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ФИЛЬТРАЦИЯ ЭФФЕКТА ГРАВИТАЦИОННОГО СДВИГА ЧАСТОТЫ ДЛЯ СИГНАЛОВ РАДИОСВЯЗИ СО СПУТНИКАМИ ЗЕМЛИ

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В статье представлен конкретный метод измерения гравитационного сдвига частоты радиосигналов связи между космическим кораблем и наземными станциями слежения. Он основан на алгоритме максимального правдоподобия и использует предел Крамера-Рао для оценки точности обнаружения параметров сигнала. Численный пример иллюстрирует эффективность метода. Оно осуществляется с использованием банка данных, полученных в ходе миссии «Радиоастрон». Внимание сосредоточено на компенсации релятивистского эффекта Доплера и частотных шумов используемых в эксперименте стандартов.

Ключевые слова: ОТО, тест ППЭ, РадиоАстрон.

FILTERING OF THE GRAVITATIONAL FREQUENCY SHIFT EFFECT FOR RADIO COMMUNICATION SIGNALS WITH THE EARTH SATELLITES

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The paper presents a some specific method of the gravitational frequency shift measurement for communication radio signals between the spacecraft and ground tracking stations. It is based on the maximum likelihood algorithm and utilizes the Cramér-Rao bound to estimate the accuracy of signal parameter determination. It is carried out with the bank of data obtained during the “Radioastron” mission. Attention is concentrated at a compensation of the relativistic Doppler effect and frequency noises of the standards used in the experiment.

Keywords: Test of general relativity, Einstein Equivalence Principle, RadioAstron.

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Introduction

A possible violation of the equivalence principle can be checked by measuring by checking the deviation of the predicted gravitational “redshift” from the measured one:

$$\frac{\Delta f_{grav}}{f} = \frac{\Delta U}{c^2}(1+\varepsilon), \tag{0.1}$$

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where $\Delta f_{grav}/f$ - is the gravitational clock shift, ΔU is the difference in the gravitational potential between them, c is the speed of light, and ε is the phenomenological parameter of GR violation. High-precision measurement of the violation parameter is the goal of the experiment described in this paper. If the epsilon parameter turns out to be zero, we can consider the theory correct. To carrying out such an experiment, the RadioAstron satellite was used[4], equipped with a highly stable frequency and time standard, as well as on-board equipment for data exchange with tracking stations. For the first time, the idea of a space experiment to measure gravitational "redshift" was implemented in a specialized Gravity-Probe A mission[3]. Compensation of the Doppler effect was carried out online, due to the simultaneous operation of the one-way and two-way modes. The result of the experiment was the confirmation of the theory with an accuracy of 1.4×10^{-4}. The best result in the space experiment was obtained in the GALILEO experiment [5][6]. In this paper we present a description of the compensation of various noise coherent effects in the measurement of the gravitational frequency shift with the Radioastron spacecraft. The hardware complex of the spacecraft allowed operating in two communication modes: "1-way" and "2-way". "1-way" (satellite → GTS (ground tracking station)) communication mode is a signal based on the onboard H-maser frequency standard. "2-way" (GTS→satellite→GTS) the mode is synchronized by the ground H-maser frequency standard. The scheme of these operating modes is shown in the figure: A spacecraft launched into an evolving high eccentricity orbit around Earth with geocentric distances

