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О ЛЕПТОНАХ В ТЕОРИИ ПРОСТРАНСТВЕННО-ВРЕМЕННОЙ ПЛЁНКИЧерницкий А. А.^{a,b,1}^a Санкт-Петербургский Государственный Химико-Фармацевтический Университет, г. Санкт-Петербург, 197376, Россия.^b Фридмановская Лаборатория Теоретической Физики, г. Санкт-Петербург, Россия.

В данной работе мы продолжаем исследование задачи нахождения тороидальных солитонных решений уравнения пространственно-временной плёнки, которые могут представлять заряженные лептоны. Введена квази-цилиндрическая комплексная тороидальная система координат с вращением и получено уравнение пространственно-временной плёнки в этой системе. Предложен способ нахождения решения в виде формального ряда по обратным степеням радиуса тороидального кольца. Обсуждается начальное приближение для этого метода.

Ключевые слова: Пространственно-временная плёнка, space-time film, лептон, lepton, полевая модель заряженного лептона, field model of charged lepton.

ABOUT LEPTONS IN SPACE-TIME FILM THEORYChernitskii A. A.^{a,b,1}^a St. Petersburg State Chemical and Pharmaceutical University, St. Petersburg, 197376, Russia.^b A. Friedmann Laboratory for Theoretical Physics, St.-Petersburg, Russia

In the present work, we continue the investigation of the problem for finding the toroidal soliton solutions of space-time film equation, which can represent the charged leptons. The quasi-cylindrical complex toroidal coordinate system with rotation is introduced and the appropriate equation of space-time film is obtained. We propose the way for finding the solutions in the form of formal series in negative powers of the radius of the toroidal ring. The initial approximation for this method is discussed.

Keywords: Space-time film, lepton, field model of charged lepton.

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Introduction

In the given work, we continue to consider possible toroidal solutions of the space-time film (STF) equation [1] associated with leptons [3, 4].

Earlier we have obtained the exact solutions of STF equation in the form of twisted or spiral solitons which are rectilinearly moving with the speed of light [1]. These spiral light-like solitons are characterized by the twist parameter m . It was shown that the solitons of the first twist order ($m = 1$) can be associated with photons. We suppose that the light-like solitons of higher twist orders $m > 1$ can be associated with various neutrinos.

Moreover, the static solution in the form of charged cylindrical shell were obtained in the work [1].

The toroidal solutions under consideration are circinate combinations of the charged cylindrical shell and the twisted light-like solitons.

¹E-mail: alexander.chernitskii@pharminnotech.com, AAChernitskii@mail.ru

1. Equation of space-time film in coordinates with rotation

The action for space-time film has the following form [1]:

$$\mathcal{A} = \int_{\bar{V}} \mathcal{L} d\bar{V}, \quad \mathcal{L} \doteq \sqrt{\left| 1 + \chi^2 \mathbf{m}^{\mu\nu} \frac{\partial \Phi}{\partial x^\mu} \frac{\partial \Phi}{\partial x^\nu} \right|} \quad (1.1)$$

where $d\bar{V} \doteq \sqrt{|\mathbf{m}|} (dx)^4$ is a four-dimensional volume element,, $\mathbf{m} \doteq \det(\mathbf{m}_{\mu\nu})$, Φ is the scalar field function, $\mathbf{m}_{\mu\nu}$, $\mathbf{m}^{\mu\nu}$ are components of metric tensor for an arbitrary coordinate system in flat space-time. The Greek indexes take the values $\{0, 1, 2, 3\}$. $x^0 \doteq \epsilon t$ is the time coordinate, where t is time and ϵ is the velocity of light in free space.

Here we consider the signature of metric $\{-, +, +, +\}$, that is we have $-\mathbf{m}^{00} = \mathbf{m}^{11} = \mathbf{m}^{22} = \mathbf{m}^{33} = 1$ in Cartesian coordinates. In the base work [1], the opposite metric $\{+, -, -, -\}$ was also considered. But the investigation of the gravitational interaction in the framework of STF theory shows that the signature must be $\{-, +, +, +\}$ [2].

