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КОСМОЛОГИЯ С ОТСКОКОМ В МОДИФИЦИРОВАННЫХ ТЕОРИЯХ ГРАВИТАЦИИ

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В настоящем исследовании мы описываем основанную на теории гравитации $f(R)$ космологическую модель Вселенной с отскоком. Для определения динамического поведения модели использовалось плоское пространство-время FLRW. В эпоху отскока геометрические параметры демонстрируют поведение сингулярности. Параметры масштабного фактора оказывают значительное влияние на поведение характера отскока.

Ключевые слова: петлевая квантовая гравитация, сценарий с отскоком.

BOUNCING COSMOLOGY IN MODIFIED THEORIES OF GRAVITY

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In this study, we describe a $f(R)$ gravity theory-based bouncing cosmological model of the universe. The flat FLRW space-time were used to determine the model's dynamical behaviour. At the bouncing epoch, the geometrical parameters exhibit singularity behaviour. The parameters of the scale factor have a significant impact on the bouncing behaviour.

Keywords: loop quantum cosmology, bouncing scenario.

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Introduction

Among other problems that General Relativity (GR) has run against in the early Universe is the initial singularity. According to Friedmann [1, 2], the evolution of the universe began at the time when the initial singularity occured. The inflationary hypothesis addressed some important problems of the early Universe, notably [3–5], it is thought that the singularity problem existed before inflation. One explanation is that after experiencing a bounce, the universe expands rather than attaining singularity during the contraction. Recent studies [1, 2, 8–12] have shown that our universe is undergoing a late time rapid expansion phase, which is explained by dark energy, time-independent vacuum energy .

The current research focuses on the bouncing model in the $f(R)$ theory in a FLRW space-time, a modified theory of gravity. It should be noted that the $f(R)$ gravity theory is a fantastic substitute for the traditional gravity model while researching dark energy cosmological theories. The Einstein Hilbert action results in Friedmann equations in FLRW metric and is expressed in terms of two terms, $f(R)$ and

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matter Lagrangian. Odintsov and Oikonomou [13] have studied a bouncing cosmology with a Type IV singularity at the bouncing point in the $f(R)$ modified gravity framework. The extended matter bounce scenario in ghost free $f(R,G)$ gravity, which is consistent with gravitational waves, has been examined by Elizalde et al. [14]. Another geometrically modified theory of gravity that has recently been developed utilising the non-metricity approach is the extended symmetric teleparallel gravity, or $f(Q)$ gravity [18], where Q represents the non-metricity. In [16–18], a number of cosmological and astrophysical features of $f(Q)$ gravity have been investigated.

Our goal is to examine the bouncing cosmological model in this research so that we can avoid the initial singularity problem with the functional form of $f(R)$. The model will examine geometrical degrees of freedom to address the late-time cosmic speed-up issue. In Sec. 1 of the study, the explanation of $f(R)$ gravity and the derivation of $f(R)$ field equations are provided. The bouncing scale factor and Hubble parameter were introduced in Sec. 2. In Sec. 3, the bouncing scale factor and functional form of $f(R)$ have been presented. The cosmographic parameters are covered in Sec. 4, while Sec. 5 provides the energy conditions for the model. The conclusions are presented in Sec. 6.

1. Field equations of $f(R)$ gravity

The gravitational action for $f(R)$ can be described as,

$$S=\int\sqrt{-g}\frac{f(R)}{2\kappa^2}d^4x+\int\sqrt{-g}d^4x\mathcal{L}_m, \tag{1.1}$$

where $\mathcal{L}_m$ be the matter Lagrangian and $\kappa^2=\frac{8\pi G}{c^4}$, G be the Newton's gravitational constant, g is determinant of the metric tensor g_{ij}. The $f(R)$ gravity field equations can be found by varying action (1.1) with respect to g_{ij}.,

$$f_R R_{ij}-\frac{f}{2}g_{ij}-(\nabla_i\nabla_j-g_{ij}\Box)f_R=\kappa^2 T_{ij}. \tag{1.2}$$

