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# О СОХРАНЯЮЩИХСЯ ВЕЛИЧИНАХ ДЛЯ ДВИЖУЩЕЙСЯ ЧЁРНОЙ ДЫРЫ В ТЕЛЕПАРАЛЛЕЛЬНОМ ЭКВИВАЛЕНТЕ ОТО*

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В рамках телепараллельного эквивалента (ТЭ) ОТО, где полевыми переменными являются компоненты тетрад, выведены масса и импульс для движущейся (равномерно относительносительно удаленных наблюдателей) черной дыры Шварцшильда (ЧДШ). Используется формализм, разработанный авторами ранее, для построения сохраняющихся величин в ТЭ ОТО, где токи и суперпотенциалы как координатно ковариантны, так и инвариантны относительно локальных лоренцевых вращений тетрад. Это преимущество достигнуто благодаря введению инерциальной спиновой связности (ИСС) и использованию теоремы Нётер с сохранением векторов смещений в окончательных выражениях. Набор пар (ИСС и тетрад), связанных гладкими преобразованиями, мы назвали калибровкой, это класс эквивалентности. Величина ИСС внешняя, поэтому мы определяем её благодаря введённому нами обобщенному принципу «выключения гравитации». Но, даже этот разумный принцип приводит к различным определениям ИСС для одной и той же тетрады, что ведет к различным результатам. Здесь, на примере движущейся ЧДШ мы 1) демонстрируем преимущества нашего полностью ковариантного формализма, 2) а также изучаем неопределенность в определении ИСС. В расчетах используются аналогии с движущимся материальным шаром в пространстве Минковского и только «статическая» калибровка. Получены ожидаемые масса и импульс. Затем сравниваются «статическая» и «движущаяся» калибровки. Найдено, что они совпадают. То есть, в случае движущейся ЧДШ, нет ожидаемой двусмысленности, и в обоих случаях получены те же масса и импульс.

*Ключевые слова*: телепараллельная гравитация, сохраняющиеся величины, черные дыры; teleparallel gravity, conserved quantities, black holes.

# ON CONSERVED QUANTITIES FOR A MOVING BLACK HOLE IN TEGR

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In the framework of the Teleparallel Equivalent of General Relativity (TEGR), where the field variables are tetrad components, mass and momentum for a moving (uniformly with respect to distant observers) Schwarzschild black hole (SBH) are constructed. A formalism developed by the authors earlier for constructing conserved quantities in TEGR, where currents and superpotentials are covariant with respect both to coordinate transformations and to local Lorentz rotations of tetrads is applied. This advantage has been reached by introducing inertial spin connection (ISC) and using the Noether theorem with preservation of a displacement vector in final expressions. A set of pairs (tetrad and related ISC) connected by smooth transformations we call as a "gauge", it is the equivalence class. The quantity ISC is an external one, therefore we define it with making the use of the introduced by us generalized "turning off gravity" principle. But, even this a reasonable principle leads to different values of ISCs for the same tetrad that leads to different results. Here, on the example of the moving SBH we 1) demonstrate advantages of our fully covariant formalism, 2) study the ambiguity in definition of ISC as well. In calculations, we the use analogies with a moving mater ball in Minkowski space only in the "static gauge".

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Expected mass and momentum have been obtained. Next we compare “static gauge” and “moving gauge”. It was found that they coincide. In the result, in the case of a moving SBH aforementioned ambiguity is absent because in both the cases the same mass and momentum are obtained.

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## Introduction

Last decades teleparallel gravity attracts a lot of attention, see, for example, [1, 2, 3] and numerous references therein. Except expanded telerarallel theories one continues actively to consider Teleparallel Equivalent of General Relativity (TEGR) [1] where the tetrad components present field variables, and non-zero torsion determines non-zero gravitational field. In many such researches, black hole solutions are the most popular models for application of various formalisms. Among such solutions, the Schwarzschild black hole (SBH) is considered more frequently than others, see [4, 5, 6, 7, 8] and references therein, and is used to calculate the mass of a black hole as a conserved global charge or derive energy density of the gravitational field measured by an observer.

Many of approaches (see, for example, [7, 8]) look as not so satisfactory ones. The reason is that they lead either to non-covariant with respect to coordinate transformations, or non-invariant with respect to local Lorentz rotations conserved currents or charges. However, using Noether's theorem, a fully covariant formalism has been developed. One can recall the series of the papers [9, 10, 11], where fully covariant conserved quantities are constructed in the formalism of differential forms. Unfortunately this approach did not obtain a development. Recently, in [12, 13, 14, 15] we have developed a fully covariant approach for constructing conserved quantities in TEGR in the more popular tensorial presentation. Namely, this formalism is applied in the present paper.

Concerning the Schwarzschild solution, for the best of our knowledge it was not considered in TEGR as a moving black hole. By this, the first goal here is to calculate the global conserved energy and momentum for the moving SBH [16] with making the use of the method [12, 13]. Of course, it is not an end in itself because such quantities can be easily obtained by other appropriate methods [17]. Here, we only demonstrate possibilities of our covariant formalism [12, 13] and its advantages. We note that in such calculations analogies with calculating the mass and momentum of a moving matter ball in Minkowski space are used.

In order to obtain the covariance of the both types one needs, first, to introduce an inertial spin connection (ISC) that is not dynamical quantity and is not determined by the theory itself. In [12, 13, 14, 15], the unified principle of “turning off” gravity is used to determine ISC for a concrete solution. It is based on the fact that in the absence of gravity, only inertial effects remain. In this case, the curvature tensor vanishes and then the Levi-Civita spin connection (L-CSC) is to be able to express only inertial effects and should be equal to the ISC. Second, to obtain the full covariance one needs to preserve in expressions a displacement vector $\xi$ after applying the Noether theorem as well. This application is based on diffeomorphisms induced by an arbitrary smooth vector field $\xi$, and then one needs to choose $\xi$ in a physically meaningful way. Thus, $\xi$ can be chosen as Killing vector fields of the reference geometry, proper vectors of observers, etc.