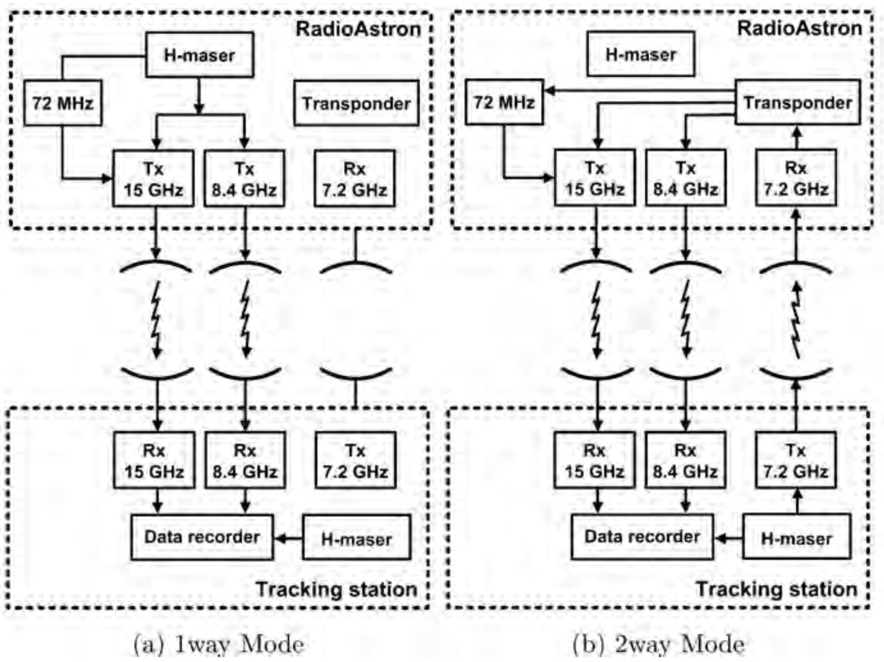


Рис. 1. Scheme of operating modes

reaching 353000km, see fig. 2.

The total duration of the operation of the satellite operation was occupied the period from the launch in 2011 to 2018. Special gravity sessions of approximately an hour duration with interleaved "1-way" and "2-way" mode were held from 2015 to 2018, when the onboard hydrogen frequency standard finished working. The presence of two signal transmission modes allowed for complete compensation of the first-order Doppler effect by interpolating the signal in 2way mode over sections of 1way mode. The signal was transmitted at two frequencies (8.4 GHz and 15GHz), the initial frequency estimation of the signal at the station was conducted using a regular frequency counter, which computed the signal spectrum with short signal interval divisions every 40 milliseconds and estimated the signal frequency with an accuracy of up to 2 mHz. These data were used in [7], in which the influence of the frequency noise of the on board frequency standard was considered. This article will consider the method of phase detection of the signal, the evaluation of its accuracy, as well as the technical features of processing gravitational sessions.

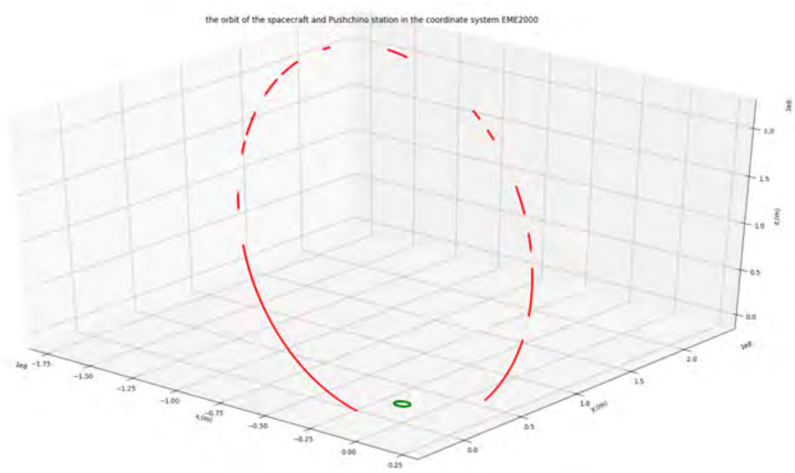


Рис. 2. The geocentric distance of the spacecraft (red), constructed using the data from the reconstructed orbit. The presence of gaps in the data is due to the absence of data from the reconstructed orbit.