Here $f=f(R)$ and $f_R=\frac{\partial f}{\partial R}$, ∇_i represents the covariant derivative, $\Box\equiv g^{ij}\nabla_i\nabla_j$ is the d'Alembert operator. The natural system of unit $8\pi G=\bar{h}=c=1$ has been used, where G, $\bar{h}$ and c respectively denote the Newtonian gravitational constant, reduced Planck constant and velocity of light in vacuum respectively. We can express the energy momentum tensor T_{ij} as,

$$T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{ij}}. \tag{1.3}$$

On contracting (1.2), we can obtain

$$Rf_R-2f+3\Box f_R=T, \tag{1.4}$$

where Ricci scalar, $R=g^{ij}R_{ij}$, trace of energy momentum tensor $T=g^{ij}T_{ij}$. We consider the flat FLRW space-time as,

$$ds^2=-dt^2+a^2(t)(dx^2+dy^2+dz^2), \tag{1.5}$$

and the energy momentum tensor as the perfect fluid,

$$T_{ij}=(\rho+p)u_iu_j+pg_{ij}. \tag{1.6}$$

Here, ρ and p are the energy density and pressure of the matter field, respectively, and u_i stands for the four fluid velocity vectors that fulfil $u^iu_i=-1$. The explicit form of the field equations (1.2) for the space time (1.5) is,

$$\kappa^2\rho = \frac{1}{2}\left[f_R R-f\right]-3H\dot{f}_R+3(1-f_R)H^2, \tag{1.7}$$

$$\kappa^2 p = -\frac{1}{2}\left[f_R R-f\right]+\ddot{f}_R+2H\dot{f}_R-(1-f_R)(2\dot{H}+3H^2). \tag{1.8}$$

The Hubble parameter, $H = \frac{\dot{a}}{a}$, can be used to formulate equations (1.7) and (1.8). To enhance our understanding of the dynamics of the Universe, we therefore require a Hubble function to determine the energy density and pressure of the matter field. In addition, the behavior of the equation of state (EoS) parameter must be examined in order to understand the problem of late time acceleration, which can be calculated as,

$$\omega = \frac{p}{\rho} = -\frac{f_R R - f - 2\ddot{f}_R - 4H\dot{f}_R + 2(1-f_R)(2\dot{H}+3H^2)}{f_R R - f - 6H\dot{f}_R + 6(1-f_R)H^2}. \tag{1.9}$$

Therefore, by taking into account the bouncing scale factor and the functional form of $f(R)$, we will analyse the bouncing scenario and late time cosmic acceleration issue of the Universe in the following sections.

2. The Scale Factor

One of the two current models of the early Universe that attempts to reconcile the theoretical inconsistencies of the Big Bang theory is inflationary cosmology, the other theory is matter bounce cosmology. Inflationary models were subject to tight constraints imposed by the most recent observational data, which supported some models while disproving others. In this article, we will examine the matter bounce scenario in $f(R)$ theory of gravity. The time evolution of the Hubble parameter, or $H = \dot{a}/a$, can be used to calculate the rate of the universe's expansion. The scale factor is subject to the following restrictions in order to obtain a bouncing scenario:

In the case of a non-singular bounce, the bouncing scenario behaves as a contracting nature described by a scale factor that decreases over time, i.e., $\dot{a} < 0$, which indicates that the Hubble parameter is negative during the phase of matter contract, i.e., $H = \dot{a}/a < 0$. Since the scale factor for the bouncing epoch contracts to a non-zero, finite critical size, the Hubble parameter disappears at bounce, making $H = 0$. The Hubble parameter changes to a positive value after the bounce, i.e., $\dot{a} > 0$, since the nature of scale factor rises over time with a positive acceleration. When the bouncing epoch is close, the Hubble parameter is true, i.e., $\dot{H} > 0$, which is appropriate for the ghost (phantom) behaviour of the model. To further understand a bouncing model, consider how equation of state (EoS) changes twice at such a phantom region: once before the bounce and once after the bounce.

As a result, our bouncing model in this instance obeys both the simultaneous dynamical behaviour and the aforementioned bouncing requirements. The Hubble parameter $H = \frac{t}{\alpha+\beta t^2}$ is then taken into account along with a bouncing scale factor $a(t) = \left(\frac{\alpha}{\beta} + t^2\right)^{\frac{1}{2\beta}}$, where α and β are positive constants. The bounce is seen to happen at $t = 0$, and the parameter α determines how steep the curve will be. A steeper slope results from a higher value of α. The bounce appears to be symmetrical; the scale factor appears to drop from a greater value at the beginning (in the negative time domain), bounce at $t = 0$, and then rise again at the end. From a higher negative value, the Hubble parameter's curve rises, crosses the bouncing point at $t = 0$, and continues to rise throughout the evolution. The parameters' behaviour encourages the occurrence of bouncing scenario, which helps to resolve the initial singularity problem.