A plenty of pairs of tetrads and related ISCs, which are connected by smooth transformations of both the types we call as a “gauge” (really it is the equivalence class). Thus, for a concrete gauge conserved quantities are the same. However, even the reasonable principle of “turning off” gravity leads to different definitions of ISCs for the same tetrad that leads to different gauges, the same to construction of different conserved quantities. This problem in detail has been studied in [14, 15] on the example of the Schwarzschild solution. Here (it is the second goal of the paper), on the example of the moving SBH we analyze the ambiguity in definition of ISC (the same, in definition of a gauge) as well. First, we introduce a so-called "static gauge". Expected mass and momentum have been obtained. Then, we introduce a so-called "moving gauge" Comparing "static gauge" and "moving gauge", we find that they coincide. In the result, in the case of a moving SBH aforementioned ambiguity is absent because in both cases the same mass and momentum are obtained.

This paper is based on our presentation at the conference PIRT-2023 [18], which unites the results of our previous works [19, 20]. The results of these papers look as very disparate ones, although evidently that they have to be given uniformly, representing a new quality. Thus, here we close this gap.

The paper is organized as follows. In section 1, a short description of elements of TEGR and of constructing fully covariant conserved quantities is given. Besides, the notion of gauges and ambiguities in their definitions are outlined. In section 2, a construction of conserved quantities for an uniformly moving matter ball in Minkowski space is presented. In section 3, a "static gauge" for the SBH solution in isotropic coordinates is introduced. In section 4, basing on the static isotropic gauge and analogy with the matter ball in Minkowski space the total energy and momentum for the moving SBH are calculated. In section 5, the fully covariant formalism in TEGR itself is applied to construct the aforementioned conserved quantities, and a "moving gauge" is defined and compared with the static gauge.

At last, in the paper, we use abbreviations as follows. GR — general relativity; TEGR — teleparallel equivalent of general relativity; SBH — Schwarzschild black hole; ISC — inertial spin connection; L-CSC — Levi-Civita spin connection; SSG — static Schwarzschild gauge; LG — Lemaitre gauge; SIG — static isotropic gauge.

## 1. Preliminaries

### 1.1. Elements of TEGR and fully covariant formalism

One of the ways to present the gravitational Lagrangian of TEGR is [1]

$$\overset{\bullet}{\mathcal{L}}= \frac{h}{2\kappa}\left(\overset{\bullet}{K}{}^{\rho}{}_{\mu\nu}\overset{\bullet}{K}{}_{\rho}{}^{\nu\mu} - \overset{\bullet}{K}{}^{\nu}{}_{\rho\nu}\overset{\bullet}{K}{}^{\mu\rho}{}_{\mu}\right), \tag{1.1}$$

that is equivalent to the Hilbert Lagrangian up to a divergence with the Einstein constant $\kappa$. Unlike metric presentation of GR, dynamical variables in TEGR are components of the tetrad field $h^a{}_\rho$, which are connected with the metric by $g_{\mu\nu} = \eta_{ab}h^a{}_\mu h^b{}_\nu$ and $h \equiv \det h^a{}_\rho$, where $\eta_{ab}$ is the Minkowski metric. Greek indexes are spacetime components, Latin indexes $a, b, c, \ldots$ are tetrad components, Latin indexes $i, j, k, \ldots$ are space components. The contortion tensor in (1.1) is defined as a difference

$$\overset{\bullet}{K}{}^{a}{}_{b\nu} =\overset{\bullet}{A}{}^{a}{}_{b\nu}- \overset{\circ}{A}{}^{a}{}_{b\nu}, \tag{1.2}$$

where

$$\overset{\bullet}{A}{}^{a}{}_{b\nu} = -h_b{}^\rho \overset{\bullet}{\nabla}_\nu h^a{}_\rho \tag{1.3}$$

is the ISC, and

$$\overset{\circ}{A}{}^{a}{}_{b\nu} = -h_b{}^\rho \overset{\circ}{\nabla}_\nu h^a{}_\rho \tag{1.4}$$

is the L-CSC. Tetrad indexes are replaced by spacetime indexes and inversely by contracting with $h^a{}_\mu$ or $h_a{}^\mu$, for example, $\overset{\bullet}{K}{}^{\rho}{}_{\mu\nu} =\overset{\bullet}{K}{}^{a}{}_{b\nu}h_a{}^\rho h^b{}_\mu$. Here and below, $\bullet$ means that a quantity is constructed with the use of the teleparallel connection $\overset{\bullet}{\Gamma}{}^{\alpha}{}_{\mu\nu}$ of zero curvature, whereas $\circ$ means that a quantity is constructed with the use of the Levi-Chivita connection $\overset{\circ}{\Gamma}{}^{\alpha}{}_{\mu\nu}$, like the covariant derivatives $\overset{\bullet}{\nabla}_\nu$ and $\overset{\circ}{\nabla}_\nu$.

Simultaneous transformations of tetrads and ISCs under local Lorentz rotations are:

$$h'^a{}_\mu = \Lambda^a{}_b h^b{}_\mu, \tag{1.5}$$

$$\overset{\bullet}{A}{}'^{a}{}_{b\mu} = \Lambda^a{}_c \overset{\bullet}{A}{}^{c}{}_{d\mu}\Lambda_b{}^d + \Lambda^a{}_c\partial_\mu\Lambda_b{}^c, \tag{1.6}$$

where $\Lambda^b{}_d(x)$ is a matrix of a local Lorentz transformation. The L-CSC $\overset{\circ}{A}{}^{a}{}_{b\nu}$ is transformed analogously to (1.6). Then it is evidently that $\overset{\bullet}{K}{}^{\rho}{}_{\mu\nu}$ defined in (1.2) is invariant under local Lorentz transformations.

