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ИССЛЕДОВАНИЕ КИРАЛЬНОЙ КОСМОЛОГИЧЕСКОЙ МОДЕЛИ $F(R, \Box R)$ ГРАВИТАЦИИ

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Изучаем модифицированную гравитацию $f(R, \Box R)$, которая может быть сведена к киральной космологической модели специального типа. Рассмотрены различные типы космологических решений, основанные на точных аналитических решениях, приближении медленного скатывания, методе суперпотенциала, включении дополнительных материальных полей, а также редукции многополевой модели к однополевой. Таким образом, в данной статье представлены актуальные методы анализа космологических моделей, основанные на эффективной многополевой интерпретации предложенной модифицированной гравитации.

Ключевые слова: киральная космологическая модель, вселенная Фридмана.

INVESTIGATION OF THE CHIRAL COSMOLOGICAL MODEL OF $F(R, \Box R)$ GRAVITY

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We study modified $f(R, \Box R)$ gravity which can be reduced to the chiral cosmological model of the special type. Various types of cosmological solutions are considered based on exact analytical solutions, the slow-roll approximation, the superpotential method, the inclusion of additional material fields and reduction multi-field model to single-field one as well. Thus, this paper presents topical methods for analyzing cosmological models based on an effective multi-field interpretation of the proposed modified gravity.

Keywords: chiral cosmological model, $f(R, \Box R)$ modified gravity, Friedman universe.

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Introduction

The need of modification of Einstein gravity closely connected with discovery of the acceleration in the expansion of the Universe. After this discovery it became clear that GR could not explain this phenomena by natural way without introduction of additional fields (dark energy). Therefore there were studied modifications of gravity theory such as the Einstein-Gauss-Bonnet theory, scalar-tensor theory of gravity, $f(R)$ gravity and $f(R)$ gravity with high derivatives.

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The article is devoted to the special kind of modified gravity with higher derivatives of the second order on scalar curvature R, namely $f(R,\Box R)$ gravity. General approach for reducing $f(R,(\nabla R)^2,(\nabla R)^2,\Box R)$ gravity to GR with additional scalar fields was proposed in the paper [1]. Then the case of truncated model when $f(R,(\nabla R)^2)=f_1(R)+X(R)R_{,\mu}R^{,\mu}$ was studied in the papers [2–4]. In the present article we made first analysis of application $f(R,\Box R)$ gravity to Friedmann cosmology [5,6]. Namely, we reduced the model under consideration to the Chiral Cosmological Model (CCM) with fixed metric of the target space and fixed potential. Such approach give us possibility to find exact and approximated solutions with specific restrictions on the modified gravity parameters [7].

In Sec. 2 we consider the method for constructing effective chiral cosmological model corresponding to cosmological model based on $f(R,\Box R)$-gravity [8]. Sec. 3 presents the equations of cosmological dynamics for the considered model and proposes a method for constructing their solutions in the slow-roll approximation. Sec. 4 discusses the method of supplementing the initial cosmological models with additional material fields, which makes it possible to obtain physically correct exact analytical solutions of the equations of cosmological dynamics [9]. The possible properties of additional material fields at the inflationary stage of the evolution of the universe are also discussed. In Section 5, we consider a method for reducing the original two-field chiral cosmological model to one-field one for constructing exact cosmological solutions. The following sections consider examples of exact solutions. Finally, we discuss the proposed methods and obtained results.

1. The chiral cosmological model of $f(R,\Box R)$ gravity

In the paper [1] there was proposed the method of the reduction of the model $f(R,(\nabla R)^2,\Box R)$ to the Einstein gravity with few scalar fields. The special case of the model when $f(R,(\nabla R)^2)=f_1(R)+X(R)R_{,\mu}R^{,\mu}$ was studied in the papers [3,4]. In the present work we study another simplification, namely when the function f does not depend on $(\nabla R)^2$, i. e. $f=f(R,\Box R)$. Thus, the action of the model is:

$$S=\int d^4x\sqrt{-g}\,[f(R,\Box R)]\,. \tag{1.1}$$

Out task is lead that model to the model of Einstein gravity with scalar fields by using the method from the paper [1].

To derive appropriate Einstein gravity with scalar fields, following by the method proposed in [1], we introduce the lagrangian multipliers $\tilde{\lambda},\tilde{\Lambda}_2$ with the corresponding additional fields ϕ,B. Thus, the action (1.1) is transformed to

$$S=\int d^4x\sqrt{-g}\left[f(\phi,B)-\tilde{\lambda}(\phi-R)-\tilde{\Lambda}_2(B-\Box R)\right]. \tag{1.2}$$

The variation of the action (1.2) with respect to fields leads to the equations

$$\frac{\partial f}{\partial\phi}-\frac{\partial\tilde{\lambda}}{\partial\phi}(\phi-R)-\tilde{\lambda}-\frac{\partial\tilde{\Lambda}_2}{\partial\phi}(B-\Box R) = 0, \tag{1.3}$$

$$\frac{\partial f}{\partial B}-\frac{\partial\tilde{\lambda}}{\partial B}(\phi-R)-\frac{\partial\tilde{\Lambda}_2}{\partial B}(B-\Box R)-\tilde{\Lambda}_2 = 0. \tag{1.4}$$

One can see that the lagrangian multipliers are determined dynamically.