3. Model

The dynamical parameters must be obtained by solving Eq. (1.7) and Eq. (1.8) before the cosmological model can be built. To do this, one must take into account a functional form for $f(R)$. We look at the form of $f(R)$ [20] as follows:

$$f(R) = R + \lambda R_0 \left[\left(1 + \frac{R^2}{R_0^2}\right)^{-n} - 1\right], \tag{3.1}$$

where R_0, λ, and n are all constants and R_0 is the characteristic curvature. To align with the Starobinsky model, we selected the exponent value of $n = 1$.

Substituting Eq. (3.1) in Eqs. (1.7), (1.8)(1.9), the matter pressure, energy density and the equation of state parameter can be obtained in the form of Hubble parameter. Further using the assumed form of the Hubble parameter, the dynamical parameters can be expressed in cosmic time.

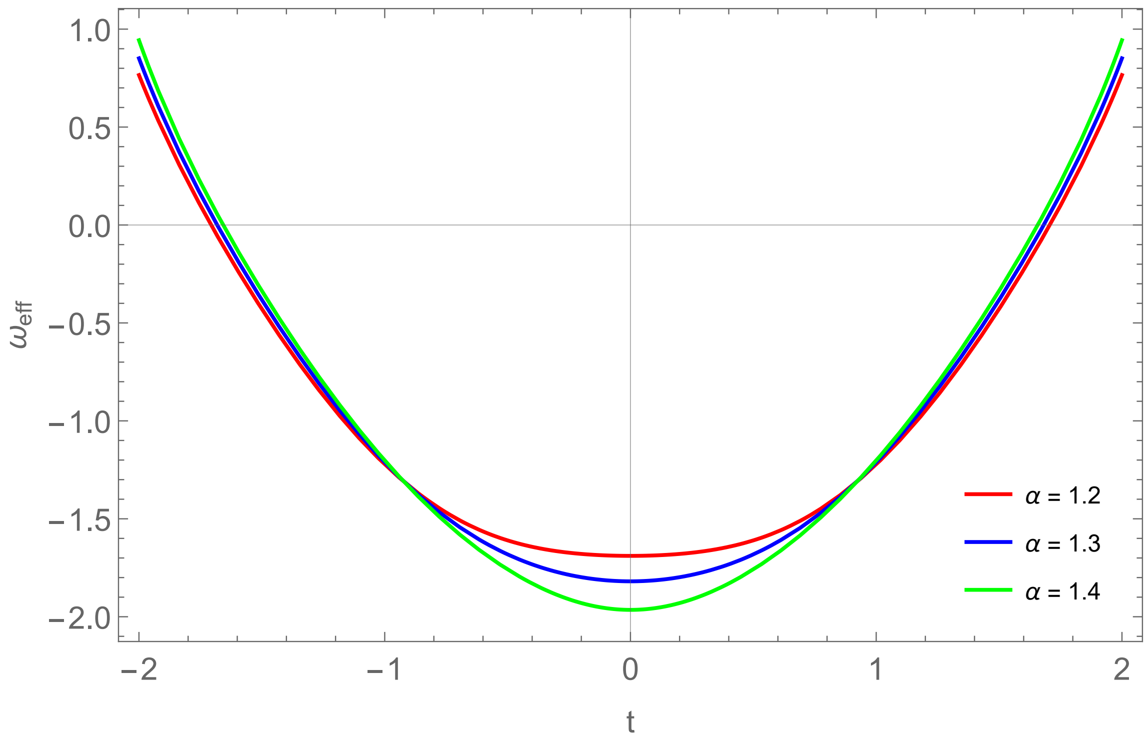

Fig. 1. Variation of equation of state parameter in cosmic time with varying $\alpha = 1.2, 1.3, 1.4$ with $\beta = 0.9$, $R_0 = 2$, $\lambda = 0.01$

The EoS parameter for the model exhibits a bounce at $t = 0$ and remains negative throughout the evolution. The well of the curve is more pronounced as α increases in magnitude. Given a lower value of α, the symmetric curve approaches being flat around the bounce epoch. Throughout, the energy density stays in the positive area. The bounce at $t = 0$ becomes more pronounced with increasing levels of α. The initial increase in energy density is followed by a kind of ditch reduction near and at the bounce, followed by a continuing decrease. Since the energy density is always positive, the matter pressure becomes always negative and it exhibits the bounce at $t = 0$. The EoS parameter further grows initially before swiftly decreasing to form the well, increasing quickly for a short period of time following the bounce.