Because $\overset{\bullet}{A}{}^c{}_{d\mu}$ represents the inertial effects it can be suppressed by (1.6) with appropriate $\Lambda^a{}_b$ [12, 13]. By the next a local Lorentz transformation $\Lambda^{*a}{}_b$ it can be represented in the form:

$$\overset{\bullet}{A}{}''^a{}_{b\mu} = \Lambda^{*a}{}_c \partial_\mu \Lambda^*{}_b{}^c\,. \tag{1.7}$$

In [12, 13], considering the invariance of (1.1) under a diffeomorphism induced by an arbitrary smooth vector field $\xi$, one derives the conservation law for the current $\overset{\bullet}{\mathcal{J}}{}^\alpha(\xi)$:

$$\partial_\alpha \overset{\bullet}{\mathcal{J}}{}^\alpha(\xi) = \overset{\circ}{\nabla}_\alpha \overset{\bullet}{\mathcal{J}}{}^\alpha(\xi) = 0. \tag{1.8}$$

Here, it is not necessary to derive a concrete structure of the current itself. It is more convenient to use its representation through the superpotential:

$$\overset{\bullet}{\mathcal{J}}{}^\alpha(\xi) = \partial_\beta \overset{\bullet}{\mathcal{J}}{}^{\alpha\beta}(\xi) = \overset{\circ}{\nabla}_\beta \overset{\bullet}{\mathcal{J}}{}^{\alpha\beta}(\xi). \tag{1.9}$$

Noether's current $\overset{\bullet}{\mathcal{J}}{}^\alpha(\xi)$ is the vector density of the weight $+1$, Noether's superpotential $\overset{\bullet}{\mathcal{J}}{}^{\alpha\beta}(\xi)$ is the antisymmetric tensor density of the weight $+1$ for which the explicit expression is

$$\overset{\bullet}{\mathcal{J}}{}^{\alpha\beta}(\xi) = \frac{h}{\kappa} \overset{\bullet}{S}{}_\sigma{}^{\alpha\beta} \xi^\sigma, \tag{1.10}$$

where the teleparallel superpotential is

$$\overset{\bullet}{S}{}_\sigma{}^{\alpha\beta} = \overset{\bullet}{K}{}^{\alpha\beta}{}_\sigma + \delta^\beta_\sigma \overset{\bullet}{K}{}^{\theta\alpha}{}_\theta - \delta^\alpha_\sigma \overset{\bullet}{K}{}^{\theta\beta}{}_\theta. \tag{1.11}$$

Both $\overset{\bullet}{\mathcal{J}}{}^\alpha(\xi)$ and $\overset{\bullet}{\mathcal{J}}{}^{\alpha\beta}(\xi)$ are locally Lorentz invariant, that invariant with respect to simultaneous transformations (1.5) and (1.6).

The conservation law (1.8) allows us to construct a conserved integral quantity:

$$\mathcal{P}(\xi) = \int_\Sigma d^3x \overset{\bullet}{\mathcal{J}}{}^0(\xi), \tag{1.12}$$

where $\Sigma$ is a hypersurface of constant time $t = x^0 = \text{const}$. In the case of spherical symmetry, when $r = x^1$, the conservation law (1.9) allow us to represent (1.12) as a conserved charge:

$$\mathcal{P}(\xi) = \oint_{\partial\Sigma} d^2x \overset{\bullet}{\mathcal{J}}{}^{01}(\xi) = \frac{1}{\kappa} \oint_{\partial\Sigma} d^2x\, h\, \overset{\bullet}{S}{}_\sigma{}^{01} \xi^\sigma, \tag{1.13}$$

where $\partial\Sigma$ is a boundary of $\Sigma$, and can be considered both at finite $r = r_0$ and at $r \to \infty$. By the construction, it is evidently that (1.12) or (1.13) are scalars with respect to the both aforementioned types of transformations. At last, the interpretation of (1.8), (1.10), (1.12) and (1.13) depend on a choice of $\xi^\sigma$.

### 1.2. Ambiguity in determining gauges

Here, we describe in more detail the aforementioned above problem related to an ambiguity in determining gauges. At the earlier stage of development of teleparallel theory the problem of non-covariance of classical pseudotensors [17] has been suggested by Møller [21] in order to construct covariant conserved quantities in GR in the tetrad form. However, it turns out that these quantities are not invariant/covariant with respect to local Lorentz rotations of tetrad vectors. As shown in the previous subsection, the incorporation of ISC and $\xi^\sigma$ into conserved quantities gives a possibility to construct them in fully covariant form [12, 13]. An ambiguity in the definition of ISC by the principle of "turning off" gravity has been outlined in Introduction.

In the framework of the covariant formalism [12, 13] the notion of "gauges" has been introduced [14]. It can be formulated as follows [20]:

*For the given tetrad-ISC pair* $(h_a{}^\mu, \overset{\bullet}{A}{}^a{}_{b\mu})$ *it is considered the equivalence class of pairs related either by smooth coordinate transformations* $(h_a{}^\mu, \overset{\bullet}{A}{}^a{}_{b\mu}) \sim (h_a{}^{\mu'}, \overset{\bullet}{A}{}^a{}_{b\mu'})$ *and/or local Lorentz transformations* $(h_a{}^\mu, \overset{\bullet}{A}{}^a{}_{b\mu}) \sim (h_{a'}{}^\mu, \overset{\bullet}{A}{}^{a'}{}_{b'\mu})$. *Any member of the equivalence class is viewed as the same* gauge *by the definition of* [14], *and any such pair leads to the same results in the calculation of conserved quantities.*

Then, the ambiguity in determining ISCs leads to an ambiguity in determining gauges themselves. This problem has been studied in detail in [14, 15] on the example of the SBH. It was clarified, that a diagonal tetrad and related ISC induced by the standard static Schwarzschild metric, *static Schwarzschild gauge* (SSG), is appropriate for calculating the total mass of SBH. On the other hand, the SSG fails in describing a freely falling observer for whom correspondence with the equivalence principle is lost. Conversely, a diagonal tetrad and related ISC induced by the Lemaitre metric, *Lemaitre gauge* (LG), is appropriate for correspondence with the equivalence principle, but it does not lead to acceptable mass for SBH. In this a concrete case, we have resolved this problem introducing a generalized Lemaitre metric and related *generalized Lemaitre gauge*. However, it is a particular case only, and more wide study of the problem is required.

Thus, here, we are continuing to study the problem of ambiguity in definition of gauges on the example of a SBH moving with a constant (with respect to distant static observers) velocity [16]. From the start, we use a calculation of the total mass for the static SBH in isotropic coordinates using an appropriate gauge (static gauge). By the aforementioned logic, calculations for the moving SBH can be carried out successfully when its own (separate) appropriate gauge (moving gauge) is found. We find it and clarify a connection between both the gauges. Besides, we compare them with SSG in [14].

