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ДЕТЕКТОРЫ ВЫСОКОЧАСТОТНЫХ ГРАВИТАЦИОННЫХ ВОЛН НА ОСНОВЕ ГРАВИТАЦИОННО-ОПТИЧЕСКОГО РЕЗОНАНСА*

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В данной работе рассматриваются возможности регистрации реликтовых гравитационных волн, предсказываемых в космологических моделях ранней Вселенной с дополнительной стадией преобладания жесткой энергии. Рассмотрены отличия спектра реликтовых гравитационных волн в данных моделях от случая стандартных инфляционных моделей. Предложен метод детектирования высокочастотных гравитационных волн на основе гравитационно-оптического резонанса в интерферометрах Фабри-Перо. Рассчитаны основные параметры детектора и определен диапазон частот реликтовых гравитационных волн, которые могут быть зарегистрированы посредством предложенного подхода.

Ключевые слова: скалярные поля, космологические возмущения, модифицированные теории гравитации.

DETECTORS OF HIGH-FREQUENCY GRAVITATIONAL WAVES BASED ON THE GRAVITATIONAL-OPTICAL RESONANCE

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In this paper, we consider the possibility of registering relic gravitational waves predicted in cosmological models of the early universe with an additional stage of stiff energy dominance. Differences in the spectrum of relic gravitational waves in these models from the case of standard inflationary models are considered. A method for detecting high-frequency gravitational waves based on gravitational-optical resonance in Fabry-Perot interferometers is proposed. The main parameters of the detector are calculated and the frequency range of relic gravitational waves, which can be registered using the proposed approach, is determined.

Keywords: scalar fields, cosmological perturbations, modified gravity theories.

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Introduction

After the first detections of gravitational waves (GWs) using ground-based laser interferometers LIGO and VIRGO [1–3] based on the method proposed in [4], it can be argued that a new instrument for precision measurements in the field of astrophysics and cosmology has appeared. Gravitational waves are space-time perturbations predicted in metric theories of gravity (including general relativity) that travel at the speed of light and can be characterized by frequency f and dimensionless amplitude h_{GW}.

Modern ground-based detectors have a maximum sensitivity in the frequency range from 10 Hz to 10 kHz [1–3], in which the development of observation technology and the abundance of astrophysical sources, such as black hole and neutron star mergers, provided the first detections of gravitational waves. The projected space laser interferometer LISA is designed for the frequency range $0.1 - 10$ Hz focusing mainly to detect gravitational waves from binary black holes and white dwarfs [5].

The motivation for the study of high-frequency gravitational waves is that there are no known astrophysical objects that are compact and dense enough to emit gravitational waves at frequencies above 10 kHz. Thus, any discovery of gravitational waves at higher frequencies will correspond either to exotic astrophysical objects (such as primordial black holes, cosmic strings or bosonic stars) [6] or they are caused by cosmological perturbations in the early universe at the inflationary stage of its evolution [7–9].

Relic gravitational waves have not been directly observed, which leads to a large number of theoretical models of cosmological inflation that explain the origin and evolution of the large-scale structure of the universe and are consistent with currently available observational data on anisotropy and polarization of the CMB [10].

Nevertheless, the specifics of the transition from the inflationary stage to the stage of radiation predominance has a significant effect on the spectrum of relic gravitational waves [11–14]. Deviations of the state parameter from the value $w = 1/3$ for radiation at a given transition correspond to a significant increase in the energy density of relic gravitational waves in the high-frequency range. These deviations are associated with the presence of an additional stage of stiff energy predominance between the end of the inflationary stage and the beginning of the radiation domination stage. The energy density of relic gravitational waves corresponding to the cutoff frequency of their spectrum f_* following from BBN constraint in such a models is estimated as $\Omega_{GW} \lesssim 10^{-6}$ [11–14].

In the framework of general relativity, such cosmological models correspond to quintessential inflation, in which a scalar field is a source of accelerated expansion of the early universe at the inflationary stage and the second observed accelerated expansion of the universe at the modern era of its evolution. The frequency of relic gravitational waves corresponding to the maximum energy density in these cosmological models is estimated as $f_* \sim 10^{10}$ Hz [15–17].

