УДК 530.122

УСТОЙЧИВОСТЬ НЕЙТРОННЫХ ЗВЕЗД В ТЕОРИИ ГРАВИТАЦИИ С НЕМИНИМАЛЬНОЙ ПРОИЗВОДНОЙ СВЯЗЬЮ СКАЛЯРНОГО ПОЛЯ И ТЕНЗОРА ЭЙНШТЕЙНА*

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В недавних исследованиях были рассмотрены линейные возмущения статических сферически симметричных нейтронных звезд в полной теории гравитации Хорндески, получены скорости распространения возмущений и условия устойчивости конфигурации нейтронных звезд. В данной работе мы применяем эти результаты к исследованию устойчивости нейтронных звезд в теории гравитации с неминимальной производной связью тензора Эйнштейна и скалярного поля, со стандартным кинетическим членом скалярного поля и космологической постоянной. В настоящей работе мы показываем, что данная модель устойчива в широком диапазоне параметров неминимальной связи ℓ и космологической постоянной ξ, и квадраты скоростей возмущений c_r^2, c_Ω^2, c_{r3}^2, $c_{\Omega\pm}^2$ и $\mathcal{K}$ положительны.

Ключевые слова: теория гравитации с неминимальной производной связью, нейтронные звезды, устойчивость.

STABILITY OF ANTI-DE SITTER NEUTRON STARS IN THE THEORY OF GRAVITY WITH NONMINIMAL DERIVATIVE COUPLING

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In recent studies linear perturbations of a static and spherically symmetric background of neutron stars have been considered in full Horndesky's theory, the propagation speeds of perturbations were derived and the stability conditions were obtained. In the present paper we applied this general stability conditions to the study of the neutron stars stability in theory of gravity with the scalar-derivative coupling of the Einstein tensor and the scalar field with the standard kinetic term and the cosmological constant. It was shown that there are stable stellar configurations in a wide class of model parameters ℓ and ξ for which all squared speeds of perturbations, c_r^2, c_Ω^2, c_{r3}^2, $c_{\Omega\pm}^2$ and $\mathcal{K}$ are positive.

Keywords: theory of gravity with nonminimal derivative coupling, neutron stars, stability.

PACS: 04.40
DOI: 10.17238/issn2226-8812.2023.1.85–90

Introduction

Horndeski theory of gravity is the most general scalar-tensor theory with a single scalar field and second-order field equations [1]. One of the subclass of shift-symmetric Horndeski theories is the theory with the scalar-derivative coupling of the scalar field and the Einstein tensor, which is described by the action (1.1). It was initially introduced to describe the inflation and the late time acceleration and it has been widely studied in various cosmological models [2]. A natural step forward in the investigation of

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this sector of Horndeski gravity is the study of black holes and neutron stars, an overview is presented in [3]. A spherically symmetric, static black hole solution in the theory (1.1) has an anti-de Sitter asymptotics, where the nonminimal derivative coupling plays the role of a negative cosmological constant [4]. Spherically symmetric neutron star solutions in the theory (1.1) with $\epsilon_1 = \Lambda_0 = 0$ and the scalar field linearly dependent on time, was constructed in [5]. Subsequently this result was extended in the case of slowly rotating star using several realistic equations of state [6]. In the work [7] we extended the result [5] without imposing any restrictions on the parameters $\epsilon_{1,2}$, ℓ and a cosmological constant Λ_0.

In the present work we consider the issue of stability of neutron stars in theory with a nonminimal scalar-derivative coupling in the general case with $\Lambda_0 \neq 0$ (nonzero bare cosmological term) and $\epsilon_1 \neq 0$ (nonzero standard kinetic term). In the next section we briefly present the result of Ref. [7]. Further in the section §2 using the relationships obtained in [8] we present plots of the squared perturbation velocities and try to find the range of model parameters for which the solution is stable. The last section will draw conclusions.