Let us transform the multipliers $(\tilde{\lambda},\tilde{\Lambda}_2)$ to (λ,Λ_2), to get the constraint equations instead of dynamical equations for them. The transformation [1] is

$$\lambda=\tilde{\lambda}+\Box\tilde{\Lambda}_2,\quad \Lambda_2=\tilde{\Lambda}_2. \tag{1.5}$$

Using the equation (1.5) for $\tilde{\lambda}$ in the action (1.1) and integrating by part (avoiding 4-divergence) we get the action

$$S=\int d^4x\sqrt{-g}\left[f(\phi,B)-\lambda(\phi-R)+\tilde{\Lambda}_2(\Box\phi-\Box R)-\tilde{\Lambda}_2(B-\Box R)\right]. \tag{1.6}$$

Opening brackets in (1.6), and changing $\tilde{\Lambda}_2$ according to (1.5), we get

$$S = \int d^4x\sqrt{-g}\left[f(\phi,B) - \lambda(\phi - R) - \Lambda_2(B - \Box\phi)\right]. \tag{1.7}$$

For the action (1.7) variation with respect to (λ, Λ_2) leads to the constraint equations

$$\begin{aligned} \phi &= R, \\ B &= \Box\phi. \end{aligned} \tag{1.8}$$

Obtained constraints can be plugged back into the action (1.7) without changing the nature of the theory. Thus, the action takes the form

$$S = \int d^4x\sqrt{-g}\left[f(\phi,\Box\phi) - \lambda(\phi - R)\right]. \tag{1.9}$$

Taking into account the constraint equations (1.8) we get

$$\frac{\partial f}{\partial\phi} = \lambda, \tag{1.10}$$

$$\frac{\partial f}{\partial B} = \Lambda_2. \tag{1.11}$$

Note that for the action (1.9) λ can not be reduced to the derivative $\frac{\partial f}{\partial\phi}$, and in general it is the function of the coordinates, i.e. the scalar field which can be determined from the dynamical equations.

Let us consider the action (1.7) again. To form the 4-divergence

$$\nabla^\mu(\Lambda_2\nabla_\mu\phi) = (\nabla^\mu\Lambda_2)\nabla_\mu\phi + \Lambda_2\Box\phi \tag{1.12}$$

we subtract and add the term $(\nabla^\mu\Lambda_2)\nabla_\mu\phi$ in the action (1.7). Then, avoiding the 4-divergence and using (1.11), we get the action

$$S = \int d^4x\sqrt{-g}\left[\lambda R - (\nabla^\mu f_B)\nabla_\mu\phi + f(\phi,\Box\phi) - f_B B - \lambda\phi\right]. \tag{1.13}$$

Let us note that $\Lambda_2 \neq const.$ (in the opposite case the relation $B = \Box\phi$ may not be valid) $f_{BB} \neq 0$. In that case we can introduce new field

$$\varphi = f_B \tag{1.14}$$

and considering $(g_{\mu\nu}, \lambda, \phi, \varphi)$ as the basis system. It is justified since the transformation $(g_{\mu\nu}, \lambda, \phi, B)$ to $(g_{\mu\nu}, \lambda, \phi, \varphi)$ locally reversibly under the condition $f_{BB} \neq 0$.

Let us transform the action (1.13) from the Jordan frame to the Einstein frame using the conformal transformation $g^E_{\mu\nu} = \Omega^2(x)g^J_{\mu\nu}$, $\Omega^2(x) = 2\lambda$ and redefinition of the scalar field $\chi = \sqrt{\frac{3}{2}}\ln\lambda$ For it introducing in the action as a canonical field. As the result in E-frame we have

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\chi_{,\mu}\chi_{,\nu} - \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}g^{\mu\nu}\varphi_{,\mu}\phi_{,\nu} + \frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\chi}\left(f(\phi,\varphi) - \varphi B(\phi,\varphi)\right) - \frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\phi\right]. \tag{1.15}$$

The integral of the action (1.15) can be considered as the three-component chiral cosmological model with the target space metric with non-zero components

$$h_{11} = 1,\ h_{23} = \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}. \tag{1.16}$$

And the potential of the interaction

$$W(\chi,\phi,\varphi) = \frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\phi - \frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\chi}\left(f(\phi,\varphi) - \varphi B(\phi,\varphi)\right). \tag{1.17}$$

2. Dynamic equations of the model

Let us use the general form of the chiral field equations in the FRW metric represented in [10]

$$-h_{CB}\left(\ddot{\phi}^B+3H\dot{\phi}^B\right)-h_{CB,D}\dot{\phi}^D\dot{\phi}^B+\frac{1}{2}h_{DB,C}\dot{\phi}^D\dot{\phi}^B-W_{,C}=0. \quad (2.1)$$

Using our designations for the fields, the metric and the potential we have the following equations of the chiral fields

$$\ddot{\chi}+3H\dot{\chi}+\frac{1}{\sqrt{6}}e^{-\sqrt{\frac{2}{3}}\chi}\dot{\phi}\dot{\varphi}-\frac{1}{2\sqrt{6}}e^{-\sqrt{\frac{2}{3}}\chi}\left(\phi-2e^{-\sqrt{\frac{2}{3}}\chi}\left[f-\varphi B\right]\right)=0, \quad (2.2)$$

$$\ddot{\varphi}+3H\dot{\varphi}-\sqrt{\frac{2}{3}}\dot{\chi}\dot{\varphi}+\frac{1}{2}\left(1-e^{-\sqrt{\frac{2}{3}}\chi}\left[f_{,\phi}-\varphi B_{,\phi}\right]\right)=0, \quad (2.3)$$

$$\ddot{\phi}+3H\dot{\phi}-\sqrt{\frac{2}{3}}\dot{\chi}\dot{\phi}-\frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}\left[f_{,\varphi}-B-\varphi B_{,\varphi}\right]=0. \quad (2.4)$$