Also, the cosmological models based of the scalar-tensor gravity theories with an additional stage of stiff energy predominance were considered in [18]. The main features of these cosmological models are the direct connection between the physical potentials of the scalar field and the known theories of scalar-tensor gravity and the satisfaction to the observational constraints for an arbitrary implementation of the inflationary scenario (arbitrary potential of a scalar field), which is impossible to implement in the cosmological models based on general relativity. However, at large times these cosmological models are reduced to the case of the ΛCDM model and gravity coincide with GR. The frequency of gravitational waves corresponding to the maximum energy density predicted in these cosmological models is estimated as $f_* \sim 10^5 - 10^7$ Hz [18].

As other sources of the stochastic gravitational wave background, we also consider cosmic strings and primordial black holes.

As a result of phase transitions in the early universe, various topological defects can arise, one of which is a cosmic strings. Cosmic strings arise in different inflationary models such as supersymmetric hybrid inflation [19] or in the brane world cosmological scenarios [20]. For frequencies $f \sim 10^{-9} - 10^{14}$

Hz, the spectrum of gravitational waves from cosmic strings closes to the flat one, and the value of the energy density is estimated as $\Omega_{GW} \lesssim 10^{-8}$ [21].

As hypothetical components of dark matter, primordial black holes are considered, which can exist both in the form of separate binary systems and binary systems in dark halos [22]. Primary binaries originate from two primordial black holes that were formed close enough to each other that the dynamics of the evolution of this system did not depend on the expansion of the universe until the equilibrium period of matter and radiation. The gravitational interaction of one or more nearby primordial black holes does not allow two black holes to merge directly, which leads to the formation of a binary system. In some cases, the binary system is quite stable, and the merger of two primordial black holes requires a time of the order of the age of the universe [22]. The frequency of gravitational waves in stochastic gravitational wave background from the primary black holes mergers is in range $f \sim 10^3 - 10^9$ Hz and its energy density is estimated as $\Omega_{GW} \lesssim 10^{-9}$ [23].

Based on the fact that high-frequency relic gravitational waves can have a higher energy density (or amplitude) than those inspired by cosmic strings or primordial black holes, we will focus on the analysis of the possibility of direct detection of high-frequency relic gravitational waves.

1. Characteristics of relic gravitational waves

To analyze the possibility of direct registration of relic gravitational waves, we firstly consider their main characteristics, taking into account the recent new constraint on the value of their amplitude compared to scalar cosmological perturbations, which follows from observations of the anisotropy and polarization of the CMB [24].

The energy density of relic gravitational waves ρ_{GW} is defined in terms of following dimensionless quantity [11–14]

$$\Omega_{GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\log f}, \tag{1.1}$$

where ρ_c is the critical density, f is the frequency of relic gravitational waves.

The spectrum of relic gravitational waves at the present time for the cosmological models with additional stiff energy domination (SD) stage can be defined by expression [13, 14]

$$\Omega_{\rm GW}(f) \simeq \Omega_{GW}^{(0)} \times \begin{cases} 1 & , \quad f \ll f_{\rm RD}, \\ 1.27 \times \left(\frac{f}{f_{\rm RD}}\right)^{\alpha_S} & , \quad f \gg f_{\rm RD}, \end{cases}$$

where the plateau of the spectrum is

$$h^2\Omega_{GW}^{(0)} \simeq r \times 10^{-15}, \tag{1.2}$$

tensor-to-scalar ratio $r < 0.032$ characterizes the contribution of relic gravitational waves to the anisotropy and polarization of the relic radiation w.r.t. scalar perturbations [24], reduced Hubble parameter is estimated as $h \simeq 0.68$ [10], and f_{RD} is the present day frequency corresponding to the horizon scale at the transition from the stiff energy domination (SD) stage to the radiation domination (RD) stage.

Also, parameter α_S is related with the state parameter w_S as follows

$$\alpha_S = 2\left(\frac{3w_S - 1}{3w_S + 1}\right). \tag{1.3}$$

The maximum value of the parameter $\alpha_S = 1$ corresponds to extremely stiff matter with state parameter $w_S = 1$, for which the speed of sound is equal to the speed of light in vacuum, the minimum value $\alpha_S = 0$ corresponds to the radiation with state parameter $w_S = 1/3$.