1. Phase detector and Cramer-Rao bound

The concept of the experiment was based on the use of two modes of SC (spacecraft)-GTS communication to compensate for the dominant Doppler effect of the 1st order. The signal was transmitted from a ground tracking station at a frequency of 7.2075 GHz, taking into account the correction for the expected 1st Doppler effect, so as to get into the SC reception band. Then, after heterodyning, the signal was sent back to the GTS already at a frequency of 8.4 GHz and 15 GHz. At the station, the signal was again subjected to heterodyn with a frequency sampling of 32 MHz and 2-bit amplitude quantization. The signal was recorded in RDF (Radioastron Data Format) format similar to the Mark5b standard. The signal in 1w and 2w mode was recorded continuously in one file for each

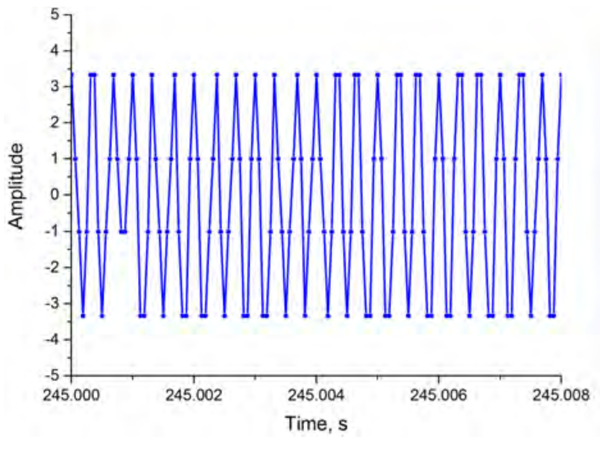


Рис. 3. Digital signal, 2-bit quantization, decrease SNR less than 1%

session, so it was necessary to divide the input recording into separate files according to their operating modes. For this task we used the library baseband[1] and information about switching the operating mode from the session's cyclogram. Recording in 1w mode lasted 80 seconds, in 2w mode 120 seconds. The phase detection algorithm assumes a narrowing of the signal band and the use of a bandpass filter. At

[1]https://doi.org/10.5281/zenodo.1214268

the output, we obtained a recording of a narrowband signal with a sampling frequency of 4 kHz and with subtracted frequency drift, which was then used in the phase detection algorithm. To extract the

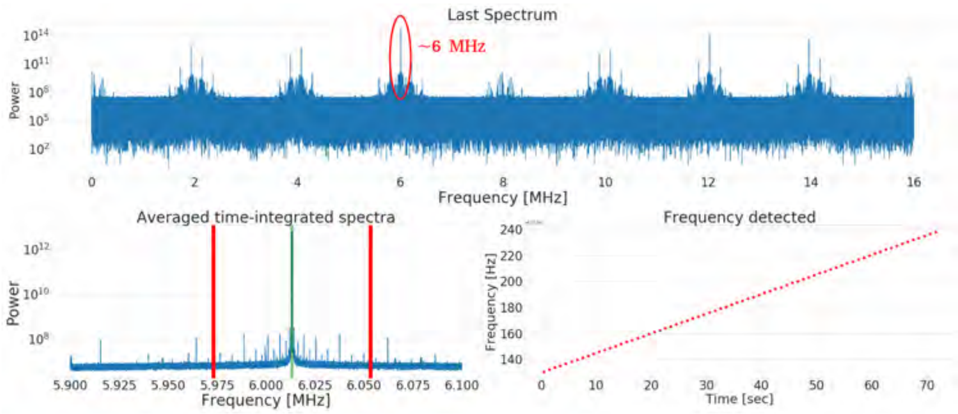


Рис. 4. The upper graph shows one of the spectra plotted on a one-second interval, at a frequency of about 6 MHz; the main tone is highlighted by which the signal frequency is determined. The lower-left graph shows the average signal spectrum over all intervals. In the red borders, we consider that there is noise, in the green borders, the signal. The lower right graph shows the dependence of frequency on time, each point corresponds to a frequency determined by the maximum in the spectrum.