4. Cosmographic Parameters

The dark energy models and modified gravity models are two families of models being researched in cosmology. Both are fundamentally distinct in that it is easy to tell these models of the two families with the identical cosmic expansion history apart from one another. Even though all of the models have the same expansion history, the growth rate of cosmic density perturbations is estimated in the normal way and distinguishes the models based on different gravity theories. One method for differentiating between modified gravity models and dark energy is to use the growth factor of the perturbation in matter density [21]. Using the state finder pair (j, s), which is mentioned in [22], is another method of separating the dark energy models. It is well known that the scale factor may be used to define the expansion rate of the universe and that the deceleration parameter (q) corresponds to the second derivative of the scale factor. The third and fourth derivatives of scale factor are represented by the jerk

(j) and snap (s) parameters, respectively, while the lerk (l) parameter represents the fifth derivative of scale factor. In the Taylor series expansion around the scale factor, these values can be described as,

$$a(t) = a(t_0) + \sum_{n=0}^{\infty} \frac{1}{n!}\frac{d^n a}{dt^n}\Bigg|_{t=t_0}(t-t_0)^n, \tag{4.1}$$

where $n = 1, 2, 3, ...,$ is an integer and t_0 is the current cosmic time. The coefficients of the expansion will give these parameters, called the cosmographic parameters. These geometrical characteristics can be derived from the scale factor as follows:

$$\begin{aligned}
q &= -\frac{\ddot{a}}{a}.\frac{1}{H^2} = \frac{\alpha}{t^2} + \beta, \\
j &= \frac{\dddot{a}}{a}.\frac{1}{H^3} = \frac{(2\beta-1)\left[t^2(\beta-1) - 3\alpha\right]}{t^2}, \\
s &= \frac{a^{(4)}}{a}.\frac{1}{H^4} = -\frac{(2\beta-1)\left[3\alpha^2 + t^4(\beta-1)(3\beta-1) + 6\alpha t^2(1-3\beta)\right]}{t^4}, \\
l &= \frac{a^{(5)}}{a}.\frac{1}{H^5} = \frac{\left(8\beta^2 - 6\beta + 1\right)\left[15\alpha^2 + t^4(\beta-1)(3\beta-1) + 10\alpha t^2(1-3\beta)\right]}{t^4}.
\end{aligned} \tag{4.2}$$

The Hubble parameter as derived from the assumed scale factor experiencing bounce at $t = 0$, the other geometrical parameters as expressed above will also having the bouncing scenario.

5. Energy conditions

The energy momentum tensor must meet a number of requirements since the causal metric and geodesic structure of space-time are addressed by Einstein's field equations in general relativity (GR). We can normalise the time-like vector u_i to be $u_i u^i = -1$ and the future directed null k^i as $k^i k_i = 0$ for the space-time $(-,+,+,+)$. According to [23–25], we can describe the energy conditions as contractions of time like or null vector fields with regard to the Einstein tensor and the energy-momentum tensor from the matter side of the Einstein's field equations. Four energy conditions that we can get are as follows:

- At each point of the space time, the energy momentum tensor should satisfy, $T_{ij}u^i u^j \geq 0$: Weak Energy Condition (WEC). So, $\rho \geq 0$, $\rho + p \geq 0$.
- For the future directed null vector k^i, $T_{ij}u^i u^j \geq 0$: Null Energy Condition (NEC). So, $\rho + p \geq 0$.
- The matter flows along time like or null line and with contracted energy momentum tensor, the quantity $T_{ij}u^{ij}$ becomes future directed time like or null like vector field: Dominant Energy Condition (DEC). So, $\rho - p \geq 0$.
- $\left(T_{ij} - \frac{1}{2}Tg_{ij}\right)u^i u^j \geq 0$ says the gravity has to be attractive: Strong Energy Condition (SEC). So, $\rho + 3p \geq 0$.

The $f(R)$ gravity and the expanded theories of gravity are simple extensions of Einstein's GR. The energy conditions should be brought up with any such expanded theory. From Eqs. (1.7) and (1.8), the expression for the energy conditions can be obtained. We have given below the graphical representation of the energy conditions.

All the energy conditions are showing the symmetric behaviour. Both the NEC and SEC are initially satisfying and before the bounce violates and subsequently after some time of the bounce again satisfies. However since it violates at and around the bouncing epoch, it supports the bouncing model. The violation of SEC has become essential in the context of modified theories of gravity. The DEC satisfies entirely. We obtain the behaviour of the energy conditions as needed in the context of the model that shows bouncing behaviour.