For $f \gg f_{RD}$ and $\alpha_S > 0$ one has $\Omega_{GW}(f) \gg \Omega^{(0)}_{GW}$, i.e the energy density of relic gravitational waves can be defined as follows [18]

$$h^2\Omega_{GW}(f) \simeq 1.27 \times r \times 10^{-15} \times \left(\frac{f}{f_{RD}}\right)^{\alpha_S}. \quad (1.4)$$

From the LIGO bound $\Omega_{GW}(f \simeq 10^2 Hz) < 10^{-9}$ [25] and expression (1.4) one can estimate the lower limit on the frequency f_{RD} as

$$f_{RD} \gtrsim 100 \times \exp\left[-\frac{13.6 + \ln\left(\frac{h^2}{r}\right)}{\alpha_S}\right] Hz. \quad (1.5)$$

To find the cutoff frequency of the spectrum of relic gravitational waves f_* one can use the BBN constraint [26]

$$\int_{f_{BBN}}^{f_*} h^2\Omega_{\rm GW}(f)\frac{df}{f} \leq 1.12 \times 10^{-6}, \quad (1.6)$$

where $f_{BBN} \simeq 1.41 \times 10^{-11} Hz$.

Taking into account that $f_* \gg f_{RD}$ and $f_* \gg f_{BBN}$ from (1.4) and (1.6) one has following constraint on the cutoff frequency

$$f_* \lesssim f_{RD} \times \left(\frac{\alpha_S}{r} \times 10^9\right)^{1/\alpha_S}. \quad (1.7)$$

Thus, from LIGO and BBN constraints we obtain lower limit on the GW's frequency corresponding to the horizon scale at the SD-to-RD transition (1.5) and upper limit on the cutoff frequency (1.7) for the cosmological models with additional stiff energy domination era.

From expressions (1.4), (1.5) and (1.7) we define following value of the energy density of relic gravitational waves corresponding to the cutoff frequency is

$$\Omega_{GW}(f_*) \lesssim 2.7 \times \alpha_S \times 10^{-6}, \quad (1.8)$$

for any cosmological model with $0 < \alpha_S \leq 1$, and for any value of the cutoff frequency f_*.

For the case $w_S = 1/3$ ($\alpha_S = 0$), the spectrum of relic gravitational waves closes to the flat one, and energy density can be defined by expression (1.2).

Also, we note, that the dimensionless amplitude of relic gravitational waves is defined as follows [9, 26]

$$h_c(f) = 1.26 \times 10^{-18}\left(\frac{Hz}{f}\right)\sqrt{h^2\Omega_{GW}(f)}. \quad (1.9)$$

This parameter can be used for assessing the possibility of direct registration of relic gravitational waves by existing and advanced GW's detectors.

2. High-frequency relic gravitational waves

We can consider the characteristics of relic gravitational waves for various models of cosmological inflation, which can be divided into two classes, namely: standard cosmological models with $\alpha_S = 0$ and models with an additional stiff energy domination era with $0 < \alpha_S \leq 1$.

For the standard cosmological models with $\alpha_S = 0$ from expression (1.2), taking into account the constraint on the tensor-to-scalar ratio $r < 0.032$, we obtain

$$\Omega_{GW} \lesssim 7 \times 10^{-17}. \quad (2.1)$$

The upper limit on the frequency of relic gravitational waves was estimated in [27] as follows

$$f_* \simeq 3 \times 10^{10} Hz. \quad (2.2)$$

For cosmological models with an additional stiff energy domination era and $0 < \alpha_S \leq 1$ from (1.8) we obtain following constraint on the energy density

$$\Omega_{GW}(f_*) \lesssim 2.7 \times 10^{-6}, \tag{2.3}$$

and the lower limit on the frequency of relic gravitational waves corresponding to maximal energy density $\Omega_{GW} = \Omega_{GW}^{(max)} \simeq 2.7 \times 10^{-6}$ (for $\alpha_S = 1$) can be obtained from (1.5) and (1.7), namely

$$f_* \gtrsim 3 \times 10^5 Hz, \tag{2.4}$$

where the frequency $f_{RD} \gtrsim 9 \times 10^{-6}$ Hz.