phase from the temporal recording of the signal, it is necessary to convert it into an analytical form, which was done using the Hilbert transform. Next, we isolated the quadrature components of the signal by computing the signal's phase and unwrapping it.[2]. As a result, we estimated the instantaneous phase of the signal, which needs to be fitting by a polynomial, the degree of which was chosen taking into account the maneuverability of the SC and the frequency noise of the SHM (spacecraft H-Maser). To calculate the errors of the coefficients of the resulting polynomial, we used the Cramer-Rao bound.[10] Let's consider a mixture of signal and noise at the input of the receiver

$$y(t) = S(t, \mathbf{a}) + n(t), \quad 0 < t < T, \tag{1.1}$$

$S(t, \mathbf{a})$ is a useful narrowband signal, the spectrum of which is concentrated in a narrow band:

$$\Delta\omega = (\omega_2 - \omega_1) \ll \omega_0 = (\omega_1 + \omega_2)/2 \tag{1.2}$$

$$< n(t)n(t+\tau) >= N\delta(\tau) - \text{gaussian white noise} \tag{1.3}$$

Primary information processing (discrete time):

$$\left.\begin{array}{ccc} y(t) & \rightarrow & ||\Psi_0 \dots \Psi_m||^T \\ \Psi_k & = & \varphi_d(t_k) + \eta(t_k) + \varphi_n(t_k) \end{array}\right\} SNR = \frac{A^2}{\sigma^2} \gg 1 \tag{1.4}$$

where $\varphi_d(t_k)$ Doppler frequency shift on the k interval, $\eta(t_k)$ - frequency noise, σ^2 - varianc of additive noise $n(t)$ in the band (ω_1, ω_2).

In matrix form:

$$\begin{array}{c} \vec{\Psi} = ||\Psi_0 \dots \Psi_m||^{\mathrm{T}} = \vec{S}_\varphi + \vec{n}_\varphi \\ \vec{S}_\varphi \quad = ||\varphi_d(t_0) \dots \varphi_d(t_m)||^T - \text{is a useful signal} \end{array} \tag{1.5}$$

depending on the vector parameter $\mathbf{a} = ||a_0, \ \dots \ , a_L||^{\mathrm{T}}$,

$$\varphi_d(t) = \sum_{i=0}^{L} a_i t^i \tag{1.6}$$

[2]https://numpy.org/doc/stable/reference/generated/numpy.unwrap.html

$\vec{n}_f$ - discrete Gaussian noise, with a correlation matrix $\vec{K}_\varphi$:

$$\vec{K}_\varphi = [K_{\varphi,ij}] = \frac{1}{2}[D(t_i) + D(t_j) - D(t_i - t_j)] + \frac{\sigma^2}{A^2}\delta_{ij}, \tag{1.7}$$

where $D(\tau)$ is a structural function, $\delta_i j$ is a Kronecker symbol. In the linear signal model , the elements of the Fisher information matrix are written:

$$I_{ij} = -\left\langle \frac{\partial^2 \ln \Lambda(y|\mathbf{a})}{\partial a_i \partial a_j} \right\rangle = \frac{1}{2}\frac{\partial^2 a}{\partial a_i \partial a_j} \tag{1.8}$$

$$I_{ij} = ||t_0^i \ldots t_m^i||\vec{K}_m \begin{Vmatrix} t_0^j \\ \vdots \\ t_m^j \end{Vmatrix} \tag{1.9}$$

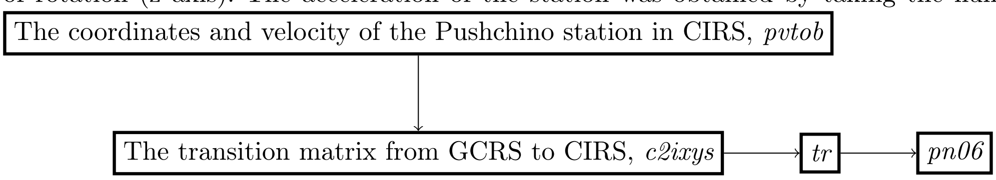
Fischer information matrix:

$$I = \begin{Vmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{Vmatrix} \rightarrow I^{-1} = \begin{Vmatrix} \sigma^{\mathbf{2}}_{\mathbf{11}} & \cdot & \cdot \\ \cdot & \sigma^{\mathbf{2}}_{\mathbf{22}} & \cdot \\ \cdot & \cdot & \sigma^{\mathbf{2}}_{\mathbf{33}} \end{Vmatrix} \tag{1.10}$$

The formulas 1.9, 1.10 define the Cramer-Rao variance of unknown parameters in the presence of frequency noise, which are taken into account in the structural function $D(\tau)$. After processing all the accumulated data that does not have artifacts in the record, sessions with an SNR greater than 50 db. they give an estimate of the variance of the coefficients of the polynomial at a level less than 2×10^{-4}, which in turn gives a potential possibility to estimate the error of ε no less than 4×10^{-5}.