Einstein – Friedman equations are

$$3H^2=\frac{1}{2}\dot{\chi}^2+\frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}\dot{\phi}\dot{\varphi}+\frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\phi-\frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\chi}\left(f(\phi,\varphi)-\varphi B(\phi,\varphi)\right), \quad (2.5)$$

$$\dot{H}=-\frac{1}{2}\dot{\chi}^2-\frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}\dot{\phi}\dot{\varphi}. \quad (2.6)$$

Note, that the equations above can be easy reduced to the slow roll approximated ones.

2.1. The slow roll solution

For the case of slow roll approximation we get

$$3H\dot{\chi}-\frac{1}{2\sqrt{6}}e^{-\sqrt{\frac{2}{3}}\chi}\left(\phi-2e^{-\sqrt{\frac{2}{3}}\chi}\left[f-\varphi B\right]\right)=0, \quad (2.7)$$

$$3H\dot{\varphi}+\frac{1}{2}\left(1-e^{-\sqrt{\frac{2}{3}}\chi}\left[f_{,\phi}-\varphi B_{,\phi}\right]\right)=0, \quad (2.8)$$

$$3H\dot{\phi}-\frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}\left[f_{,\varphi}-B-\varphi B_{,\varphi}\right]=0. \quad (2.9)$$

Einstein – Friedman equations are

$$3H^2=\frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\phi-\frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\chi}\left(f(\phi,\varphi)-\varphi B(\phi,\varphi)\right), \quad (2.10)$$

$$\dot{H}=0. \quad (2.11)$$

From (2.11) we find $H=H_*=const$ Then with necessity we get $\phi=\phi_*=const,\ \ \varphi=\varphi_*=const$. From (2.10) we can find the relation

$$\left(f(\phi,\varphi)-\varphi B(\phi,\varphi)\right)=e^{\sqrt{\frac{2}{3}}\chi}\left(\phi_*-12H_*^2e^{\sqrt{\frac{2}{3}}\chi}\right). \quad (2.12)$$

Integrating (2.7) we find

$$\chi=\sqrt{\frac{3}{2}}\ln\left(e^{\frac{4H_*^2}{3}t}+\frac{1}{24H_*^2}\right). \quad (2.13)$$

With this solution (2.12) reduced to

$$\left(f(\phi,\varphi)-\varphi B(\phi,\varphi)\right)=\left(e^{\frac{4H_*^2}{3}t}+\frac{1}{24H_*^2}\right)\left(\phi_*-12H_*^2\left(e^{\frac{4H_*^2}{3}t}+\frac{1}{24H_*^2}\right)\right). \quad (2.14)$$

The potential (1.17) for this slow roll solution equals to constant

$$W=3H_*^2. \quad (2.15)$$

Thus we find the essential difference from Friedmann inflation where $H=const$ leads to $V=const,\ \ \phi=const$. We find that one of the fields for the model (1.15) is dynamical one.

3. Additional material field and cosmological solutions

Let us consider the ansatz

$$f - \varphi B = \phi e^{\sqrt{2/3}\chi}. \tag{3.1}$$

In that case the potential $W = 0$ and the CCM dynamical equations with the additional material field take the following form

$$\ddot{\chi} + 3H\dot{\chi} + \frac{1}{\sqrt{6}}\left(\dot{\phi}\dot{\varphi} + \frac{1}{2}\phi\right)e^{-\sqrt{\frac{2}{3}}\chi} = 0, \tag{3.2}$$

$$\ddot{\varphi} + 3H\dot{\varphi} - \sqrt{\frac{2}{3}}\dot{\chi}\dot{\varphi} = 0, \tag{3.3}$$

$$\ddot{\phi} + 3H\dot{\phi} - \sqrt{\frac{2}{3}}\dot{\chi}\dot{\phi} = 0, \tag{3.4}$$

$$3H^2 = \rho_m + \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}\dot{\phi}\dot{\varphi}, \tag{3.5}$$

$$3H^2 + 2\dot{H} = -\frac{1}{2}\dot{\chi}^2 - \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}\dot{\phi}\dot{\varphi} - p_m, \tag{3.6}$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0. \tag{3.7}$$

If $\rho_m = 0$, $p_m = 0$, $\phi = 0$ and $H(t) = 1/3t$ we have the exact solution:

$$f(0,\varphi) = \varphi B(0,\varphi), \tag{3.8}$$

$$\chi(t) = \pm\sqrt{\frac{2}{3}}\ln(t) + c_1, \tag{3.9}$$

$$\varphi(t) = c_2 \exp\left[\pm\frac{2}{3}\left(\ln(t) - \sqrt{\frac{3}{2}}c_1\right)\right] + c_3, \tag{3.10}$$

Where c_1, c_2 и c_3 – are the integration constants.

Thus, the obtained solution corresponds to the expansion of the universe according to the law $a(t) \propto t^{1/3}$ and the universe expansion is driven solely by evolution of the geometrical chiral fields φ and χ.

Let us note that this solution can be determined from the general equations (2.2)-(2.6) with another ansatz, namely, $f - \varphi B = \phi$ and $\phi = 0$. We have also one more solution $\phi = 0$, $H(t) = 0$, $\chi = c_1$ и $\varphi = c_2 t + c_3$ which corresponds to the stationary universe.