1. Anti-de Sitter neutron stars in the theory of gravity with nonminimal derivative coupling

In our previous work [7] we numerically constructed the static and spherically symmetrical neutron stars configurations in the subclass of Horndeski gravity represented by models with a nonminimal derivative coupling of a scalar field with the Einstein tensor and the cosmological constant Λ_0:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa}(R - 2\Lambda_0) - \frac{1}{2}\left(\varepsilon_1 g_{\mu\nu} + \varepsilon_2 \ell^2 G_{\mu\nu}\right)\nabla^\mu\phi\nabla^\nu\phi\right] + S^{(m)}, \tag{1.1}$$

where R and $G_{\mu\nu}$ are the Ricci scalar and the Einstein tensor, $\kappa = 8\pi G/c^4$ is the Einstein gravitational constant, coefficients $\varepsilon_{1,2}$ are dimesionless parameter, ℓ has the dimension of length and Λ_0 is a ‘bare’ cosmological constant[1]. $S^{(m)}$ is the action for a perfect fluid, and the energy density ϵ and pressure p are related by the polytropic equation of state $p = K\rho_0^\Gamma$, $\epsilon = \rho_0 c^2 + \frac{p}{\Gamma-1}$, where ρ_0 is a baryonic mass density. We will consider the case $\Gamma = 2$ and $K = 1.79 \times 10^5$, since this models leads to compact objects with accepted mass and radius of neutron stars [5]. A general static spherically symmetric spacetime metric has the following form:

$$ds^2 = -A(r)c^2dt^2 + \frac{dr^2}{B(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{1.2}$$

The scalar field ϕ, the energy density ϵ, and the pressure p depend only on r. Note, that in order to provide a regularity of solutions, we need to make a choice $\varepsilon = \varepsilon_1/\varepsilon_2 = -1$, and a dimensionless cosmological constant $\xi = \Lambda_0\ell^2$ takes the values $-3 < \xi < 1$ [7]. As a result of numerical calculations, we obtained the behavior of the metric functions, scalar field, baryonic density and the mass-radius diagram for different values of the model parameters ℓ and ξ [7]. The case $\xi = -1$ was analyzed in more detail, because in this case the vacuum solution has the particularly simple form of the Schwarzschild-anti de Sitter black hole. Applying observable restrictions for mass and radius of the neutron star, we get the restrictions on the parameter ℓ, specifically $10 \leqslant \ell \leqslant 50$ km.

2. Stability of anti-de Sitter neutron stars

An investigation of stability of solutions describing relativistic compact objects is an important and rather complicated problem. Perturbations and quasi-normal modes of static and spherically symmetric black holes and neutron stars in full Horndeski theory have been intensively studied in Refs. [8], in this work the propagation speeds of perturbations were derived and the stability conditions were obtained. In subsequent works [9,10] it was shown the instability of compact stars [5] in the theory with a nonminimal scalar-derivative coupling (1.1) without a standard kinetic term $\epsilon_1 = 0$, $\Lambda_0 = 0$ and with $\phi'(r) \neq 0$.

[1] An observed negative cosmological constant Λ_{AdS} appears as a certain combination of Λ_0 and the parameter of nonminimal derivative coupling ℓ [7].

Shortly, the essence of the method consists in the following: To study the stability of relativistic compact objects, one considers metric perturbations $g_{\mu\nu}$, such that $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ where $\bar{g}_{\mu\nu}$ is the background metric of the spherically symmetric and static spacetime. Analogously, scalar and matter fields are perturbed. All perturbations are divided into scalar, vector, and tensor ones, expanded into spherical harmonics, and then odd- and even-parity modes are analyzed separately. In such the approach, stability conditions could be considered as a positivity of squared propagation speeds of odd- and even-parity perturbations along the radial and angular directions. The values crucial for the stability analysis are the following: c_r and c_Ω are speeds of the only propagating degree of freedom in odd-parity sector in radial and angular directions, respectively; c_{r3} is the propagation speed of scalar-field even-parity perturbations $\delta\phi$ propagating the radial direction; $c_{\Omega-}$ and $c_{\Omega+}$ are the propagation speed of the scalar-field even-parity perturbations $\delta\phi$ propagating in the angular direction, which corresponds to the degree of freedom arising from the metric perturbations. A necessary and sufficient condition for the absence of Laplacian instability is the non-negativity of the squared speeds of perturbations:

$$c_r^2 \geqslant 0, \quad c_\Omega^2 \geqslant 0, \quad c_{r3}^2 \geqslant 0, \quad c_{\Omega\pm}^2 \geqslant 0. \tag{2.1}$$