Thus, in general case, for the models with an additional stiff energy domination era ($0 < \alpha_S \leq 1$) we can consider the following constraint on the cutoff frequency of relic gravitational waves

$$3 \times 10^5 Hz \lesssim f_* \lesssim 3 \times 10^{10} Hz, \tag{2.5}$$

corresponding to the maximal energy density.

The maximal amplitude of relic gravitational waves in such a models can be estimated from (1.9), (2.3) and (2.4) as

$$3 \times 10^{-27} Hz \lesssim h_c \lesssim 3 \times 10^{-32} Hz, \tag{2.6}$$

which is significantly less than the amplitude of low-frequency gravitational waves [18].

As possible method for detecting high-frequency relic gravitational waves let us consider the use of the gravitational-optical resonance effect in the multi-beam Fabry-Perot interferometers [28–30].

3. Gravitational-optical resonance

Gravitational-optical resonance in a multi-beam interferometer occurs if the condition is satisfied that an integer number of half-waves of gravitational radiation fits on the length L of the resonator [28–30]

$$L = \frac{nc}{2f}, \quad n = 1, 2, 3, ... \tag{3.1}$$

where c is the speed of light.

We also note that when using a multi-beam interferometer to register high-frequency gravitational waves, it is not required to create a complex system for decoupling mirrors used for the ground-based gravitational antennas operating in the low-frequency part of the spectrum. This is due to the fact that the frequency of mechanical vibrations of the interferometer mirrors in this case turns out to be significantly less than the frequency of the gravitational wave.

4. The sensitivity of the detector

Consider the case of tuning a multi-beam resonator at $n = 1$. Then the response $\delta W(t)$ of such a gravitational wave detector of the high-frequency gravitational waves can be estimated by expression [28–30]

$$\delta W(t) = \frac{QL}{\lambda_e} W_0 h_c(t), \tag{4.1}$$

where δW is the power variation of the laser radiation transmitted by the interferometer, Q is the quality factor of the Fabry-Perot interferometer, λ_e is a laser wavelength, W_0 is the power of laser radiation at the entrance to the interferometer. We also note that expression (4.1) depends on the tuning point of the Fabry-Perot interferometer.

On Fig. 1 a scheme of a gravitational-wave detector based on a Michelson interferometer, in the arms of which two Fabry-Perot interferometers are placed, is shown.

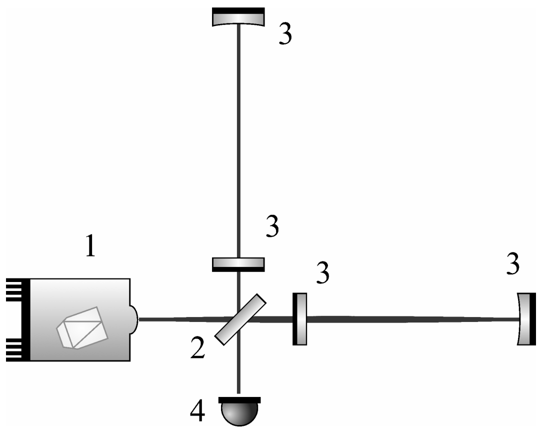

Fig. 1. A scheme of a high-frequency gravitational-wave detector based on the Michelson interferometer: 1 - laser, 2 - beam splitter, 3 - mirrors of Fabry-Perot interferometers, 4 - photodetector.