Now, let us consider the cosmological models with non-zero additional material field (fields) which we consider as perfect barotropic fluid.

If $\phi = 0$ we have two classes of the exact solutions. The first one corresponds to $\chi = const$

$$\rho_m = 3H^2, \tag{3.11}$$

$$p_m = -3H^2 - 2\dot{H}, \tag{3.12}$$

$$\varphi(t) = c_1\int a^{-3}dt + c_2, \tag{3.13}$$

where $a = a(t)$ – is the scale factor.

The second class of the solutions for any field $\chi \neq const$ can be represented as follow

$$H(t) = -\frac{1}{3}\frac{\ddot{\chi}}{\dot{\chi}}, \qquad a(t) = c_3\dot{\chi}^{-1/3}, \quad \dot{\chi} = c_3 a^{-3}, \tag{3.14}$$

$$\varphi(t) = c_4 \exp\left(\sqrt{\frac{2}{3}}\chi(t)\right) + c_5, \tag{3.15}$$

$$\rho_m=\frac{1}{3}\left(\frac{\ddot{\chi}}{\dot{\chi}}\right)^2-\frac{1}{2}\dot{\chi}^2, \quad (3.16)$$

$$p_m=-\left(\frac{\ddot{\chi}}{\dot{\chi}}\right)^2-\frac{1}{2}\dot{\chi}^2+\frac{2}{3}\left(\frac{\dddot{\chi}}{\dot{\chi}}\right). \quad (3.17)$$

Thus, determining the dynamic $H(t)$ or $a(t)$ we can generate the exact solutions of the first class. The exact solutions of the second class generate with the determination of the dynamic $H(t)$ (or $a(t)$) or evolution of the scalar field $\chi(t)$.

3.1. The properties of additional material field for quasi de Sitter expansion

Now, we consider the properties of additional material field for the solutions obtained based on the state parameter $w=p_m/\rho_m$.

For solutions (3.11)–(3.12) one has

$$w=-1-\frac{2}{3}\frac{\dot{H}}{H^2}, \quad (3.18)$$

which corresponds to the case usual canonical scalar field.

For quasi de Sitter accelerated expansion $H\simeq const$ one has vacuum-like state of this field $w\simeq-1$.

For solutions (6.7)–(3.17) one has

$$w=-\frac{4\dot{H}+6H^2+(c_3/a)^6}{6H^2-(c_3/a)^6}. \quad (3.19)$$

Thus, for quasi de Sitter accelerated expansion $H\simeq const$ one has $w\simeq-1$ for $6H^2\gg(c_3/a)^6$ and $w\simeq1$ for $6H^2\ll(c_3/a)^6$. Therefore, the accelerated expansion of the universe can be induced by the different types of the material fields (from vacuum-like to extremely hard matter) for this modification of GR.

We also note that the evolution of the state parameter of the material field can be reconstructed for the chosen dynamics of the universe $a=a(t)$ based on the expression (3.19).

A promising direction of research is the generation of cosmological solutions for system (2.2)–(2.6) with an additional material field.

4. Construction of the one-field model

Let us follow by procedure introduced in the article [11]. To apply the standard method of cosmological parameters calculation (power spectrum, spectral indexes, tensor-to-scalar ratio) we reduce three-field model to single field inflationary model assuming linear dependence between fields:

$$\phi(t)=k_\phi\chi(t),\quad \varphi(t)=k_\varphi\chi,\quad k_\phi=const,\quad k_\varphi=const. \quad (4.1)$$

The action (1.15) after simple algebra reduced to the following

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R-\frac{1}{2}g^{\mu\nu}\chi_{,\mu}\chi_{,\nu}\left(1+e^{-\sqrt{\frac{2}{3}}\chi}k_\phi k_\varphi\right)+\frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\chi}\left(f(\chi)-k_\varphi\chi B(\chi)\right)-\frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}k_\phi\chi\right]. \quad (4.2)$$

Let us note that transition to new canonical field

$$\Psi=\int\left(1+e^{-\sqrt{\frac{2}{3}}\chi}\right)^{1/2}d\chi$$

does not perspective because of complicated dependence Ψ on χ:

$$\Psi=\frac{1}{x}\left[\sqrt{\frac{1+x}{x}}\left(x\ln(1+2x+2\sqrt{x(1+x)})\right)-x\ln2-2\sqrt{x(1+x)}\right],$$

where $x=e^{\sqrt{\frac{2}{3}}\chi}$.

4.1. The one-field model equations

Starting from the action (1.15) one can derive by standard way gravitational and scalar field equations. Using the FRW metric, one can write down the Einstein-Friedmann equations for the one-field model:

$$3H^2 = \frac{1}{2}\left(1+e^{-\sqrt{\frac{2}{3}}\chi}\right)k_\phi k_\varphi \dot{\chi}^2 + W(\chi), \tag{4.3}$$

$$\dot{H} = -\frac{1}{2}\left(1+e^{-\sqrt{\frac{2}{3}}\chi}\right)k_\phi k_\varphi \dot{\chi}^2. \tag{4.4}$$

The field equation takes the following form

$$3H\omega\dot{\chi} + \partial_t(\omega\dot{\chi}) - \frac{1}{2}\frac{\partial\omega}{\partial\chi}\dot{\chi}^2 + \frac{\partial W}{\partial\chi} = 0, \tag{4.5}$$

where

$$\omega = \left(1+e^{-\sqrt{\frac{2}{3}}\chi}\right)k_\phi k_\varphi. \tag{4.6}$$