Besides these conditions, there is the additional one, $\mathcal{K} > 0$, which can be regarded as the no-ghost condition in the presence of matter (see [8,10]). The model (1.1) is corresponded to the following choice of arbitrary functions in the Horndesky theory $G_2 = \epsilon_1 X - \Lambda_0/\kappa$, $G_3 = 0$, $G_4 = 1/(2\kappa)$, $G_5 = \epsilon_2 \ell^2 \phi/2$, where $X = -(\nabla\phi)^2/2$. Following Refs. [8], we considered values of the squared propagation speeds of odd- and even-parity perturbations for the internal and the external vacuum solution.

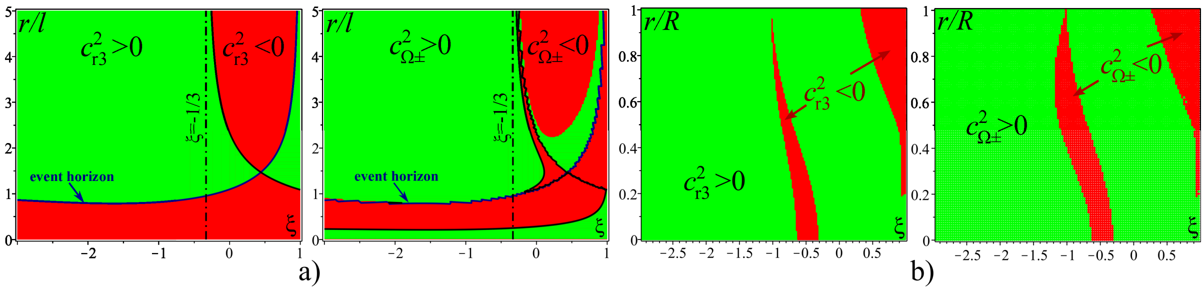

Fig. 1. a) The region $(r/\ell;\xi)$ shown in green corresponds to the fulfillment of condition $c_{r3}^2 > 0$ and $c_{\Omega\pm}^2 > 0$ for the vacuum solution. The blue curve corresponds to the event horizon r_h: $A(r_h) = 0$. In this case $r_g/\ell = 1$, where $r_g = 2GM/c^2$, M is an asymptotic mass [7]. For other values of r_g the figure looks similar. b) The area $(r/R;\xi)$ in which conditions $c_{r3}^2 > 0$ and $c_{\Omega\pm}^2 > 0$ are fulfilled for the interior neutron star solution is shown in green. In this case $\ell = 10km$, $\rho_{0c} = 10^{15} g/cm^3$.

Outside the star one has a scalar «vacuum» ($p = 0$) with a nontrivial configuration of the scalar field, the vacuum solution was described in the § 3 of the Ref. [7]. The propagation speeds of the odd and even-parity perturbations in the vacuum case have the form:

$$c_\Omega^2/c^2 = 1, \quad c_r^2/c^2 = \frac{(1-\xi)\,r^2 + 2\ell^2}{(\xi+3)\,r^2 + 2\ell^2}, \quad c_{r3}^2/c^2 = \frac{\left(2 + x^2(1-\xi)\right)\left(x^2+1\right)^2}{YB}, \tag{2.2}$$

$$c_{\Omega\pm}^2/c^2 = -B_1 \pm \sqrt{B_1^2 - B_2}, \quad \mathcal{K}\kappa^3/\ell^2 = \frac{2\,(1+\xi)^2\,BYx^6}{(x^2+1)^4}, \tag{2.3}$$