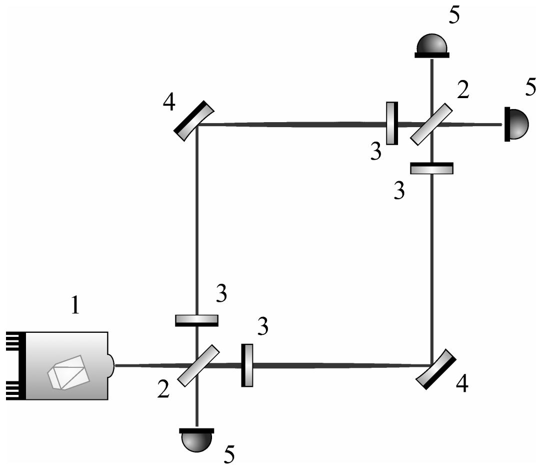

Fig. 2. A scheme of a high-frequency gravitational-wave detector based on the Mach-Zehnder interferometer: 1 - laser, 2 - beam splitters, 3 - mirrors of Fabry-Perot interferometers, 4 - mirrors for bending the beam inside Fabry-Perot interferometers, 5 - photodetectors.

On Fig. 2 an alternative scheme of the high-frequency gravitational wave detector based on a Zander-Mach interferometer, which contains one main and two auxiliary photodetectors is shown.

Since gravitational waves stochastic background is often characterized by its spectral density [26,31]

$$S_h(f) = \frac{h_c^2(f)}{2f}, \tag{4.2}$$

one can rewrite expression (4.1) in terms of the spectral density of power variations $S_{\delta W}$ as follows

$$S_{\delta W}(f) = \frac{Q^2 L^2}{\lambda_e^2} W_0^2 S_h(f). \tag{4.3}$$

To increase the sensitivity of the gravitational wave detector, it is possible to use the averaging procedure of the spectral density of power variations $S_{\delta W}(f)$ over the time period T. This makes it possible to increase the sensitivity of this detector by a factor [30]

$$K = \sqrt{\frac{f}{Q}T} = \sqrt{\frac{cT}{QL}}. \tag{4.4}$$

In this case, expression (4.3) for the spectral density of power variations is

$$\tilde{S}_{\delta W}(f) = K S_{\delta W}(f) = \frac{\sqrt{cT} Q^{3/2} L^{3/2}}{\lambda_e^2} W_0^2 S_h(f), \tag{4.5}$$

and, therefore, one can define the spectral density of detectable gravitational waves as

$$S_h(f) = \frac{\lambda_e^2}{\sqrt{cT}Q^{3/2}L^{3/2}W_0^2}\tilde{S}_{\delta W}(f). \tag{4.6}$$

Thus, from expressions (1.9) and (4.6) one has following amplitude of detectable gravitational waves

$$h_c = 2\left(\frac{Q}{T}\right)^{1/4}\left(\frac{\lambda_e}{QW_0c}\right)\tilde{S}_{\delta W}^{1/2}(f)f^{5/4}, \tag{4.7}$$

corresponding to the sensitivity of the detector.

Nevertheless, the sensitivity of such a GW detectors is restricted by the shot photon noise and radiation pressure noise, since the laser light arrives on a mirrors of the Frabri-Perot resonators and the photodetector as discrete quanta which produce the additional power fluctuations δW, that can be registered by the photodetector [31, 32].

In this case, we will consider the shot photon noise and radiation pressure noise as the factors, which restrict the sensitivity of such a gravitational waves detectors, assuming the influence of other factors is significantly less than these ones [31, 32].

5. Photon shot noise

Now, we estimate the minimal detectable amplitude of gravitational waves due to reducing the power of laser radiation incident on the photodetector to decrease the photon shot noise.

The average power measured at the photodetector during the observation time is [31]

$$W_{ph} = \alpha W_0 = \frac{1}{T}N_\gamma\hbar\omega_e = \frac{2\pi c\hbar}{T\lambda_e}N_\gamma, \tag{5.1}$$

where $\hbar$ is the reduced Planck constant, α is a factor that determines the difference between the power of the laser radiation that measured at the photodetector W_{ph} and the power of laser radiation at the entrance to the interferometer W_0.