Making substitution of ω from (4.6) and simple transformations, the equations (4.3)-(4.5) are reduced to the following

$$3H^2 = \frac{1}{2}\gamma\omega\dot{\chi}^2 + W(\chi), \tag{4.7}$$

$$\dot{H} = -\frac{1}{2}\gamma\omega\dot{\chi}^2, \tag{4.8}$$

$$\ddot{\chi} + 3H\dot{\chi} - \frac{1}{2}\frac{\partial\ln\omega}{\partial\chi} + \frac{1}{\omega}\frac{\partial W}{\partial\chi} = 0. \tag{4.9}$$

Here $\gamma = 1$ for canonical and $\gamma = -1$ for phantom scalar field χ.

Substituting (4.1) into (1.17) we obtain the reduced potential in the following form

$$W = \frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\left(k_\phi\chi - e^{-\sqrt{\frac{2}{3}}\chi}\left(f(\chi) - k_\varphi\chi B(\chi)\right)\right). \tag{4.10}$$

The derivative of the potential by the field χ is

$$\frac{\partial W}{\partial\chi} = \frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\chi}\left[2\sqrt{\frac{2}{3}}\left(f(\chi) - k_\varphi\chi B(\chi)\right) - \left(\frac{\partial f(\chi)}{\partial\chi} - k_\varphi B(\chi) - k_\varphi\chi\frac{\partial B(\chi)}{\partial\chi}\right) + e^{\sqrt{\frac{2}{3}}\chi}\left(k_\phi - \sqrt{\frac{2}{3}}k_\phi\chi\right)\right]. \tag{4.11}$$

5. Special solutions

Let us consider possibility to find exact solutions under special suggestions about models functions.

5.1. Zero potential $W = 0$

If we set potential equal to zero: $W(\chi) = 0$, then we get the special ansatz

$$f(\chi) = k_\phi\chi e^{\sqrt{\frac{2}{3}}\chi} + k_\varphi B(\chi). \tag{5.1}$$

Using the consequence of (4.7) and (4.8)

$$3H^2 + \dot{H} = W(\chi) = 0 \tag{5.2}$$

we find the solution for Hubble parameter

$$H = \frac{1}{3(t-t_*)} \tag{5.3}$$

and the scale factor

$$a(t) = a_0(t-t_*)^{1/3}. \tag{5.4}$$

Substitution $H(t)$ into field equation (4.8) and integration the resulting equation gives the expression defining the solution for the field χ

$$\frac{1}{4}\sqrt{3k_\phi k_\varphi}\left[2\sqrt{1+e^{-\sqrt{\frac{2}{3}}\chi}}-\ln\left(\frac{\sqrt{1+e^{-\sqrt{\frac{2}{3}}\chi}}+1}{\sqrt{1+e^{-\sqrt{\frac{2}{3}}\chi}}-1}\right)\right]=\sqrt{t}+const. \tag{5.5}$$

It is impossible to express explicitly the field χ as the function of t.

5.2. The constant potential $W = W_* = const.$

If $W = W_* = const$, using (4.10), we find key relation

$$(f(\chi)-k_\varphi\chi B(\chi))=k_\phi\chi e^{\sqrt{\frac{2}{3}}\chi}-4W_*e^{2\sqrt{\frac{2}{3}}\chi}. \tag{5.6}$$

That means that when we find χ as the function on time we know the relation between model's functions $f(\chi)$ and $B(\chi)$ which include W_*.

Setting $W = W_* = const$ we conclude that the derivative W_* on χ: $\frac{\partial W}{\partial \chi}$ becomes equal to zero

$$\frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\left[2\sqrt{\frac{2}{3}}(f(\chi)-k_\varphi\chi B(\chi))-\left(\frac{\partial f(\chi)}{\partial\chi}-k_\varphi B(\chi)-k_\varphi\chi\frac{\partial B(\chi)}{\partial\chi}\right)+k_\phi e^{\sqrt{\frac{2}{3}}\chi}\left(1-\sqrt{\frac{2}{3}}\chi\right)\right]=0. \tag{5.7}$$

The solution for Hubble parameter we find from the equation

$$3H^2+\dot{H}=W_* \tag{5.8}$$

The solutions are

1. Exponential rate of a scale factor

$$H=H_*=const,\quad a(t)=a_0e^{H_*t},\quad W_*=3H_*^2,\quad \chi=\chi_*\to\infty, \tag{5.9}$$

where a_0 is the scale factor at the beginning of inflation. It is clear that the solution of (5.14) is achieved at the fixed moment $(t-t_*)=\sqrt{3W_*}\ln(\pi/2)$.

2. Expansion defined by hyperbolic functions

$$H(t)=\sqrt{\frac{W_*}{3}}\tanh\left(\sqrt{3W_*}(t-t_*)\right),\qquad a(t)=a_0\cosh^{1/3}\left(\sqrt{3W_*}(t-t_*)\right). \tag{5.10}$$

where $\gamma=-1$, and

$$H(t)=\sqrt{\frac{W_*}{3}}\coth\left(\sqrt{3W_*}(t-t_*)\right),\quad a(t)=a_0\sinh^{1/3}\left(\sqrt{3W_*}(t-t_*)\right). \tag{5.11}$$

where $\gamma=1$.