## 2. A moving matter ball

It turns out that in order to demonstrate advantages of our fully covariant formalism [12, 13] it is very fruitful to use analogies with properties of special relativity. We recall them beginning from the Minkowski space with metric:

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2. \tag{2.1}$$

We denote $(t, x, y, z) = (x^0, x^i) = (x^\alpha)$, where $i = 1, 2, 3$. To define a reference frame, we add to (2.1) static observers with proper vectors

$$\xi^\alpha = (-1, 0, 0, 0). \tag{2.2}$$

Assume that the matter in the Minkowski space has energy-momentum tensor $\Theta^\alpha{}_\beta$, which is differentially conserved, $\partial_\alpha \Theta^\alpha{}_\beta = 0$. Then, if one defines the current $\mathcal{J}^\alpha = \Theta^\alpha{}_\beta \xi^\beta$, one finds that it is conserved, $\partial_\alpha \mathcal{J}^\alpha = 0$, as well. Its components present the energy density $\mathcal{J}^0 = \Theta^0{}_0 \xi^0$ and the momentum density $\mathcal{J}^i = \Theta^i{}_0 \xi^0$ measured by the introduced above observers (2.2). To define integral (global) conserved quantities on has to integrate $\mathcal{J}^0$ to obtain the total energy (mass)

$$E = \int_\Sigma dx dy dz \mathcal{J}^0, \tag{2.3}$$

and integrate $\mathcal{J}^i$ to obtain the total momentum

$$P^i = \int_\Sigma dx dy dz \mathcal{J}^i, \tag{2.4}$$

over the space section in (2.1) $\Sigma := t = \text{const}$. The current in TEGR defined as $\overset{\bullet}{\mathcal{J}}{}^\alpha(\xi)$ generalizes the simplest definition in Minkowski space and its components have the analogous interpretation for observers with proper vectors $\xi^\alpha$.

Assuming the static spherically symmetric distribution of matter, one has for the current

$$\mathcal{J}^\alpha_s = [\rho(r),\ 0,\ 0,\ 0]\,, \tag{2.5}$$

(subscript '$s'$ means 'static') with $\rho(r) = \mathcal{J}^0_s(r) = \Theta^0{}_0(r)\xi^0$. It is just the energy density, where $r^2 \equiv x^2 + y^2 + z^2$ with

$$x = r\sin\theta\cos\phi; \quad y = r\sin\theta\sin\phi; \quad z = r\cos\phi. \tag{2.6}$$

Let the matter distribution on the hypersurface $\Sigma$ be bounded by $\partial\Sigma$ that presents a sphere $r = r_0$. Then the total mass (energy) of such an object is calculated as

$$E_s = \int_\Sigma dxdydz\mathcal{J}^0_s(r) = \int_\Sigma dxdydz\rho(r) = \int_0^{2\pi}\int_0^{\pi}\int_0^{r_0} d\phi d\theta dr\sin\theta\, r^2\rho(r) = \mathcal{M}. \tag{2.7}$$

Let an absolutely identical matter ball be moving with the constant velocity $v$ along the axis $\mathbf{x}$ relatively to the frame $\{x^\alpha\}$ connected with (2.1). The proper coordinates of the moving object are connected with those in (2.1) by the Lorentz transformation:

$$\overline{t} = \gamma(t - vx); \quad \overline{x} = \gamma(x - vt); \quad \overline{y} = y; \quad \overline{z} = z, \tag{2.8}$$

where, as usual, $\gamma \equiv (1 - v^2)^{-\frac{1}{2}}$. In analogy with the reference frame $\{x^\alpha\}$ determined by (2.1) the moving ball has a *proper* (its own) reference frame $\{\overline{x}^\alpha\}$.

Let us give the simplest illustration before real calculations. Let the moving sphere be filled by $N$ point particles with masses $m$ at the rest in the proper frame $\{\overline{x}^\alpha\}$. Then the total mass in $\{\overline{x}^\alpha\}$ is $\mathcal{M}_s = Nm$. After that, let us find the mass and momentum of such a moving object in the frame $\{x^\alpha\}$. First, the moving sphere undergoes relativistic compression and its volume decreases $\gamma$ times. Second, by effects of special relativity, energy and momentum of each particle with mass $m$ becomes $\gamma m$, and $\gamma v m$. At last, third, because the number of particles $N$ is conserved the concentration of particles increases in $\gamma$ times. It is evidently that the first and third factors are compensated. Then, the total mass and momentum of the moving object becomes $\mathcal{M}_m = N(\gamma m) = \gamma\mathcal{M}_s$ and $\mathcal{P}^1_m = N(\gamma v m) = \gamma v\mathcal{M}_s$.

Turning to the case of continuous matter distribution one can carry out the same. In the proper frame $\{\overline{x}^\alpha\}$ of the moving ball the current has the form:

$$\mathcal{J}^{\overline{\alpha}}_s = [\rho(\overline{r}),\ 0,\ 0,\ 0]\,, \tag{2.9}$$

where $\overline{r}^2 = \overline{x}^2 + \overline{y}^2 + \overline{z}^2$ and $\rho(\overline{r})$ is the same function like in (2.5). Now, let us transform from the frame $\{\overline{x}^\alpha\}$ to the frame $\{x^\alpha\}$. Then, first, the factor of the relativistic compression of the sphere is to be taken into account in boundaries of integration. Second, the components of the vector (2.9) after Lorentz transformations (2.8) become

$$\mathcal{J}^\alpha_m = [\gamma\rho(\overline{r}), \gamma v\rho(\overline{r}),\ 0,\ 0] \tag{2.10}$$

in the frame $\{x^\alpha\}$ in coordinates $(t, x, y, z)$, where $\overline{r}^2 = \gamma^2(x - vt)^2 + y^2 + z^2$ (subscript '$m'$ means 'moving'). Third, due to the relativistic compression the densities (that is the components of (2.10)) have to be multiplied by $\gamma$ under the integration in the compressed boundaries. Again the first and third factors are compensated.