The fluctuation in a large number of photons is [31]

$$\delta N_\gamma = \sqrt{N_\gamma} \tag{5.2}$$

since the Poisson distribution is reduced to the Gaussian distribution for a large number of photons, and one can define the produced fluctuations in the observed power as follows

$$(\delta W)_{shot} = \frac{2\pi c\hbar}{T\lambda_e}N_\gamma^{1/2} = \sqrt{\frac{2\pi c\hbar}{T\lambda_e}\alpha W_0}. \tag{5.3}$$

The signal-to-noise ratio can be defined as [31]

$$\frac{S}{N} = \frac{\delta W}{(\delta W)_{shot}} = \frac{QL}{\lambda_e}\left(\frac{T\lambda_e W_0}{2\pi c\hbar\alpha}\right)^{1/2}h_c = \frac{Q}{2f}W_0\left(\frac{T}{W_{ph}}\right)^{1/2}\left(\frac{c}{2\pi\lambda_e\hbar}\right)^{1/2}h_c. \tag{5.4}$$

Thus, to increase the signal-to-noise ratio it is possible to reduce the power of laser radiation arriving at the photodetector W_{ph}, which will be restricted by the detectable spectral density of power variations $\tilde{S}_{\delta W}(f)$.

To analyze this possibility we consider the photon shot noise registered by the photodetector in more detail.

The photon shot noise dispersion is defined as follows

$$\sigma_w = \sqrt{S_{\delta W}(f)\Delta f} = \sqrt{\frac{S_{\delta W}(f)f}{Q}}, \tag{5.5}$$

where $\Delta f = f/Q$ is the resonant line width, since the detector perceives the gravitational wave signal only in a narrow range of the spectrum.

If averaging time is $T > 1/\Delta f$, then the noise dispersion σ_w is reduced to $\tilde{\sigma}_w = \sigma_w/K$, and the power fluctuations are

$$(\delta W)_{shot} = \frac{1}{K}\sqrt{\tilde{S}_{\delta W}(f)\Delta f} = \sqrt{\frac{\tilde{S}_{\delta W}(f)}{T}}. \tag{5.6}$$

From expressions (5.3) and (5.6) we obtain the power of a laser radiation

$$W_{ph}^{(min)} = \alpha W_0 = \frac{\tilde{S}_{\delta W}(f)\lambda_e}{2\pi c\hbar}, \tag{5.7}$$

corresponding to the minimal detectable spectral density of laser radiation variations $\tilde{S}_{\delta W}(f)$.

Thus, from (5.7) one has following factor

$$\alpha = \frac{W_{ph}^{(min)}}{W_0} = \frac{\tilde{S}_{\delta W}(f)\lambda_e}{2\pi c\hbar W_0}, \tag{5.8}$$

corresponding to minimal power of a laser radiation restricted due to the sensitivity of the photodetector.

Further, from expressions (4.1) and (5.6) the signal-to-noise ratio is

$$\frac{S}{N} = \frac{\delta W}{(\delta W)_{shot}} = \frac{QLW_0 h_c}{\lambda_e\sqrt{\frac{\tilde{S}_{\delta W}(f)}{T}}}. \tag{5.9}$$

Thus, under condition $S/N = 1$, from (3.1) and (5.9) one has

$$h_c^{(min)} = 2f\left(\frac{\lambda_e}{QW_0 c}\right)\left[\frac{\tilde{S}_{\delta W}(f)}{T}\right]^{1/2}, \tag{5.10}$$

and from (4.7) and (5.10) we obtain

$$h_c = (QTf)^{1/4}\, h_c^{(min)}. \tag{5.11}$$

Thus, from expression (5.11) and condition $h_c = h_c^{(min)}$ (or, otherwise, $S/N = 1$) one has $T^{(min)} = (Qf)^{-1}$.

Therefore, for observation time

$$T > \frac{1}{\Delta f} = \frac{Q}{f}, \tag{5.12}$$

one has $T > T^{(min)}$ for $Q > 1$, i.e. under condition (5.12) photon short noise can be reduced below the sensitivity of such a gravitational wave detector (4.7).

6. Radiation pressure noise

The radiation pressure noise of a Frabri-Perot resonator is derived from fluctuations of the mirror positions due to fluctuations of the laser power.