3. Expansion defined by trigonometric functions

$$H(t)=-\sqrt{\frac{W_*}{3}}\tan\left(\sqrt{3W_*}(t-t_*)\right),\qquad a(t)=a_0\cos^{1/3}\left(\sqrt{3W_*}(t-t_*)\right). \tag{5.12}$$

where $\gamma=1$.

The solution (5.12) can be represented in another form:

$$H(t)=\sqrt{\frac{W_*}{3}}\cot\left(\sqrt{3W_*}(t-t_*)\right),\quad a(t)=a_0\sin^{1/3}\left(\sqrt{3W_*}(t-t_*)\right). \tag{5.13}$$

where $\gamma=1$.

We note that for the solution (5.10) the dependence χ on time t can be find from the equation

$$\frac{3}{4}\sqrt{-k_\phi k_\varphi}\left[-2\sqrt{1+e^{-\sqrt{\frac{2}{3}}\chi}}+\ln\left(\frac{\sqrt{1+e^{-\sqrt{\frac{2}{3}}\chi}}+1}{\sqrt{1+e^{-\sqrt{\frac{2}{3}}\chi}}-1}\right)\right]=\arctan\left(\exp[\sqrt{3W_*}(t-t_*)]\right)+const. \tag{5.14}$$

As one can see again, it is impossible to express explicitly the field χ as the function of t.

Working with cosmological parameters we find that the solutions (5.10)-(5.13) conflict with observation data and they are not verifiable. Therefore it needs to take into account quantum fluctuation on the early stage of universe evolution, including, say, Gauss-Bonnet term into the model action (1.1). Another possibility is to find solutions with nonconstant potential $W(\phi)\neq const$. Next section is devoted to search of new solution with nonconstant potential.

6. Solutions for special form of the potential

We looking for the solutions using some freedom for the functions f and B.

6.1. Selection of the full square form of the potential *W*

To represent the potential as the square expression let us suggest

$$f(\chi)=e^{\sqrt{\frac{2}{3}}\chi}f_1(\chi),\quad B(\chi)=e^{\sqrt{\frac{2}{3}}\chi}k_\varphi\chi B_1(\chi). \tag{6.1}$$

The chain of transformations is showing how the potential reducing to the expression in square form. Using (6.1) we obtain

$$k_\phi\chi-e^{-\sqrt{\frac{2}{3}}\chi}\left(f(\chi)-k_\varphi\chi B(\chi)\right)=k_\phi\chi-e^{-\sqrt{\frac{2}{3}}\chi}e^{\sqrt{\frac{2}{3}}\chi}\left(f_1-k_\varphi\chi B_1\right)=k_\phi\chi-\left(f_1-k_\varphi\chi B_1\right). \tag{6.2}$$

Evident transformation to the full square

$$k_\phi\chi-(f_1-k_\varphi\chi B_1)=\sqrt{k_\phi\chi}-\frac{f_1}{2\sqrt{k_\phi\chi}}\sqrt{k_\phi\chi}+k_\varphi\chi B_1=\left(\sqrt{k_\phi\chi}-\frac{f_1}{2\sqrt{k_\phi\chi}}\right)^2 \tag{6.3}$$

leads for the relation between f and B

$$\left(\frac{f_1}{2\sqrt{k_\phi\chi}}\right)^2=k_\varphi\chi B_1. \tag{6.4}$$

Therefore B_1 can be defined over f_1 and χ by the following way:

$$B_1=\frac{f_1^2}{4k_\phi k_\varphi\chi^2}. \tag{6.5}$$

Thus the potential reads:

$$W=\frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\left(\sqrt{k_\phi\chi}-\frac{f_1}{2\sqrt{k_\phi\chi}}\right)^2. \tag{6.6}$$

Such form of the potential simplify definition of Hubble parameter in slow roll approximation and when the superpotential method is applied.

6.2. The slow roll regime

Standard conditions in inflationary Friedmann cosmology with a canonical scalar field for the slow roll are: $\dot\chi^2\ll 1$, $\ddot\chi\ll 1$. In our case we deal with additional kinetic term as multiplier for kinetic term of the scalar field. Therefore we may simplify dynamic equations (4.7)-(4.9) by suggestions $\frac{1}{2}\omega\dot\chi^2\ll W(\chi)$ and $\ddot\chi\ll 1$.

The cosmological dynamic equations become

$$3H^2 \simeq W, \tag{6.7}$$

$$\dot{H} = \frac{1}{2}\omega\dot{\chi}^2, \tag{6.8}$$

$$3H\omega\dot{\chi} + \frac{\partial W}{\partial \chi} \simeq 0. \tag{6.9}$$

Using (6.7) we find H:

$$H = \sqrt{\frac{W}{3}} = \frac{1}{2\sqrt{3}}e^{-\frac{1}{2}\sqrt{\frac{2}{3}}\chi}\left(\sqrt{k_\phi\chi} - \frac{f_1}{2\sqrt{k_\phi\chi}}\right). \tag{6.10}$$

The potential (6.6) can be represented in the following form

$$W = \frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\left(k_\phi\chi - f_1 + \frac{k_\varphi}{4k_\phi}f_1^2\right), \tag{6.11}$$

$$\frac{\partial W}{\partial \chi} = -\frac{1}{4}\sqrt{\frac{2}{3}}e^{-\sqrt{\frac{2}{3}}\chi}\left(k_\phi\chi - f_1 + \frac{k_\varphi}{4k_\phi}f_1^2\right) + \frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\left(k_\phi - f_{1,\chi} + \frac{k_\varphi}{2k_\phi}f_1 f_{1,\chi}\right). \tag{6.12}$$

