УДК 115.4

О ПРОБЛЕМЕ ВРЕМЕНИ В КВАНТОВОЙ КОСМОЛОГИИ *

Фильченков М. Л.[a,1], Лаптев Ю. П.[a,2]

[a] Учебно-научный институт гравитации и космологии РУДН, г. Москва, 117198, Россия.

Рассмотрена проблема времени в квантовой космологии в рамках квантовой геометродинамики. Хотя время явно не присутствует в квантовой космологии, оно появляется в классической космологии. Классический мир оказывается запрограммированным на квантовом уровне.

Ключевые слова: геометродинамика, космология.

ON THE TEMPORAL PROBLEM IN QUANTUM COSMOLOGY

Fil'chenkov M. L.[a,1], Laptev Yu. P.[a,2]

[a] Institute of Gravitation and Cosmology, Peoples' Friendship University of Russia, 6 Miklukho-Maklay Street, Moscow 117198, Russia.

The time problem in quantum cosmology has been considered in the framework of quantum geometrodynamics. Although time is not present in quantum cosmology explicitly, it emerges in classical cosmology. The classical world proves to be programmed on the quantum level.

Keywords: geometrodynamics, cosmology .

PACS: 98.80.-k

DOI: 10.17238/issn2226-8812.2023.3-4.89-92

Introduction

The four-dimensional space-time is split into time and a three-dimensional of instantaneous configurations, forming a a space-time of 3-geometries being considered in quantum cosmology. On the other hand, its classical limit describes the Universe's time. Therein lies the temporal problem in quantum cosmology, which was considered by philosophers, mainly on the level of interpretations [1]. Consider it in the framework of quantum geometrodynamics.

1. Wheeler-DeWitt's Equation

In quantum cosmology, the Universe's wave function is described in space of 3-geometries [2], i.e. $\frac{\partial\psi(^3G)}{\partial t} = 0$ is assumed. Hence we obtain Wheeler-DeWitt's equation $\hat{H}\psi = 0$. Since each 3-geometry describes a spatial configuration at a certain instant, time is present in 3-geometry implicitly. The combination of 3-geometries describes an implicit dependence of the superspace on time. The 3-geometries are space-loke cross-sections of the4-dimensional space-time, whose realization probability is determined by the absolute value squared of the wave function.

For a homogeneous isotropic Universe Friedmann's first equation

$$\frac{1}{2}\left(\frac{da}{d\eta}\right)^2 - \frac{4\pi G\varepsilon a^2}{3c^2} = -\frac{kc^2}{2} \tag{1.1}$$

is describable in the form of Hamiltonian connection

$$H = \frac{p_a^2}{2} + \frac{ka^2}{2} - \frac{4\pi G\varepsilon a^4}{3c^2} = 0, \tag{1.2}$$

*This research was founded by RUDN University scientific project No. FSSSF2023-0003.

[1]E-mail: fmichael@mail.ru

[2]E-mail: yplaptev@rambler.ru

where ε is the energy density, a is the scale factor, $k = 0, \pm 1$ the model parameter, $p_a = \frac{da}{d\eta}$ the generalized momentum, η the conformal time, related to synchronous one t by the formula $cdt = ad\eta$.

Hence it follows that the Lagrangian

$$L = \frac{p_a^2}{2} - \frac{ka^2}{2} + \frac{4\pi G\varepsilon a^4}{3c^2} = 0 \tag{1.3}$$

and the generalized momentum

$$p_a = \sqrt{\frac{8\pi G\varepsilon a^4}{3c^4} - ka^2}. \tag{1.4}$$

Replacing the quantity p_a in the Hamiltonian connection by the operator $\hat{p}_a = \frac{l_{pl}^2}{i}\frac{d}{da}$, we obtain Wheeler-DeWitt's equation in the minisuperspace of scale factors [3]

$$\frac{d^2\psi}{da^2} - V(a)\psi = 0, \tag{1.5}$$

where

$$V(a) = \frac{1}{l_{pl}^4}\left(ka^2 - \frac{8\pi G\varepsilon a^4}{3c^4}\right). \tag{1.6}$$

From the relation $Ld\eta = p_a da$ we find the dependence of the synchronous time on the scale factor

$$t = \frac{1}{cl_{pl}^2}\int \frac{ada}{\sqrt{-V}}. \tag{1.7}$$

2. WKB Approximation of Quantum Geometrodynamics

Consider a WKB approximation of quantum geometrodynamics. The WKB wave function has the form $\psi \sim e^{\frac{iS}{\hbar}}$, where the action reads

$$S = \hbar\int \sqrt{-V}da. \tag{2.1}$$

Find a relation between $t(a)$ and $S(a)$ in the form

$$t = \frac{\hbar}{cl_{pl}^2}\int \frac{ada}{\frac{dS}{da}}. \tag{2.2}$$

Since time is determined by the WKB wave function, the classical world proves to be programmed on the quantum level [3].