Finally for the total mass of the moving matter ball one has

$$E_m = \int_\Sigma dxdydz\left(\gamma\mathcal{J}^0_m(\overline{r})\right) = \gamma\int_\Sigma dx'dydz\rho(r') = 4\pi\gamma\int_0^{r_0} dr'\, r'^2\rho(r') = \gamma\mathcal{M}, \tag{2.11}$$

where the boundary $\partial\Sigma$ of $\Sigma$ is defined as $\gamma^2x^2 + y^2 + z^2 = r_0^2$. Without the loss of generality we set $t = 0$ . After the simple redefinition $x' = \gamma x$ one has $x'^2 + y^2 + z^2 = r'^2$ and the boundary $\partial\Sigma$ is defined as usual $r' = r_0$, thus the last integration in (2.11) repeats exactly (2.7). Keeping in mind (2.4) and following the logic in calculations (2.11) one finds for the total momentum

$$P^1_m = \int_\Sigma dxdydz\left(\gamma\mathcal{J}^1_m(\overline{r})\right) = \gamma v\int_\Sigma dx'dydz\rho(r') = 4\pi\gamma v\int_0^{r_0} dr'\, r'^2\rho(r') = \gamma v\mathcal{M}. \tag{2.12}$$

The results (2.11) and (2.12) are in the full correspondence with the conclusions of special relativity. The analogies with the above calculus based on the covariant formalism of [12, 13] will be used to calculate the global mass and momentum of the moving SBH.

## 3. Static isotropic gauge for Schwarzschild solution

Before studying a moving SBH it is more convenient to consider the Schwarzschild metric in isotropic coordinates, like in [16]:

$$ds^2 = -\alpha^2(r)dt^2 + \psi^4(r)(dx^2 + dy^2 + dz^2), \tag{3.1}$$

where $\alpha(r) \equiv (1 - \frac{M}{2r})/(1 + \frac{M}{2r})$, $\psi(r) \equiv 1 + \frac{M}{2r}$ and again $x^2 + y^2 + z^2 = r^2$.

Let us derive the necessary TEGR expressions. The most convenient is to choose the tetrad in diagonal form:

$$h^a{}_\mu = \text{diag}\left[\alpha(r), \psi^2(r), \psi^2(r), \psi^2(r)\right]. \tag{3.2}$$

Non-zero components of L-CSC (1.4) calculated for (3.1) and (3.2) are:

$$\overset{\circ}{A}{}^{\hat{0}}{}_{\hat{\imath}0} = -\overset{\circ}{A}{}^{\hat{\imath}}{}_{\hat{0}0} = \frac{Mx^i}{r^3}\frac{1}{\psi^4(r)}; \quad \overset{\circ}{A}{}^{\hat{\imath}}{}_{\hat{k}i} = -\overset{\circ}{A}{}^{\hat{k}}{}_{\hat{\imath}i} = \frac{Mx^k}{r^3}\frac{1}{\psi(r)}, \tag{3.3}$$

where the indexes with "hat" are tetrad components and indexes without "hat" are spacetime components; here, $x^i \equiv (x^1, x^2, x^3) \equiv (x, y, z)$. "Turning-off gravity" for L-CSC (3.3) and related curvature leads to $M \to 0$. Then L-CSC vanishes giving for all the components of ISC

$$\overset{\bullet}{A}{}^a{}_{b\mu} = 0. \tag{3.4}$$

For the L-CSC (3.3) and zero ISC (3.4) the formulae (1.2) and (1.11) give the teleparallel superpotential, non-zero components of which are:

$$\overset{\bullet}{S}{}_0{}^{0i} = -\overset{\bullet}{S}{}_0{}^{i0} = \frac{2Mx^i}{r^3}\frac{1}{\psi^5(r)}; \quad \overset{\bullet}{S}{}_i{}^{ik} = -\overset{\bullet}{S}{}_i{}^{ki} = \frac{M^2x^k}{2r^4}\frac{1}{\alpha(r)\psi^6(r)}. \tag{3.5}$$

To calculate the total mass of the Schwarzschild black hole, it is more convenient to take the spherical coordinates. Therefore, let us provide the standard coordinate transformation (2.6) after that the metric (3.1) acquires the form:

$$ds^2 = -\alpha^2(r)dt^2 + \psi^4(r)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \tag{3.6}$$

Again we chose the diagonal tetrad, this time for the metric (3.6):

$$h^a{}_\mu = \text{diag}\left[\alpha(r), \psi^2(r), r\psi^2(r), r\psi^2(r)\sin\theta\right]. \tag{3.7}$$

For the metric (3.6) and tetrad (3.7), the non-zero components of L-CSC (1.4) are

$$\begin{array}{c}\overset{\circ}{A}{}^{\hat{0}}{}_{\hat{1}0} = \overset{\circ}{A}{}^{\hat{1}}{}_{\hat{0}0} = \frac{M}{r^2}\frac{1}{\psi^4(r)}; \; \overset{\circ}{A}{}^{\hat{1}}{}_{\hat{2}2} = -\overset{\circ}{A}{}^{\hat{2}}{}_{\hat{1}2} = \frac{M}{r}\frac{1}{\psi(r)} - 1; \\ \overset{\circ}{A}{}^{\hat{1}}{}_{\hat{3}3} = -\overset{\circ}{A}{}^{\hat{3}}{}_{\hat{1}3} = -\alpha(r)\sin\theta; \; \overset{\circ}{A}{}^{\hat{2}}{}_{\hat{3}3} = -\overset{\circ}{A}{}^{\hat{3}}{}_{\hat{2}3} = -\cos\theta,\end{array} \tag{3.8}$$

where now $x^i \equiv (x^1, x^2, x^3) \equiv (r, \theta, \phi)$.

"Turning off" gravity by $M \to 0$ in (3.8) gives ISC, non-zero components of which are:

$$\overset{\bullet}{A}{}^{\hat{1}}{}_{\hat{2}2} = -\overset{\bullet}{A}{}^{\hat{2}}{}_{\hat{1}2} = -1; \quad \overset{\bullet}{A}{}^{\hat{1}}{}_{\hat{3}3} = -\overset{\bullet}{A}{}^{\hat{3}}{}_{\hat{1}3} = -\sin\theta; \quad \overset{\bullet}{A}{}^{\hat{2}}{}_{\hat{3}3} = -\overset{\bullet}{A}{}^{\hat{3}}{}_{\hat{2}3} = -\cos\theta. \tag{3.9}$$