The relationship between the fluctuation of the mirror position δx and the laser power fluctuations δW is [31, 32]

$$\delta x = \frac{\delta W}{2Mc\pi^2 f^2}, \tag{6.1}$$

where M is the mass of the mirror of the Frabri-Perot resonator, and the expression for the laser power fluctuations can be obtained from (5.4) for $\alpha = 1$, namely

$$\delta W = \sqrt{\frac{2\pi c\hbar}{T\lambda_e}}W_0. \tag{6.2}$$

The response of detector on the radiation pressure noise can be defined as follows [31, 32]

$$h_c^{(rad)} = \frac{\delta x}{L}. \tag{6.3}$$

Thus, from expressions (3.1) and (6.1)–(6.3) one has following restriction on the radiation pressure noise

$$h_c \geq h_c^{(rad)} = \frac{1}{Mf}\left(\frac{2W_0\hbar}{\pi^3 c^3 \lambda_e T}\right)^{1/2}, \tag{6.4}$$

where condition $h_c^{(rad)} = h_c$ defines the mirror's mass $M^{(min)}$ corresponding to signal-to-noise ratio $S/N = 1$, and for $M > M^{(min)}$ the radiation pressure noise can be reduced below the sensitivity of such a gravitational wave detector (4.7).

7. Parameters of GW's detector

Now, we consider the possibility of detection of relic gravitational waves by proposed method for the following parameters: the laser wavelength $\lambda_e = 1.064\,\mu m$, the quality factor of the Fabry-Perot resonator $Q = 10^6$, the power of a laser radiation at the entrance to the interferometer $W_0 = 10^3$ W.

Also, we consider for the photodetector DET10N2 with following parameters: an operating spectral range $500 - 1700$ nm, the sensitivity band up to 70 MHz, and an equivalent noise power 2×10^{-14} $W/Hz^{1/2}$ [30].

For this photodetector, the minimal detected spectral density of laser radiation variations is [30]

$$\tilde{S}_{\delta W} = 4 \times 10^{-28}\, W^2/Hz. \tag{7.1}$$

From expression (5.8) one has

$$\alpha = \frac{W_{ph}^{(min)}}{W_0} = \frac{\tilde{S}_{\delta W}\lambda_e}{2\pi c\hbar W_0} = 2.2 \times 10^{-12}, \tag{7.2}$$

i.e. the reduced power of laser radiation on the photodetector must be equal to $W_{ph}^{(min)} = 2.2 \times 10^{-9}$ W.

From expressions (1.9) and (4.7) one has following observation time

$$T^{1/4} = \left(\frac{1.6 \times 10^{18}}{Hz}\right)\left(\frac{\lambda_e}{W_0 hc}\right)\left[\frac{\tilde{S}_{\delta W}^2 f^9}{Q^3 \Omega_{GW}^2}\right]^{1/4}, \tag{7.3}$$

which it is necessary to detect the gravitational waves.

Also, from (5.12) and (7.3) one has following condition on the frequency of detectable gravitational waves

$$f > 5.2 \times 10^{-8} \times \left(\frac{h^2\Omega_{GW}}{\tilde{S}_{\delta W}}\right)^{1/5}\left(\frac{QW_0 c}{\lambda_e} Hz\right)^{2/5}. \tag{7.4}$$

Thus, for proposed detector's parameters the lower limit on the frequency of detectable gravitational waves can be estimated as follows $f > 2 \times 10^6$ Hz.

To estimate the upper limit on the frequency of gravitational waves which can be detected by this method we consider the time of detection for GW's with energy density $\Omega_{GW} \simeq 10^{-6}$ closed to the maximum value.

In Tab. 1 the observation time and the length of the Fabri-Perot resonator, which is necessary to detect of the high-frequency relic gravitational waves are represented. Also, we note that realistic masses of the mirrors of the Frabri-Perot resonators are $M \gg M^{(min)}$, thus, in this case, the short photon noise can be considered as the main factor that restricts sensitivity of the detector.

Table 1. The parameters of the Fabri-Perot resonator for detectable gravitational waves with energy density $\Omega_{GW} \simeq 10^{-6}$.

f, Hz	h_c	T, sec	L, m	$M^{(min)}, kg$
3×10^6	3×10^{-28}	9	50	6×10^{-6}
5×10^6	2×10^{-28}	879	30	5×10^{-6}
1×10^7	1×10^{-28}	4.6×10^5	15	2×10^{-8}
2×10^7	5×10^{-29}	2.4×10^8	7.5	1×10^{-9}

As one can see, the minimal length of the Fabri-Perot resonator can be estimated as $L^{(min)} \approx 15\,m$ with corresponding observation time $T \approx 4.6 \times 10^5\,sec \simeq 5$ days, since in the latter case for $L = 7.5\,m$ m the observation time is $T = 2.4 \times 10^8\,sec \simeq 7.6$ years.