2. Relativistic Doppler effect

To calculate the second-order Doppler effect, it is necessary to know the coordinates, speed and acceleration of the Pushchino station and spacecraft in the EME2000 system. Since these data are provided in the ITRF system, it was necessary to transform to the EME2000 coordinate system, in which the coordinate and velocity of the spacecraft are given. Geodetic coordinates have been converted to EME2000 using the SOFA library [11]. We used the pysofa2 python library[3] which had to be supplemented with the necessary function wrappers. Using the *pvtob* function, the coordinates and velocity of the Pushchino station in CIRS were obtained, then the transition matrix from GCRS to CIRS was called using the *c2ixys* function, transposed using the *tr* function built into the sofa library and multiplied by the coordinates and velocities of the Pushchino station in CIRS. We get the coordinates in GCRS, and then we get the matrix (frame bias) using the *pn06* function, multiply by the coordinates in GCRS and get the coordinates in EME2000. The EME2000 coordinate system is close to GCRS, the difference between these coordinate systems on the Earth's surface is 0.7 m. The difference consists in determining the position of the earth's axis of rotation (z axis). The acceleration of the station was obtained by taking the numerical derivative.



The compensatory switching scheme allowed for the compensation of the effect without taking the acceleration of the spacecraft into account.[12] In the 1-way mode, the received signal on the GTS had a frequency offset:

$$\frac{\Delta f_{1w}}{f_{nominal}} = -\frac{\dot{D}}{c} + \frac{\Delta U}{c^2} + \frac{|\vec{v}_e - \vec{v}_s|^2}{2c^2} + \frac{\vec{D} \cdot \vec{a}_s}{c^2} + \frac{\Delta f_{ion}}{f^2} + \frac{\Delta f_{3rd}}{f_{nominal}} \tag{2.1}$$

[3]https://pypi.org/project/pysofa/

where $\dot{D}$ is the rate of change in the magnitude of the range vector $\vec{D}$, also called range rate, and $\vec{a}_s$ is the acceleration of the spacecraft. $\frac{\Delta f_{3rd}}{f_{nominal}}$ includes effects of the third order of smallness, including the effect of the movement of the phase center.

In the "2-way" mode, the formula for the signal frequency offset had the form:

$$\frac{\Delta f_{2w}}{f_{nominal}} = -2\frac{\dot{D}}{c} + 2\frac{|\vec{v}_e - \vec{v}_s|^2}{2c^2} + 2\frac{\vec{D}\cdot\vec{a}_s}{c^2} - 2\frac{\vec{D}\cdot\vec{a}_e}{c^2} + \frac{\Delta f_{2w,ion}}{f^2} + \frac{\Delta f_{2w,3rd}}{f_{nominal}} \tag{2.2}$$

where $\vec{a}_e$ acceleration of the ground tracking station and $\frac{\Delta f_{2w,3rd}}{f_{nominal}}$ small effects similar to 2.1. The characteristic of this mode is the absence of the gravitational frequency shift we need, while the signal "2-way" undergoes a double Doppler and troposphere frequency shift compared to "1-way", and also contains doubled contributions of propagation atmospheric noise and a number of instrumental effects. This makes it possible to compensate for these effects using complex data processing "1-way" and "2-way" and, that avoid the need to evaluate these effects from orbital data.