Then, formulae (1.2) and (1.11) for the L-CSC (3.8) and ISC (3.9) give non-zero components of $\overset{\bullet}{S}{}_\sigma{}^{\alpha\beta}$:

$$\overset{\bullet}{S}{}_0{}^{01} = -\overset{\bullet}{S}{}_0{}^{10} = -\frac{2M}{r^2}\frac{1}{\psi^5(r)}; \; \overset{\bullet}{S}{}_2{}^{12} = -\overset{\bullet}{S}{}_2{}^{21} = \overset{\bullet}{S}{}_3{}^{13} = -\overset{\bullet}{S}{}_3{}^{31} = -\frac{M^2}{2r^3}\frac{1}{\alpha(r)\psi^6(r)}. \tag{3.10}$$

It is necessary to compare the pair (3.2) and (3.4) with the pair (3.7) and (3.9). Let us apply the transformations (1.5) and (1.6) to the tetrad (3.7) and ISC (3.9), where

$$\Lambda^a{}_b = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ 0 & \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ 0 & \cos\theta & -\sin\theta & 0 \end{pmatrix}. \tag{3.11}$$

Then the tetrad (3.7) goes to the tetrad (3.2) (after the coordinate transformations (2.6)), whereas the transformed ISC vanishes that is becomes (3.4). Thus, in the framework of the fully covariant formalism [12, 13] in terminology of [14, 15] these pairs represent the same gauge, we call it the "static isotropic gauge" (SIG).

It is useful to compare the SIG introduced here with the SSG in [14]. The isotropic coordinates in (3.6) are connected with the static Schwarzschild coordinates in [14], with $R$ radial coordinate, by the relation $R = r(1 + M/2r)^2$. Applying this transformation to the components of the tetrad (3.7), one obtains the components of the diagonal tetrad in [14] in the SSG; the components of the ISC (3.9) do not change and coincide with those in [14]. Thus the SIG here and the SSG in [14] are the same gauge presented in different coordinates.

## 4. Calculations in analogy with the matter ball in Minkowski space

Let us calculate the global mass of the SBH. First of all, it is necessary to determine the observers in the same way as (2.2) in the Minkowski space. A spacetime with metric (3.1), or (3.6), and with the 4-vectors of static observers

$$\xi^\sigma = \left[-\alpha^{-1}(r),\ 0,\ 0,\ 0\right] \tag{4.1}$$

presents a static reference frame $\{x^\alpha\}$. Then, (1.10) with (3.10) gives the non-zero component of the Noether superpotential

$$\overset{\bullet}{\mathcal{J}}{}^{01}_{s} = -\overset{\bullet}{\mathcal{J}}{}^{10}_{s} = 2\kappa^{-1} M\psi(r)\sin\theta; \tag{4.2}$$

and (1.9) gives the Noether current in the form:

$$\overset{\bullet}{\mathcal{J}}{}^{\alpha}_{s} = \left[2\kappa^{-1} M\psi'(r)\sin\theta,\ 0,\ 0,\ 0\right]. \tag{4.3}$$

Because $\overset{\bullet}{\mathcal{J}}{}^{\alpha}_{s}$ is a vector density, see (1.8) and (1.9), the energy density in (4.3) presented in spherical coordinates can be rewritten as $\overset{\bullet}{\mathcal{J}}{}^{0}_{s} = \overset{\bullet}{\rho}(r) r^2 \sin\theta$, where $\overset{\bullet}{\rho}(r)$ is the energy density in the Cartesian coordinates of (3.1), compare with (2.5). Thus, substituting (4.3) and (4.2) into (1.12) and (1.13) we get

$$E_s = \lim_{r_0\to\infty} \int_\Sigma dx dy dz \overset{\bullet}{\rho}(r) = \lim_{r_0\to\infty} \int_\Sigma dr d\theta d\phi \overset{\bullet}{\mathcal{J}}{}^{0}_{s} = \lim_{r_0\to\infty} \oint_{\partial\Sigma} d\theta d\phi \overset{\bullet}{\mathcal{J}}{}^{01}_{s} = M, \tag{4.4}$$

where the boundary $\partial\Sigma$ of $\Sigma$ presents a sphere $r = r_0$ again, and then one takes the limit $r_0 \to \infty$. The result (4.4) can be interpreted as the global mass of the static SBH, since at $r_0 \to \infty$ the 4-vector (4.1) asymptotically tends to the timelike Killing vector in the form (2.2). If the charge (4.4) is calculated at finite $r = r_0$, it can be interpreted as the energy measured by observers resting at $r = r_0$ and inside $r = r_0$. The acceptable result (4.4) shows us that the choice of the gauge as SIG corresponds to the problem of calculating the global mass. It is not surprisingly because SIG and SSG being identical ones give the same result $M$.

Following [16] we construct the related description of the SBH moving with a constant velocity $v$ with respect to distant observers. It has its own frame $\{\bar{x}^\alpha\}$ with the barred metric (3.1). Then, to describe the moving SBH in the frame $\{x^\alpha\}$ the authors of [16] apply the transformations (2.8) to the *barred* metric (3.1) and obtain the metric:

$$ds^2 = -\frac{\alpha^2\psi^4}{\gamma^2(\psi^4 - \alpha^2 v^2)} dt^2 + \gamma^2(\psi^4 - \alpha^2 v^2)(dx + \beta dt)^2 + \psi^4(dy^2 + dz^2). \tag{4.5}$$

Here, $\beta = -v(1 - \alpha^2/\psi^4)(1 - \alpha^2 v^2/\psi^4)^{-1}$, where $\alpha = \alpha(\bar{r})$ and $\psi = \psi(\bar{r})$ with $\bar{r}^2 = \bar{x}^2 + \bar{y}^2 + \bar{z}^2 = \gamma^2(x - vt)^2 + y^2 + z^2$. Thus, $\bar{r} = \text{const}$ presents a compressed sphere (ellipsoid) moving in the frame $\{x^\alpha\}$ with the constant velocity $v$ in direction $x$.

Let us turn to the *proper* reference frame of the moving SBH $\{\overline{x}^\alpha\}$ defined by the *barred* metric (3.1) and related observers analogous to (4.1). Repeating all the steps done for the static SBH and preserving the SIG (that has to be accented), we get in the coordinates $(\bar{t}, \bar{x}, \bar{y}, \bar{z})$:

$$\overset{\bullet}{\mathcal{J}}{}_{s}^{\overline{\alpha}}(\overline{r}) = \left[\overset{\bullet}{\rho}(\overline{r}),\ 0,\ 0,\ 0\right], \tag{4.6}$$

where the dependence $\overset{\bullet}{\rho}(\overline{r})$ is exactly the same as defined for (4.3) and $\overline{r}^2 = \overline{x}^2 + \overline{y}^2 + \overline{z}^2$. We emphasize that in the frame $\{\overline{x}^\alpha\}$ we, of course, repeat the result (4.4): $\overline{E}_s = M$ for the global mass.