Thus, the proposed approach in principle makes it possible to detect relic gravitational waves in the frequency range $f \simeq 3 - 10$ MHz, if condition of gravitational-optical resonance (3.1) in the Fabri-Perot resonator with length $L \simeq 50 - 15\,m$ is met for a certain GW's frequency in this range.

We also note that relation (7.3) implies $T \sim \Omega_{GW}^{-2}$, for this reason, if the energy density of stochastic gravitational wave background is less than $\Omega_{GW} \simeq 10^{-6}$, the detection time T increases significantly (by two orders w.r.t. detection time presented in Tab. 1). For this reason, frequency range $f \simeq 3 - 5$ MHz is more optimal for detection than the wider one $f \simeq 3 - 10$ MHz.

Conclusion

In this paper, we analyzed the possibility of detecting high-frequency relic gravitational waves by using the effect of gravitational-optical resonance in Fabry-Perot interferometers.

Also, we considered the characteristics of relic gravitational waves for standard inflationary models and cosmological models with an additional stage of stiff energy domination. For the second class of cosmological models, the energy density of relic gravitational waves can significantly exceed the energy density for standard inflation, but the amplitude of high-frequency gravitational waves is much less than the amplitude of low-frequency gravitational waves. This fact causes interest in the low-frequency part of the spectrum of gravitational waves in the design of advanced detectors such as LISA, DECIGO, BBO etc. [33].

Nevertheless, the high-frequency part of the spectrum of relic gravitational waves is interesting in that their direct detection, because it make possible to determine the correctness of a class of cosmological models with presence of an additional stage between the end of inflation and the beginning of the radiation domination era.

Based on the restrictions associated with the presence of short photon noise and radiation pressure in Fabry-Perot interferometers, we determined the optimal frequency range for detecting relic gravitational waves using the proposed approach as $f \simeq 3 - 5$ MHz with corresponding cavity length $L \simeq 50 - 30$ m.

A significant limitation of this approach is the need for exact fulfillment of relation (3.1), which is necessary for the appearance of gravitational-optical resonance. Nevertheless, there are substantial theoretical grounds for analyzing high-frequency gravitational waves in this frequency range both in the context of modern cosmological models [18], and in the case of a stochastic gravitational wave background from cosmic strings [21] and primordial black holes [23] as well.

We also note a recent report [34] on the detection of high-frequency gravitational waves with a frequency $f = 5.5$ MHz and amplitude $h_c \approx 2.5 \times 10^{-16}$ (two rare events) by using cryogenic Bulk Acoustic Wave cavity as high frequency gravitational wave antenna. As a possible source of gravitational waves, a merger of primordial black holes with masses $M_{PBH} \sim 10^{-4} M_{\odot}$ is considered.

This result met with significant criticism, see, for example, in papers [23, 35]. In [23] it was noted both the low probability of such events $\mathcal{R} \sim 10^{-24}$ and the fact that it is necessary to consider the stochastic gravitational wave background of the primary black holes mergers, for which the energy density and the corresponding amplitude of gravitational waves are much lower than stated in [34].

Also in work [35] it was noted that high-frequency gravitational waves from primordial black holes mergers should be accompanied by low-frequency gravitational waves due to the effect of gravitational-wave memory, and the high-frequency GW's with amplitude $h_c \approx 2.5 \times 10^{-16}$ imply low-frequency gravitational waves, which should have been registered by LIGO.

Since the frequency of gravitational waves $f = 5.5$ MHz is close to the optimal range for observation $f \simeq 3 - 5$ MHz obtained in the framework of our approach to detecting high-frequency gravitational waves one can directly verify the existence of gravitational wave signals at this frequency with a possible lower amplitude.

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