Substitution into equation (6.9) gives

$$2\sqrt{3}e^{-\frac{1}{2}\sqrt{\frac{2}{3}}\chi}\left(\sqrt{k_\phi\chi} - \frac{f_1}{2\sqrt{k_\phi\chi}}\right)\left(1 + e^{-\sqrt{\frac{2}{3}}\chi}\right)k_\phi k_\varphi\dot{\chi} + \\ + e^{-\sqrt{\frac{2}{3}}\chi}\left(-\sqrt{\frac{2}{3}}\left(k_\phi\chi - f_1 + \frac{k_\varphi}{4k_\phi}f_1^2\right) + k_\phi - f_{1,\chi} + \frac{k_\varphi}{2k_\phi}f_1 f_{1,\chi}\right) = 0. \tag{6.13}$$

Thus we have got a general solution in integral form

$$t = -\int\left[\frac{4\sqrt{3}\left(\sqrt{k_\phi\chi} - \frac{f_1}{2\sqrt{k_\phi\chi}}\right)\cosh\left(\frac{1}{2}\sqrt{\frac{2}{3}}\chi\right)k_\phi k_\varphi}{-\sqrt{\frac{2}{3}}\left(k_\phi\chi - f_1 + \frac{k_\varphi}{4k_\phi}f_1^2\right) + k_\phi - f_{1,\chi} + \frac{k_\varphi}{2k_\phi}f_1 f_{1,\chi}}\right]d\chi. \tag{6.14}$$

Knowing the function f_1 one can consider the integral (6.14).

6.3. The example of solutions

Let us return to the exact equation (4.8). Starting from

$$H = \frac{\sqrt{W}}{\sqrt{3}} = \frac{1}{2\sqrt{3}}e^{-\frac{1}{2}\sqrt{\frac{2}{3}}\chi}\left(\sqrt{k_\phi\chi} - \frac{f_1}{2\sqrt{k_\phi\chi}}\right) \tag{6.15}$$

we get

$$\dot{H} = \frac{1}{4\sqrt{3}}e^{-\frac{1}{2}\sqrt{\frac{2}{3}}\chi}\left[-\sqrt{\frac{2}{3}}\left(\frac{2k_\phi\chi - f_1}{2\sqrt{k_\phi\chi}}\right) + \left(\frac{2k_\phi\chi - 2f_{1,\chi}\chi + f_1}{2\chi\sqrt{k_\phi\chi}}\right)\right]\dot{\chi}. \tag{6.16}$$

Substituting the result into (4.8) we find

$$\dot{\chi} = \frac{e^{-\frac{1}{2}\sqrt{\frac{2}{3}}\chi}}{2\sqrt{3}\left(1 + e^{-\sqrt{\frac{2}{3}}\chi}\right)k_\phi k_\varphi}\left[\frac{f_{1,\chi} - \left(\frac{1}{2\chi} + \frac{\sqrt{2}}{2\sqrt{3}}\right)f_1 + k_\phi\left(\sqrt{\frac{2}{3}}\chi - 1\right)}{\sqrt{k_\phi\chi}}\right]. \tag{6.17}$$

Or under some manipulations

$$\dot{\chi} = \frac{1}{8\sqrt{3}}\frac{1}{k_\phi k_\varphi}\left(\cosh\frac{1}{2}\sqrt{\frac{2}{3}}\chi\right)^{-1}\frac{1}{\sqrt{k_\phi\chi}}\left(f_{1,\chi} - \left(\frac{1}{2\chi} + \frac{\sqrt{2}}{2\sqrt{3}}\right)f_1 + k_\phi\left(\sqrt{\frac{2}{3}}\chi - 1\right)\right). \tag{6.18}$$

A.

Let us suggest

$$1+e^{-\sqrt{\frac{2}{3}}\chi}=\left[\sqrt{\frac{2}{3}}\left(\frac{2k_\phi\chi-f_1}{2\sqrt{k_\phi\chi}}\right)-\left(\frac{2k_\phi\chi-2f_{1,\chi}\chi+f_1}{2\chi\sqrt{k_\phi\chi}}\right)\right]. \tag{6.19}$$

Then we can define the field χ from equation

$$\dot{\chi}=\frac{e^{-\frac{1}{2}\sqrt{\frac{2}{3}}\chi}}{2\sqrt{3}k_\phi k_\varphi}. \tag{6.20}$$

The result is

$$\chi=\sqrt{6}\ln\left[\frac{\sqrt{2}(t-t_*)}{12k_\phi k_\varphi}\right]. \tag{6.21}$$

Our next task is to find f_1 from (6.19) as the function on χ. The equation (6.19) can be reduced to the form

$$f_{1,x}-f_1\left(\frac{1+x}{2x}\right)+\sqrt{\frac{3}{2}}k_\phi x-\sqrt{\frac{3}{2}}k_\phi-\sqrt[4]{\left(\frac{3}{2}\right)^3}\sqrt{k_\phi x}\left(1+e^{-x}\right)=0. \tag{6.22}$$

The solution is

$$f_1(x)=\sqrt{6}k_\phi x-(54)^{1/4}\sqrt{k_\phi x}\left[\frac{1}{3}e^{-x}+1-\frac{4}{3}e^{x/2}-C_1,e^{x/2}\right] \tag{6.23}$$

where $x=\sqrt{\frac{2}{3}}\chi$.