Because the gauge is already chosen, and the solution (4.5) is obtained from the barred metric (3.1) with the use of (2.8), the covariant formalism [12, 13] allows us to transform the components of the current (4.6) with the use of (2.8) to the frame $\{x^\alpha\}$:

$$\overset{\bullet}{\mathcal{J}}{}_{m}^{\alpha}(\overline{r}) = \left[\gamma\overset{\bullet}{\rho}(\overline{r}),\ \gamma v\overset{\bullet}{\rho}(\overline{r}),\ 0,\ 0\right]. \tag{4.7}$$

Formally (4.7) coincides with the current (2.10) for a matter ball in Minkowski space. Likewise, the integration for the components of (4.7) actually repeats the integration in (2.11) and (2.12). The only difference is that according to (1.13) one has to go to the surface integration like in (4.4), and then take the limit $r' = r_0 \to \infty$. Finally, we get the global mass for the moving black hole:

$$E_m = \gamma M \tag{4.8}$$

and the global momentum for the moving black hole

$$P^1_m = \gamma v M \tag{4.9}$$

the same as (2.11) and (2.12) for the moving matter ball. Note that inner surfaces are ignored in all surface integrations. This position is in a correspondence with Einstein's point of view [1] that energy of an isolated system is determined by external boundary conditions only.

## 5. A direct application of the fully covariant formalism in TEGR and a "moving gauge"

Up to now, to construct conserved quantities for the moving SBH we have used analogies in Minkowski space. By this, it was taken into account the fully covariance of our formalism [12, 13], but the formalism itself was not applied totally. In this section, we do it.

In the proper frame $\{\bar{x}^\alpha\}$ for the related SIG, the components $\overset{m}{S}{}_{\bar{\sigma}}{}^{\bar{\alpha}\bar{\beta}}$ of $S$-tensor in (1.10) are exactly the components (3.5) in the *barred* form only. Due to the fully covariant formalism we represent the components $\overset{m}{S}{}_{\bar{\sigma}}{}^{\bar{\alpha}\bar{\beta}}$ as $\overset{m}{S}{}_{\sigma}{}^{\alpha\beta}$ in the frame $\{x^\alpha\}$ with the use of the coordinate transformation (2.8). Of course, the components of $\overset{m}{S}{}_{\sigma}{}^{\alpha\beta}$, being very cumbersome ones, differ from the components of $\overset{\bullet}{S}{}_{\sigma}{}^{\alpha\beta}$ in (3.5) if both of them are in the same frame $\{x^\alpha\}$.

To calculate total energy for the moving SBH in the frame $\{x^\alpha\}$ we use again the general formulae (1.12) and (1.13). Note that under integration we use only (1.12) with zero curent component, not (2.4) or (2.12). Thus

$$E_m = \lim_{r\to\infty} \int_{\Sigma_r} dxdydx\, \overset{\bullet}{\mathcal{J}}{}_{m}^{0}(\xi) = \lim_{r\to\infty} \int_{\Sigma_r} dxdydz\, \partial_k \overset{\bullet}{\mathcal{J}}{}_{m}^{0k}(\xi). \tag{5.1}$$

Here, for the sake of simplicity we use $r$ instead of $r_0$ under limits. To evaluate this expression, we exploit the following. First, we have already carried out the easier calculation for the static case in isotropic coordinate system in SIG gauge. Second, since we consider $r \to \infty$, terms which make no contribution to this limit may be neglected. Thus, the integration (5.1) takes place on a $t = \text{const}$ slice, and without the loss of generality we set $t = 0$, after that on this slice we make the coordinate transformation $x = \bar{x}/\gamma, y = \bar{y}, z = \bar{z}$. From here the relations $r^2 = \bar{r}^2/\gamma^2 + v^2(\bar{y}^2 + \bar{z}^2)$ and $\bar{r}^2 = r^2 + \gamma^2 v^2 x^2$ follow easily. Therefore one can replace $r \to \infty$ by $\bar{r} \to \infty$ and similarly on the slice $t = \text{const}$, and, thus, (5.1) is rewritten as

$$E_m = \lim_{\bar{r}\to\infty} \frac{1}{\gamma} \int_{\Sigma_{\bar{r}}} d\bar{x}d\bar{y}d\bar{z}\, \partial_k \overset{\bullet}{\mathcal{J}}{}_{m}^{0k}(\xi). \tag{5.2}$$

Here, the limit is carried out for the surface $\partial\Sigma_{\bar r}$ defined by $\bar r = \text{const}$. The integrand in (5.2) is rewritten as

$$\partial_k \overset{\bullet}{\mathcal{J}}{}^{0k}_{m}(\xi) = \partial_\beta \overset{\bullet}{\mathcal{J}}{}^{0\beta}_{m}(\xi) = \frac{1}{\kappa}\partial_\beta\left(h(x^\alpha)\overset{m}{S}_\sigma{}^{0\beta}\xi^\sigma\right) = \frac{1}{\kappa}\partial_{\bar\beta}\left(h(\bar r)\overset{m}{S}_\sigma{}^{0\bar\beta}\xi^\sigma\right). \tag{5.3}$$

Because determinant of the Lorentz transformations (2.8) is equal to unit one can transform $h(x^\alpha) = \det h^a{}_\mu$ simply to $h(\bar r) = \det h^a{}_{\bar\mu}$. Thus the last equality is holding due to the covariance of the divergence which involves the correct density weight. As an observer we can choose again the static observer with the proper vector (4.1) for which $\xi^\sigma \to (-1,0,0,0)$ at $r, \bar r \to \infty$. Finally, to carry out the calculation of (5.2), we need the asymptotic behavior at $r,\bar r \to \infty$ of the quantity

$$\overset{m}{S}_0{}^{0\bar\beta}\xi^0 = \overset{m}{S}_{\bar\sigma}{}^{\bar\alpha\bar\beta}\frac{\partial x^0}{\partial \bar x^{\bar\alpha}}\frac{\partial \bar x^{\bar\sigma}}{\partial x^0}\xi^0 = -\overset{m}{S}_{\bar 0}{}^{\bar 0\bar\beta}\frac{\partial t}{\partial \bar t}\frac{\partial \bar t}{\partial t} + \text{neglectable terms}. \tag{5.4}$$