where c is the speed of light, $x = r/\ell$ is a dimensionless coordinate, $Y(x) = 8 - 2(3\xi - 2)x^2 - (3\xi+1)x^4$,

$$B_1 = \frac{1}{8YB}\left[\,(6\xi+2)\,x^6 + ((6B+13)\,\xi + 2B - 1)\,x^4 + ((21B+7)\,\xi - 17B - 9)\,x^2 - 34B - 6\right],$$

$$B_2 = -\frac{1}{4YB^2(x^2\xi + 3x^2 + 2)}\Big[(1+\xi)^2 x^{10} + \big(3 + 3\xi^2 + (8B+6)\,\xi\big)\,x^8 + \big(\,(3B^2 + B + 3)\,\xi^2 + \big(-2B^2 + 26B + 6\big)\,\xi + 3B^2 - 11B + 3\big)x^6 + \big(\,(6B^2 + B + 1)\,\xi^2 + \big(-4B^2 + 26B + 2\big)\,\xi + 2B^2 - 35B + 1\big)x^4 - 8B\,((B-1)\,\xi + B + 9/2)\,x^2 - 4B^2 - 12B\Big].$$

c_Ω^2 and c_r^2 are positive for all $\xi \in (-3; 1)$. In the case $r \to \infty$ the remaining functions

$$c_{r3}^2/c^2 = -\frac{3(1-\xi)}{\xi + 1/3} + O\left(\frac{1}{x^2}\right), \quad c_{\Omega\pm}^2/c^2 = 1 + O\left(\frac{1}{x^2}\right) \tag{2.4}$$

are positive in the case $\xi < -\frac{1}{3}$. The value $\mathcal{K}$ is positive at the same time as c_{r3}^2. In general the region of $(\xi; r)$ under which $c_{r3}^2 > 0$ and $c_{\Omega\pm}^2 > 0$ are marked in green at the Fig. 1a. At the event horizon $r = r_h$ function $B(r)$ and c_{r3}^2 (2.2) changes the sign at the same time. Thus in general the vacuum solution is unstable at least near the event horizon for all $\xi \in (-3; 1)$. However for an external vacuum solution of a neutron star whose radius is greater than the event horizon r_h of the star, the stability conditions will be satisfied in the case $\xi < -\frac{1}{3}$.