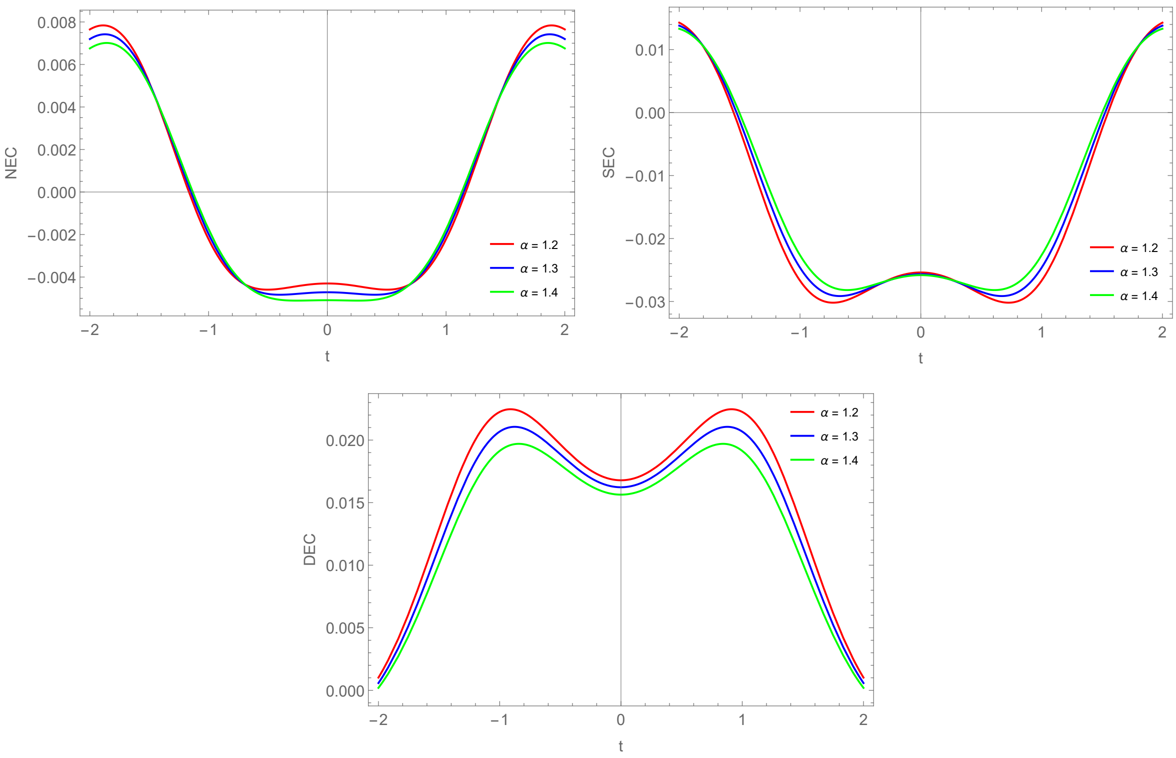

Fig. 2. Variation of null energy condition (left panel), strong energy condition (right panel) and dominant energy condition (lower panel) in cosmic time with varying $\alpha = 1.2, 1.3, 1.4$ with $\beta = 0.9$, $R_0 = 2$, $\lambda = 0.01$

6. Conclusion

In the $f(R)$ theory of gravity, the bouncing cosmological model of the universe has been proposed. A minimally coupled function in R is used in place of the typical Ricci scalar in the action that leads to the $f(R)$ theory of gravity. With a bouncing scale factor, well-known versions of the $f(R)$ function have been taken into consideration. At $t = 0$, the model displays the bouncing behaviour. Additionally, for the selected bouncing scale factor, the Hubble radius diverges at the place of the bounce, falls monotonically on both sides of the bounce, and then asymptotically contracts to zero, pointing to an expanding universe in the late stages of the expansion. Additionally, for the provided $f(R)$ theory to be compatible with Planck constraints and produce the necessary perturbation modes close to the bounce, such scale factor behaviour is necessary. Throughout evolution, the matter pressure and energy density have remained, respectively, negative and positive. To support the bouncing behaviour, the EoS parameter curve crosses the phantom-divide line twice. The behaviour of the EoS and deceleration parameters further verified the models' accelerated expansion. The singularity in the EoS parameter is eliminated if there is a finite non-zero value of R_0 during the bouncing epoch. Additionally, the scale parameter of the scale factor affects how the EoS parameter behaves. The NEC and SEC violations are verified. In the framework of a bouncing and accelerating cosmic model, these breaches are unavoidable. It should be noted that the failure of the null energy criteria could lead to the phantom phase developing in the model with a positive Hubble parameter slope. We conclude by stating that this may provide additional insight into overcoming the original singularity problem.

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