The stationary condition for the action (1.1) gives the following equation:

$$\frac{1}{\sqrt{|\mathbf{m}|}} \frac{\partial}{\partial x^\mu} \frac{\sqrt{|\mathbf{m}|} \mathbf{m}^{\mu\nu}}{\mathcal{L}} \frac{\partial \Phi}{\partial x^\nu} = 0, \quad (1.2)$$

Let us consider the class of four-dimensional space-time coordinate systems having the azimuthal coordinate or azimuth angle φ , called also horizontal angle. Let us introduce the following coordinate transformation for such coordinate systems:

$$\check{\theta} = \varphi - \tilde{\omega} x^0, \quad \check{\vartheta} = \varphi + \tilde{\omega} x^0. \quad (1.3)$$

The coordinates $\check{\theta}$ and $\check{\vartheta}$ can be called right and left phase coordinates accordingly. The positive parameter $\tilde{\omega} > 0$ meaning of the angular velocity. It is evident that a function depending from one phase coordinate $\check{\theta}$ or $\check{\vartheta}$ is right- or left-rotating field configuration relatively vertical axis x^3 or z .

The densities of energy \mathcal{E} and vertical component of angular momentum \mathcal{J}_z have the following form in the coordinate system with rotation:

$$\mathcal{E} = \frac{1}{4\pi} \left(\frac{\tilde{\omega}^2}{\mathcal{L}} \left(\frac{\partial \Phi}{\partial \check{\theta}} - \frac{\partial \Phi}{\partial \check{\vartheta}} \right)^2 + \frac{1}{\chi^2} (\mathcal{L} - 1) \right), \quad (1.4a)$$

$$\mathcal{J}_z = \frac{\tilde{\omega}}{4\pi \mathcal{L}} \left(\left(\frac{\partial \Phi}{\partial \check{\theta}} \right)^2 - \left(\frac{\partial \Phi}{\partial \check{\vartheta}} \right)^2 \right), \quad (1.4b)$$

Here we consider the rational toroidal coordinate system [3, 4]. This system is obtained from the usual toroidal one $\{x^0, \kappa, \nu, \varphi\}$ with the help of the following change of the variable κ ($0 \leq \kappa \leq \infty$): $\bar{\kappa} = e^\kappa - 1$. This system is convenient because its components of metrical tensor are the rational functions of the variable $\bar{\kappa}$:

$$\mathbf{m}_{00} = -1, \quad \mathbf{m}_{\bar{\kappa}\bar{\kappa}} = \frac{\rho_0^2}{4\bar{\mathcal{K}}^2}, \quad \mathbf{m}_{\nu\nu} = \frac{(\bar{\kappa} + 1)^2 \rho_0^2}{4\bar{\mathcal{K}}^2}, \quad \mathbf{m}_{\varphi\varphi} = \frac{\bar{\kappa}^2 (\bar{\kappa} + 2)^2 \rho_0^2}{16\bar{\mathcal{K}}^2}, \quad (1.5a)$$

$$\bar{\mathcal{K}} \doteq \frac{1}{4} (2 + \bar{\kappa} (\bar{\kappa} + 2) - 2(\bar{\kappa} + 1) \cos \nu) = (\bar{\kappa} + 1) \sin^2 \left(\frac{\nu}{2} \right) + \frac{\bar{\kappa}^2}{4}. \quad (1.5b)$$

where ρ_0 is the radius of the singular ring of the coordinate system.

We will consider, in particular, the field configurations with singularity on the toroidal surface $\kappa = \check{\kappa} = \text{const}$ or $\bar{\kappa} = \check{\bar{\kappa}} = \text{const}$. The large $\check{\rho}_b$ and small $\check{\bar{\rho}}$ radii of the toroid is given by the formulas

$$\check{\rho}_b = \rho_0 \coth \check{\kappa} = \rho_0 \left(\frac{1}{\check{\nu}} + \frac{1 + \check{\kappa}}{2 + \check{\kappa}} \right), \quad \check{\bar{\rho}} = \rho_0 \operatorname{csch} \check{\kappa} = \rho_0 \left(\frac{1}{\check{\kappa}} + \frac{1}{2 + \check{\nu}} \right). \quad (1.6)$$

As we see, the large radius of the toroid $\check{\rho}_b$ is not coincide with the radius of the singular ring ρ_b of toroidal coordinate system.

Then we use the rational toroidal coordinates with rotation $\{\check{\theta}, \check{\vartheta}, \bar{\kappa}, \nu\}$. The metrical tensor for this coordinate system is not diagonal. But the metrical tensor for the truncated systems $\{\check{\theta}, \bar{\kappa}, \nu\}$ and $\{\check{\vartheta}, \bar{\kappa}, \nu\}$ is diagonal.