The last terms are not important due to asymptotic behaviour in (3.5) presented in barred coordinates. Taking into account (2.8) we rewrite (5.3) as

$$\partial_k \overset{\bullet}{\mathcal{J}}{}^{0k}_{m}(\xi) = \gamma^2 \partial_{\bar k}\overset{\bullet}{\mathcal{J}}{}^{\bar 0\bar k}_{s}(\xi) + \text{neglectable terms} \tag{5.5}$$

and substitute it into (5.2)

$$E_m = \gamma \lim_{\bar r\to\infty}\int_{\Sigma_{\bar r}} d\bar x d\bar y d\bar z\, \partial_{\bar k}\overset{\bullet}{\mathcal{J}}{}^{\bar 0\bar k}_{s}(\xi) = \gamma \lim_{\bar r\to\infty}\oint_{\partial\Sigma_{\bar r}} d\bar\theta d\bar\phi\, \overset{\bullet}{\mathcal{J}}{}^{\bar 0\bar 1}_{s}(\xi) = \gamma E_s = \gamma M. \tag{5.6}$$

Calculations are carried out with the use of barred coordinates $(\bar x, \bar y, \bar z)$ introduced now on the slice $t = \text{const}$. The last integral is written in spherical coordinates, in fact, repeating the calculation (4.4), that gives the energy $E_s$ of the static SBH.

The formulae (1.12) and (1.13) for defining global conserved quantities are quite universal and a choice of vector $\xi^\alpha$ determines their interpretation. Thus, formulae (5.1)-(5.3) are left universal up to the choice of vector $\xi^\alpha$. In order to calculate the momentum expression of for the moving SBH we choose $\xi^\alpha = (0,\xi^1,0,0)$ when $\xi^1 \to 1$ at $r\to\infty$ that specifies an $x$-translation Killing vector at $r\to\infty$. In this case we need to derive an asymptotics of the expression:

$$\overset{m}{S}_1{}^{0\bar\beta}\xi^1 = \overset{m}{S}_{\bar\sigma}{}^{\bar\alpha\bar\beta}\frac{\partial x^0}{\partial \bar x^{\bar\alpha}}\frac{\partial \bar x^{\bar\sigma}}{\partial x^1}\xi^1 = \overset{m}{S}_{\bar 0}{}^{\bar 0\bar\beta}\frac{\partial t}{\partial \bar t}\frac{\partial \bar t}{\partial x} + \text{neglectable terms}. \tag{5.7}$$

Again, the last terms are not important due to asymptotic behaviour in (3.5) presented in barred coordinates. Note that sign ‘minus’ in barred (3.5) for $\overset{m}{S}_{\bar 0}{}^{\bar 0\bar i}$ and sign ‘minus’ in $\partial\bar t/\partial x = -v\gamma$ are compensated, and the leading term in (5.7) is positive. Taking into account (2.8), we rewrite (5.3) for the asymptotically $x$-translation vector $\xi$ as

$$\partial_k \overset{\bullet}{\mathcal{J}}{}^{0k}_{m}(\xi) = v\gamma^2 \partial_{\bar k}\overset{\bullet}{\mathcal{J}}{}^{\bar 0\bar k}_{s}(\xi) + \text{neglectable terms}. \tag{5.8}$$

Thus, analogously to (5.6) one has for the total momentum of the moving SBH:

$$P_m = v\gamma \lim_{\bar r\to\infty}\int_{\Sigma_{\bar r}} d\bar x d\bar y d\bar z\, \partial_{\bar k}\overset{\bullet}{\mathcal{J}}{}^{\bar 0\bar k}_{s}(\xi) = v\gamma \lim_{\bar r\to\infty}\oint_{\partial\Sigma_{\bar r}} d\bar\theta d\bar\phi\, \overset{\bullet}{\mathcal{J}}{}^{\bar 0\bar 1}_{s}(\xi) = v\gamma E_s = v\gamma M. \tag{5.9}$$

One can see that the results (5.6) and (5.9) are acceptable from the point of view of relativistic theory.

Returning to gauges introduced in [14], we recall that both SSG (connected with a static tetrad) and different from it LG (connected with a freely falling tetrad) are not in the same equivalence class, but nonetheless are both obtained by the “switching off” gravity principle. However, in [15], we have shown that the static tetrad of SSG *after a radial boost* together with the *unchanged* related ISC leads to the LG itself. Here, to construct the ‘moving gauge’ in a more economical way we follow the logic of [15] and provide the boost

$$(\Lambda_{boost})^{a'}{}_b = \begin{pmatrix} \gamma & v\gamma & 0 & 0\\ v\gamma & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}, \tag{5.10}$$

for the tetrad (3.2) that transforms it to the tetrad moving correspondingly to the moving SBH. At the same time we *preserve* zero ISC. On the other hand, to be staying in the framework of SIG, simultaneously with changing the tetrad we have to apply the global boost (5.10) to zero ISC in correspondence with (1.6). We see that the zero ISC is left to be zero. Thus, the "moving gauge" and the SIG are the same unique gauge. This means that the acceptable results for both the total energy and the total momentum for the moving SBH, in fact, have been obtained in the framework of the related appropriate "moving gauge". Of course, the aforementioned "moving gauge" can be obtained by the "switching off" gravity principle for the moving (boosted) tetrad.

**Conclusion**

The formalism for constructing conserved quantities in TEGR [12, 13] has been applied to the SBH moving with a constant velocity with respect to distant static observers. These conserved quantities are calculated by two ways: first, in analogy with the matter ball model in Minkowski space; second, by the technology of the TEGR itself. The expected energy and momentum are obtained in both the cases. The acceptable results follow due to the fully covariance of the formalism. However there is ambiguity in the definition of gauges. To avoid an ambiguity in such calculations an appropriate gauge defined by a pair of tetrad and related inertial spin connection has to be defined [14, 15]. Such a gauge is found for the moving SBH and we stress that the SIG is the unique gauge both for the static and moving tetrads, and SIG is just the SSG introduced in [14].

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