The combination of "1-way" and "2-way" communication modes gives:

$$\frac{\Delta f_{1w}}{f_{nominal}} - \frac{1}{2}\cdot\frac{\Delta f_{2w}}{f_{nominal}} = \frac{\Delta U}{c^2} - \frac{|\vec{v}_e - \vec{v}_s|^2}{2c^2} + \frac{\vec{D}\cdot\vec{a}_s}{c^2} + \frac{\Delta f_{ion}}{f^2} + \frac{\Delta f_{3rd}}{f_{nominal}} \tag{2.3}$$

In the last term, the residual effects of the troposphere, ionosphere, the movement of the phase center and the Doppler effect of the third order of smallness remain. To achieve the desired accuracy, it is necessary to take into account the relative frequency shifts up to 10^{-15}, including the third-order terms of $\frac{v}{c}$

To compensate for the relativistic Doppler effect, we need to know the parameters that are given in this formula:

$$\frac{\Delta f_{Doppler}}{f} = -\frac{|\vec{v}_e - \vec{v}_s|^2}{2c^2} + \frac{(\vec{D}\cdot\vec{a}_e)}{c^2} \tag{2.4}$$

The calculation of the effect for one complete orbit allows us to estimate the limiting values for the contribution of the relativistic Doppler effect to the frequency shift see 5.

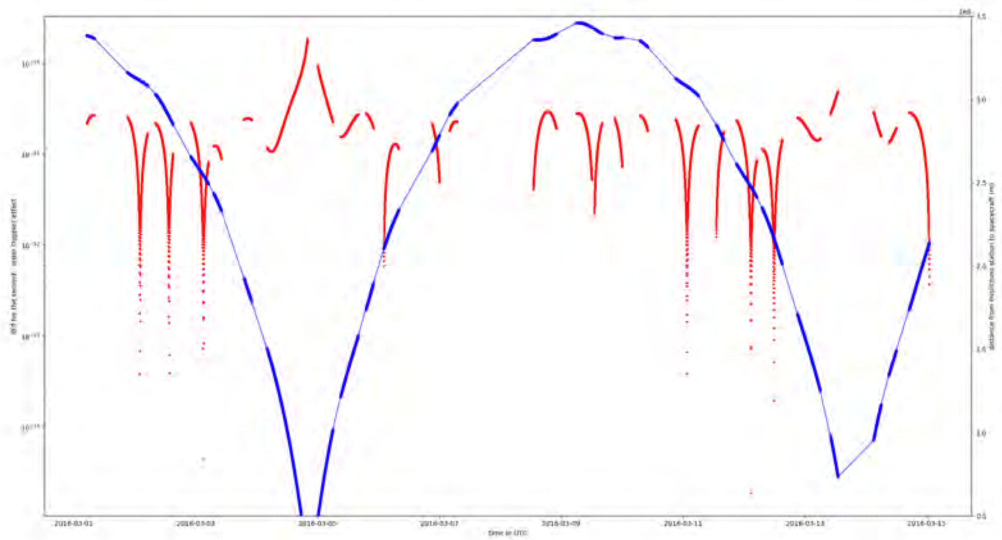


Рис. 5. Relativistic Doppler effect (red) against the background of geocentric distance (blue), gravity sessions were chosen in such a way as to avoid measurements near the perigee of the orbit.

The error in determining the velocity (1.4 mm/s) and coordinates (300 m) SC [13] allows us to compensate for the effect with accuracy $\Delta f/f < 10^{-15}$.

3. Measurements and conclusions

Result of processing some selected gravitational sessions are presented in the Table 1:

Таблица 1. Results of individual gravity sessions

session code	SNR, db	"Redshift" offset, Hz	Cramer-Rao error, Hz	relative accuracy
raks17bb	55.0	4.641	0.00041514	8.945e-05
raks17bj	56.0	5.002	0.00036288	7.255e-05
raks17bk	55.8	5.608	0.00036805	6.563e-05

The frequency of the signal obtained by this method requires further post-processing, which will include compensation for the shift of the ionosphere [14], the drift of frequency standards. The results presented in Table 1 demonstrate for the selected sessions the possibility of measuring the raw redshift effect with relative accuracy exceeding 10^{-4}. After accumulating and processing all the measurement sessions conducted, this result could be improved, as expected. Combining sessions conducted in one orbit period provides a potential possibility to compensate for the drift of frequency standards and reduce the bound of the GR violation parameter to the level of 10^{-5}, however, it has to be addressed.

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