In the present work, we take the following relation between the parameter of angular velocity and the large radius of the toroid:

$$\tilde{\omega} = \frac{1}{\rho_b}. \quad (1.7)$$

Let us introduce also the coordinates $\{\check{\rho}, \check{\varphi}, \check{z}\}$, which can be named quasi-cylindrical toroidal (QCT) ones. We will consider two variants of such coordinates in accordance with the following formulas:

$$\check{\rho} = \frac{2\rho_b}{\bar{\kappa}}, \quad 0 \leq \check{\rho} < \infty, \quad \check{\varphi} = -\nu, \quad \check{z} = \rho_b \varphi, \quad (1.8a)$$

$$\check{\rho} = \rho_b \operatorname{sech} \kappa, \quad 0 \leq \check{\rho} \leq \rho_b, \quad \check{\varphi} = -\nu, \quad \check{z} = \rho_b \varphi. \quad (1.8b)$$

The coordinate systems (1.8a) and (1.8b) can be called unlimited and limited QCT accordingly.

Metrical tensor for these coordinate system passes to metrical tensor of cylindrical system when $\rho_b \rightarrow \infty$.

Also we introduce the complex quasi-cylindrical toroidal coordinates by the formulas

$$\check{\xi} \doteq \check{\rho} e^{i\check{\varphi}}, \quad \check{\xi}^* \doteq \check{\rho} e^{-i\check{\varphi}}. \quad (1.9)$$

Next step is the transformation to phase-complex coordinates $\{\check{\theta}, \check{\vartheta}, \check{\xi}, \check{\xi}^*\}$ in the nonlinear equation of the model. The obtained equation is reduced to the following form:

$$\left(1 + 2\chi^2 \frac{\partial \Phi}{\partial \check{\xi}} \frac{\partial \Phi}{\partial \check{\xi}^*}\right) \frac{\partial^2 \Phi}{\partial \check{\xi} \partial \check{\xi}^*} - \chi^2 \left(\left(\frac{\partial \Phi}{\partial \check{\xi}}\right)^2 \frac{\partial^2 \Phi}{(\partial \check{\xi})^2} + \left(\frac{\partial \Phi}{\partial \check{\xi}^*}\right)^2 \frac{\partial^2 \Phi}{(\partial \check{\xi}^*)^2} \right) = \sum_{l=1}^{1_{max}} \frac{1}{\rho_b^l} Q_l \left(\chi, \check{\xi}, \check{\xi}^*, \frac{\partial \Phi}{\partial q_i}, \frac{\partial^2 \Phi}{\partial q_j \partial q_k} \right), \quad (1.10)$$

where $\{q_i\} = \{\check{\theta}, \check{\vartheta}, \check{\xi}, \check{\xi}^*\}$. For variant (1.8a) $1_{max} = 14$, the functions Q_l depend on integral and half-integral powers of variables $\{\check{\xi}, \check{\xi}^*\}$, they are polynomials of the rest arguments. For variant (1.8b) $1_{max} = 7$, the functions Q_l are polynomials of all arguments.

It is evident that the right side of the equation (1.10) vanishes for $\rho_b \rightarrow \infty$. Notable that the left side of the equation does not contain the derivatives with respect to variables $\{\check{\theta}, \check{\vartheta}\}$. This property is the consequent of the condition (1.7), because that the derivatives with respect to phases are small for $\rho_b \rightarrow \infty$.

Here we will consider a rotated field configurations depending from the three variables $\{\check{\theta}, \bar{\kappa}, \nu\}$. For such solutions we have the following notable expressions:

$$\mathcal{E} = \tilde{\omega} \mathcal{J}_z + \frac{1}{4\pi \chi^2} (\mathcal{L} - 1), \quad \mathcal{J}_z = \frac{\tilde{\omega}}{4\pi \mathcal{L}} \left(\frac{\partial \Phi}{\partial \check{\theta}} \right)^2 \quad (1.11)$$

Energy and angular momentum of a soliton-particle are

$$\mathbb{E} \doteq \int_V \mathcal{E} dV, \quad \mathbb{J} \doteq \int_V \mathcal{J}_z dV, \quad (1.12)$$

where V is the three-dimensional space without of the interior of the singular shell, in particular, toroidal one.

2. Charged lepton as toroidal soliton

Here we use the following dimensionless function and parameter for a solution:

$$\underline{\Phi} \doteq \frac{\rho_b}{\bar{e}} \Phi, \quad \varepsilon \doteq \frac{\bar{e}^2 \chi^2}{\rho_b^4}, \quad (2.1)$$

where \bar{e} is the elementary electrical charge. The parameter ε used here differs by sign from one introduced in works [3, 4], where the field model with two metric signatures were considered.