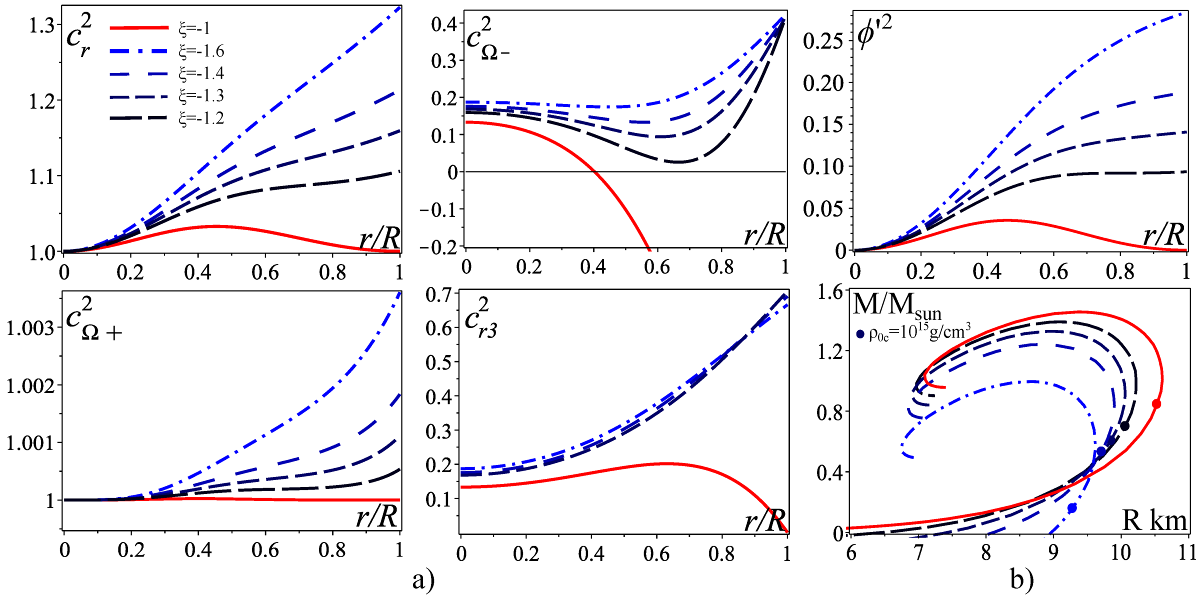

Fig. 2. a) Graphs of squared speeds of perturbations c_r^2, c_{r3}^2, $c_{\Omega\pm}^2$ depending on a reduced radial coordinate r/R, where R is the radius of a star. The red solid curve corresponds to $\xi = -1$. Blue curves correspond to the domain $\xi \in (-1.67, -1.18)$: $\xi = -1.6; -1.4; -1.3; -1.2$. b) The mass-radius diagram and square of a scalar field depending on a reduced radial coordinate r/R.

For the interior solution of a neutron star the stability conditions were investigated numerically. Particularly in the case $\ell = 10$ km (characteristic scale of the nonminimal derivative coupling) and $\rho_{0c} = 10^{15}$ g/cm^3 (central baryonic mass density) the Fig. 1b shows the range of values $(r/R; \xi)$ under which conditions $c_{r3}^2 > 0$ and $c_{\Omega\pm}^2 > 0$ are performed, the values c_r^2 and c_Ω^2 are positive in this case and is not represented at the diagram. In the Fig. 2a we demonstrate examples of star configurations for which all squared speeds of perturbations, c_r^2, c_Ω^2, c_{r3}^2, $c_{\Omega\pm}^2$, and also $\mathcal{K}$ are positive. The examples are given for $\rho_{0c} = 10^{15}$ g/cm^3 and $\ell = 10km$. In this case the stability conditions are realized within domain of the parameter $\xi \in (-1.67, -1.18)$. Note additionally that $c_\Omega^2/c^2 = 1$ and $\mathcal{K} > 0$ within these domains, and also the asymptotic mass is positive and the scalar field is real (Fig. 2b). In this range, the external vacuum solution also satisfies the stability conditions (2.1), since $\xi < -1/3$. Hence, in principle, the neutron star configurations with such the set of parameters are free from the Laplacian and ghost instabilities. Note also that the value $\xi = -1$ lays outside these domains. For comparison, we also include into plots graphs of squared speeds corresponding to the case $\xi = -1$. It is clearly seen that $c^2_{\Omega-}$ takes negative values inside the star, that is the neutron star configurations with $\xi = -1$ are unstable.

Conclusion

We studied in details the stability of neutron star configurations in theory of gravity with the scalar-derivative coupling of the Einstein tensor and the scalar field with the standard kinetic term and the cosmological constant and found that there exists a wide class of model parameters ξ, ℓ for which all stability conditions are fulfilled. In particular, in Figs. 1 and 2 we demonstrated regions of the model parameters and examples of star configurations for which all squared speeds of perturbations, c^2_r, c^2_Ω, c^2_{r3}, $c^2_{\Omega\pm}$, and also $\mathcal{K}$ are positive, and so such the configurations are free from the Laplacian and ghost instabilities.

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Просьба ссылаться на эту статью следующим образом:
Кашаргин П. Е., Сушков С. В. Устойчивость нейтронных звезд в теории гравитации с неминимальной производной связью скалярного поля и тензора Эйнштейна. *Пространство, время и фундаментальные взаимодействия.* 2023. № 1. С. 85–90.

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Please cite this article in English as:
Kashargin P. E., Sushkov S. V. Stability of anti-de Sitter neutron stars in the theory of gravity with nonminimal derivative coupling. *Space, Time and Fundamental Interactions*, 2023, no. 1, pp. 85–90.