For the multicomponent medium with

$$\varepsilon(a) = \varepsilon_0 \sum_n B_n \left(\frac{r_0}{a}\right)^n, \tag{2.3}$$

where $n = 3(1+w)$, $\sum_n B_n = 1$, $\frac{1}{r_0^2} = \frac{8\pi G\varepsilon_0}{3c^4}$, r_0 is de Sitter's horizon. In the case of a barotropic equation of state we have $p = w\varepsilon$. Consider the dependence of the scale factor on time and the corresponding ones for the WKB wave function phase on the scale factor for one-component media with $k = 0$. The scale factor

$$a(t) = r_0\left(\frac{nct}{2r_0}\right)^{\frac{2}{n}-1} \text{ for } n \neq 0, \tag{2.4}$$

the wave function phase

$$\frac{S}{\hbar} = \frac{r_0^{\frac{n}{2}} a^{3-\frac{n}{2}}}{(3-\frac{n}{2})l_{pl}^2} \text{for } n \neq 6, \tag{2.5}$$

Consider these formulae for de Sitter's vacuum, i.e. for $w = -1,\ n = 0$:

$$a(t) = r_0 e^{\frac{ct}{r_0}},\ \frac{S}{\hbar} = \frac{a^3}{3r_0 l_{pl}^2}. \tag{2.6}$$

De Sitter's vacuum responsible for the first inflation is unstable, since the sound velocity $v_s = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$ is imaginary in this case.

At $t \sim 10^{-33}s$ from the singularity there occurs the Big Bang being accompanied by creation of ultrarelativistic particles and radiation with the equation of state $w = \frac{1}{3},\ n = 1$. In this case we obtain:

$$a(t) = r_0\sqrt{\frac{2ct}{r_0}},\ \frac{S}{\hbar} = \frac{r_0 a}{l_{pl}^2}. \tag{2.7}$$

3. Time Emergence in Quantum Cosmology

Reduce Wheeler-DeWitt's equation in minisuperspace of 3-geometries, allowing for the relation

$$V(a) = \frac{2m_{pl}}{\hbar^2}[U(a) - E], \tag{3.1}$$

to the equation of stationary Schrödinger type

$$\frac{d^2\psi}{da^2} - \frac{2m_{pl}}{\hbar^2}[U(a) - E] = 0, \tag{3.2}$$

where

$$E = \frac{m_{pl}c^2}{2}\left(\frac{r_o}{l_{pl}}\right)^2 B_4. \tag{3.3}$$

This equation describes the Universe, behaving as an ultrarelativistic planckeon in the field formed by types of matter with $w \neq \frac{1}{3}$, to which corresponds the potential energy $U(a)$. The Universe's birth from de Sitter's vacuum, as a result of quantum fluctuation, is interpreting as a tunnelling of the planckeon from the pre-de Sitter's stage through a potential barrier [4].

The tunnelling probability is given by Gamow's formula

$$D = \exp\left(-|\frac{2}{\hbar}\int\limits_{a_1}^{a_2}\sqrt{E - U}da|\right), \tag{3.4}$$

where $U(a_1) = U(a_2) = E$.

4. Conclusion

In quantum geometrodynamics, the Universe wave function is implicitly dependent on time in the minisuperspace of scale factors, whereas the WKB wave function phase depends on the scale factor, describing the Universe evolution. A relation between the a certain instant, dependence of the scale factor on time and the dependence of the wave function phase on the scale factor has been found for one-component media. Each 3-geometry describes the spatial configuration at a certain instant, which means that the superspace of scale factors contains time implicitly.

Acknowledgement

We are grateful to the Organizing Committee of PIRT-2023 Conference for the invitation. This research was founded by RUDN University scientific project No. FSSSF2023-0003

Список литературы/References

1. A.Yu. Sevalnikov, *Metaphysics*No. 1(17), p. 136 (2013).
2. J.A. Wheeler, *Einsteins Vision.* Berlin: Springer–Verlag, 1968.
3. M.L. Fil'chenkov, Yu.P. Laptev. *Quantum Gravity.* Moscow: Lenand, 2016.
4. E.P. Tryon, *Nature*, v. 246, 396 (1973).

Авторы

Фильченков Михаил Леонидович, д.ф.-м.н., доцент Учебно-научного института гравитации и космологии РУДН, Россия.
E-mail: fmichael@mail.ru

Лаптев Юрий Павлович, к.ф.-м.н.
E-mail: yplaptev@rambler.ru

Просьба ссылаться на эту статью следующим образом:
Фильченков М. Л., Лаптев Ю. П. О проблеме времени в квантовой космологии. *Пространство, время и фундаментальные взаимодействия.* 2023. № 3-4. С. 89–92.

Authors

Fil'chenkov Mikhail Leonidovich, D.Sc.
E-mail: fmichael@mail.ru

Laptev Yuri Pavlovich, Ph.D.
E-mail: yplaptev@rambler.ru

Please cite this article in English as:
Fil'chenkov M. L., Laptev Yu. P. On The Temporal Problem In Quantum Cosmology. *Space, Time and Fundamental Interactions*, 2023, no. 3-4, pp. 89–92.