Using these designations and relation (1.7), we have the following expressions for spin and energy densities for a rotating solution:

$$\mathcal{J}_z = \frac{\bar{e}^2}{4\pi \rho_b^3 \mathcal{L}} \left(\frac{\partial \underline{\Phi}}{\partial \underline{\theta}} \right)^2, \quad \mathcal{E} = \tilde{\omega} \mathcal{J}_z + \frac{\bar{e}^2}{4\pi \rho_b^4 \varepsilon} (\mathcal{L} - 1). \quad (2.2)$$

We take the following empirical conditions for leptons:

$$\mathbb{E} = \hbar \omega, \quad \mathbb{J} = \frac{\hbar}{2}. \quad (2.3)$$

Also we take that the length of the singular ring of the toroidal coordinate system is multiple to Compton wave-length λ_C of a charged lepton:

$$2\pi \rho_b = n \lambda_C \implies \rho_b = \frac{n}{\omega} \implies \omega = n \tilde{\omega}. \quad (2.4)$$

Using (2.3) and (2.4) we have the following conditions for charged leptons:

$$\tilde{\mathbb{J}} \doteq \frac{1}{\bar{e}^2} \mathbb{J} = \frac{\bar{e}^2}{4\pi \rho_b^3} \int_V \frac{1}{\mathcal{L}} \left(\frac{\partial \underline{\Phi}}{\partial \underline{\theta}} \right)^2 dV = \frac{\alpha^{-1}}{2}, \quad (2.5a)$$

$$\tilde{\mathcal{A}} \doteq \frac{1}{\bar{e}^2} (\rho_b \mathbb{E} - \mathbb{J}) = \int_V \frac{1}{4\pi \rho_b^3 \varepsilon} (\mathcal{L} - 1) dV = \alpha^{-1} \left(n - \frac{1}{2} \right), \quad (2.5b)$$

where α is fine structure constant, $\alpha^{-1} \approx 137$, $\tilde{\mathbb{J}}$ is dimensionless spin, $\tilde{\mathcal{A}}$ is dimensionless time density of regularized action.

Using relations(2.5) let us write the following expression for energy of a charged toroidal lepton:

$$\mathbb{E} = \frac{\bar{e}^2}{\rho_b} (\tilde{\mathbb{J}} + \tilde{\mathcal{A}}) = \frac{n \bar{e}^2}{\alpha \rho_b}. \quad (2.6)$$

3. Initial approximation to toroidal solution for charged lepton

We can try to find a solution in the form of formal series in the negative powers of the radius ρ_b of the toroidal ring. The equation of space-time film in phase-complex quasi-cylindrical toroidal coordinates (1.10) is suitable for this purpose. The left side of this equation is coincide with the equation for which the exact solutions were obtained in work [1].

For the solution associated with a charged lepton, it is naturally to take the initial approximation in the form of combination of the charged tubular shell and twisted lightlike soliton.

The exact solutions in [1] were obtained with the help of transformation to new complex coordinates (tilde ones), in which the equation becomes linear. The transformation matrix to new coordinates depends on the solution. The obtained elementary solutions have singular cylindrical surface in the tilde coordinates. But a combination of the elementary solutions have more complicated singular surface in the tilde coordinates. And the singular surfaces transformed to the base cylindrical coordinates have a complicated form even for the elementary solutions. Thus the singular surface of the desired solution is not an exact toroid. In this case we must consider the small toroid radius $\bar{\rho}$ in (1.6) as some mean quantity.

If we take the initial approximation in the form of referred combination of exact solutions for initial nonlinear equation in (1.10), then the behaviour of such field configuration at space infinity will not be correct. It is natural to consider that the desired solution at infinity is similar to an appropriate solution of linear equation. But dynamical solutions of wave equation in toroidal coordinates are not known. We can consider the rotating static solution as an approximation to such solution.

