{B}^y = B^y \left(\frac{1 + h_+}{\sqrt{1 - h_+^2}} \right), \tag{3.5}$$

$$\tilde{B}^z = B^z \left(\frac{1 + h_+}{\sqrt{1 - h_+^2}} \right). \tag{3.6}$$

Thus, the relative change in the magnetic field

$$\frac{\delta B}{B} = \frac{\tilde{B} - B}{B} = -1 + \frac{1 + h_+}{1 - h_+} \simeq 2h_+ \sim h_0. \tag{3.7}$$

Taking into account the sensitivity of SQUID magnetometers $(\delta B)_{\min} \sim 10^{-18}\ T$ [22, 23] and the magnetic field $B \simeq 10\ T$, from (2.13) and (3.7) we can estimate the frequency the detectable decoupled gravitational waves as follows

$$f_g = \tilde{f} \leq 2 \times 10^{-2}\ Hz. \tag{3.8}$$

Thus, the main problem in implementing this approach is the formation of the system of standing electromagnetic waves in the resonator at the difference frequency $\tilde{f} \leq 2 \times 10^{-2}\ Hz$ and generating sufficiently strong magnetic fields.

Similar estimates can be made for other methods of detecting gravitational waves [24]. However, increasing the power of laser radiation and the sensitivity of gravitational wave detectors make it possible in the future to increase the difference frequency $\tilde{f} = f_g$ and consider such experiments as feasible ones.

Conclusion

In this work, we considered coupled gravitational and electromagnetic waves in the Fabri-Perot resonators. It has been shown that coupled gravitational waves can appear not as gauge artifacts generated by a special choice of coordinate system, but as actual physical effects.

To study such gravitational-electromagnetic waves, a method for detecting decoupled gravitational waves in the surrounding space was proposed. A system of standing electromagnetic waves and associated gravitational waves was considered as a source of decoupled gravitational waves. For realistic parameters of the experiment on the generation and observation of such decoupled gravitational waves, restrictions

$\tilde{f} \leq 2 \times 10^{-2}\ Hz$ were obtained on the difference frequency of standing electromagnetic waves, which leads to the significant difficulties in implementation.

Nevertheless, this result can be interpreted as technical, rather than fundamental, problems in implementing such experiment based on the proposed approach, which can be solved both by using opportunities to increase the characteristics of the source and improving the sensitivity of gravitational wave detectors.

Also, as a perspective for studying the considered states of coupled gravitational and electromagnetic waves, we can note the analysis of their propagation in the expanding universe and their possible influence on the stochastic gravitational waves background at the present era.

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