Thus from (6.15) we can find the Hubble parameter

$$H(t)=\frac{\sqrt{2}}{12}\left[(s(t-t_*))^{-3}+(s(t-t_*))^{-1}-4-3C_1\right],\quad s=\frac{\sqrt{2}}{12k_\phi k_\varphi}. \tag{6.24}$$

B.

Let us suggest

$$\left[\sqrt{\frac{2}{3}}\left(k_\phi\chi-\frac{1}{2}f_1\right)-\left(\frac{1}{2}-f_{1,\chi}+\frac{f_1}{\chi}\right)\right]=A\sqrt{\chi},\quad A=const. \tag{6.25}$$

The equation on f_1 is reduced to

$$f_{1,x}-f_1\left(\frac{1+x}{2x}\right)+\sqrt{\frac{3}{2}}k_\phi x-\sqrt{\frac{3}{2}}k_\phi-A\sqrt[4]{\left(\frac{3}{2}\right)^3}\sqrt{x}=0. \tag{6.26}$$

The solution for $f_1(\chi)$ is

$$f_1(x)=-xk_\phi\sqrt{3\pi}e^{x/2}erf\left(\sqrt{\frac{x}{2}}\right)+\sqrt{6}k_\phi x+C_1\sqrt{x}e^{x/2}. \tag{6.27}$$

The solution for the field χ is

$$\chi=\sqrt{6}\sinh^{-1}\left[\frac{t-t_*}{12\sqrt{2}}\right]. \tag{6.28}$$

Thus from (6.15) we can find the Hubble parameter

$$H(x)=\frac{\sqrt[4]{6}}{2}\sqrt{k_\phi x}\left(\sqrt{2}-\sqrt{\pi}e^{x/2}erf(x/2)\right)+\frac{C_1}{2\sqrt{k_\phi}}\sqrt[4]{\frac{2}{3}}. \tag{6.29}$$

7. The superpotential approach

Let us define the superpotential as

$$S_W = W(\chi) + \frac{1}{2}\omega\dot{\chi}^2 = \frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\left(k_\phi\chi - e^{-\sqrt{\frac{2}{3}}\chi}\left(f(\chi) - k_\varphi\chi B(\chi)\right)\right) + \frac{1}{2}U(\chi)^2, \tag{7.1}$$

where $U^2(\chi) = \omega\dot{\chi}^2$.

Taking U in the form

$$U(\chi) = \frac{\sqrt{2}}{4\sqrt{k_\phi}}e^{-\sqrt{\frac{3}{2}}\chi}\left(f(\chi) - k_\varphi\chi B(\chi)\right)(\chi)^{-1/2} \tag{7.2}$$

we can represent S_W as

$$S_W = 2e^{\frac{2}{3}\chi}\left[\sqrt{\frac{k_\phi}{2}}e^{-\sqrt{\frac{3}{2}}\chi}\chi^{1/2} + U(\chi)\right]^2. \tag{7.3}$$

Knowing $U = \sqrt{\omega}\dot{\chi}$ we can define χ from (7.2)

$$\chi = \left(\frac{3}{2}K_1\right)^{2/3}(t - t_*)^{2/3}, \tag{7.4}$$

taking into account the ansatz

$$\left(f(\chi) - k_\varphi\chi B(\chi)\right) = e^{-\sqrt{\frac{3}{2}}\chi}\left(1 + e^{\sqrt{\frac{3}{2}}\chi}\right)^{-1}. \tag{7.5}$$

In (7.4)

$$K_1 = \pm\frac{\sqrt{2}}{4\sqrt{k_\varphi k_\phi}}. \tag{7.6}$$

The Hubble parameter now is

$$H(\chi(t)) = \sqrt{\frac{2}{3}}\left[\sqrt{\frac{k_\phi}{2}}\sqrt{\chi} + e^{\frac{1}{2}\sqrt{\frac{2}{3}}\chi}U(\chi)\right], \tag{7.7}$$

$$U(\chi) = \frac{\sqrt{2}}{4\sqrt{k_\phi}}e^{-\frac{5}{2}\sqrt{\frac{2}{3}}\chi}\chi^{-1/2}\left(1 + e^{\sqrt{\frac{2}{3}}\chi}\right)^{-1/2}. \tag{7.8}$$

Conclusion

We study chiral cosmological model with the action (1.15) as an equivalent of $f(R,\Box R)$ modified gravity. Examples of exact solutions with additional material field were found and their properties for quasi de Sitter expansion were investigated. To find the way of verification of considered cosmological model we construct the one-field model using linear connection between chiral fields (4.1). For this model we study two special cases for zero and constant potential and found set of exact solutions. Further we search for new solutions for special form of the potential. Considering the slow roll regime we find the general solution in integral form (6.14). Further we found two special exact solutions with functional fixing of the modified gravity parameters (6.1), (6.23), (6.27). Considering the superpotential approach we found the example of exact solution imposing ansatz relation on model's parameters.

Thus, in this paper we propose various methods for analyzing effective chiral cosmological model based on $f(R,\Box R)$ gravity. It was also shown that the proposed interpretation of these modifications of Einstein gravity implies a wide class of cosmological models with different possible dynamics of the accelerated expansion of the early universe and different types of potentials of scalar fields as well.

In spite of complexity of obtained in the article solutions, it may be possible to verify them for correspondence to observation data in another separate work.

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