The necessary static solution of linearized equation of the field model under consideration follows from the known solution of the Laplace equation in toroidal coordinates [5]:

$$\underline{\Phi} = \sqrt{\bar{\kappa}} \dot{\Phi}_{nm} \left(C_1 \cos(n\varphi) + C_2 \sin(n\varphi) \right) \left(C_3 \cos(mv) + C_4 \sin(mv) \right), \quad (3.1a)$$

$$\dot{\Phi}_{nm} \doteq \frac{2^{1-2|m|+|n|} \bar{\kappa}^{|n|}}{(\bar{\kappa}+1)^{|m|} (\bar{\kappa}+2)^{1-2|m|+|n|}} \mathcal{F}_{\frac{1}{2}-|m|, \frac{1}{2}-|m|+|n|; |n|+1} \left(\frac{\bar{\kappa}^2}{(\bar{\kappa}+2)^2} \right) \quad (3.1b)$$

$$= \bar{\kappa}^{|n|} \left(1 - \frac{|n|+1}{2} \bar{\kappa} + \mathcal{O}(\bar{\kappa}^2)_{\bar{\kappa} \rightarrow 0} \right), \quad (3.1c)$$

where n and m are integer numbers, $\varphi = (\check{\theta} + \bar{\theta})/2$, $\mathcal{F}_{\alpha, \beta; \gamma}(z) \doteq {}_2F_1(\alpha, \beta; \gamma; z)$ is a hypergeometric function, $\{C_1, C_2, C_3, C_4\}$ are arbitrary real constants.

Let us consider the following rotating static solution as an approximation to dynamical solution of the linear wave equation in toroidal coordinates:

$$\underline{\Phi}_{(0)} = \sqrt{\bar{\kappa}} \left(\dot{\Phi}_{00} + a_{nm} \dot{\Phi}_{nm} \sin(n\check{\theta} - mv) \right), \quad (3.2)$$

where the functions $\dot{\Phi}_{nm}$ are defined in (3.1b).

Here in (3.2), the function $\dot{\Phi}_{00}$ is the static part which defines the charge of the particle. The dynamic part with amplitude a_{nm} defines the wave or quantum behaviour of the particle.

Let us consider the asymptotic behaviour of the dynamic part in (3.2) near ring of the quasi-cylindrical toroidal coordinates (1.8). We have the following asymptotic expression for this dynamic part:

$$\underline{\Phi} \sim \frac{1}{\check{\rho}^{|m|}} \sin(n\check{\theta} + m\check{\varphi}), \quad \check{\rho} \rightarrow 0. \quad (3.3a)$$

where $|m| > 0$, $|n| > 0$, and according to (1.3), (1.7), (1.8) we have

$$\check{\theta} = \tilde{\omega} (\check{z} - x^0). \quad (3.3b)$$

This is a solution of the linear wave equation in cylindrical coordinates $\{x^0, \check{\rho}, \check{\varphi}, \check{z}\}$. This twisted wave propagates with the speed of light along the ring of the toroidal system.

Let us consider the asymptotic behaviour for the dynamic part of the rotating static solution near the origin of the cylindrical coordinates system $\{\rho, \varphi, z\}$ with the vertical axis z and the horizontal angle φ . In these coordinates we have the following asymptotic expression for the dynamic part:

$$\underline{\Phi} \sim \rho^{|n|} \sin\left(n\varphi + \tilde{\omega} (2mz - nx^0)\right), \quad \rho \rightarrow 0, \quad z \rightarrow 0. \quad (3.4)$$

where $|m| > 0$, $|n| > 0$.

The function (3.4) is also the asymptotic form of a solution of the linear wave equation in cylindrical coordinates $\{x^0, \rho, \varphi, z\}$. This twisted wave propagates with the phase velocity $n/(2m)$ along z axis and presents so called Bessel beam.

For the special case

$$n = 2m, \quad (3.5)$$

the function (3.4) is a twisted wave propagating with the speed of light. This case needs a separate consideration.

4. Conclusions

In the present work, we have continued the investigation of the problem for finding the toroidal soliton solutions of space-time film equation, which can represent the charged leptons.

The quasi-cylindrical complex toroidal coordinate system with rotation is introduced and the appropriate equation of space-time film is obtained. We propose the way for finding the solutions in the form of formal series in negative powers of the radius of the toroidal ring. The initial approximation for this method is discussed.

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Авторы

Черницкий Александр Александрович, к.ф.-м.н., доцент, Санкт-Петербургский Государственный Химико-Фармацевтический Университет, ул. Проф. Попова, д. 14, г. Санкт-Петербург, 197376, Россия.

E-mail: alexander.chernitskii@pharminnotech.com, AAChernitskii@mail.ru

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Authors

Chernitskii Alexander Aleksandrovich, Ph.D., Associate Professor, St. Petersburg State Chemical and Pharmaceutical University, Prof. Popov st., 14, St. Petersburg, 197376, Russia.

E-mail: alexander.chernitskii@pharminnotech.com, AAChernitskii@mail.ru

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