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EXACT SOLUTIONS IN FRIEDMANN COSMOLOGY WITH SCALAR FIELDS *

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The method of exact analysis of cosmological dynamics at the early inflation stage of the evolution of the Friedman Universe, which is determined by the dynamics of the scalar field for the case of minimal and nonminimal interaction of the field and curvature is considered in this work. The basis of the method is to reduce the equations of dynamics to the same form for all models. The proposed approach allows us to compare models based on the different theories of gravity.

Keywords: scalar field, inflation, modified gravity theories.

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Introduction

Despite the fact that standard inflationary scenarios based on Einstein's gravity theory explain successfully the origin of the large scale structure, the anisotropy of background radiation and the mechanism for the formation of elementary particles [1–5], that is, they give a consistent method of explaining the origin of the universe and its further evolution, there are problems that go beyond this approach, for example, the nature of dark energy [6, 7] at the stage of the repeated expansion of the universe or construction of the theory of quantum gravity. For this reason, at the present time, cosmological models based on the modification of Einstein's gravity are considered [7–9]. In the framework of this approach, among others, scalar-tensor theories of gravitation and Einstein-Gauss-Bonnet gravity are considered as well (see, for example, [8, 9]).

We note that the scalar-tensor gravity theories make it possible to explain both stages of accelerated expansion without attracting dark energy. Also, Gauss-Bonnet scalar arises in the low-energy limit of the supergravity action for superstrings [8] and Einstein-Gauss-Bonnet gravity can be considered as an effective theory of quantum gravity.

New methods of exact solutions for cosmological models with scalar-tensor gravity theories and with Einstein-Gauss-Bonnet gravity were presented earlier in papers [10–14], in which, also, a comparison with standard models for both: at the level of dynamics and on the parameters of cosmological perturbations was made.

When analyzing the evolution of the universe at the stage of cosmological inflation, the scalar field potential plays an important role, which determines the nature of inflationary processes and the mechanism of generation of elementary particles after the end of inflation, thus, the main task of this work is to find the main parameters characterizing the modifications of GR for the case of known physical potentials which were considered in standard cosmology. For this purpose, a method is proposed that allows us to bring the equations of dynamics in the case of modified gravity theories to the standard equations of cosmological inflation with Einstein's gravity.

In the first part of the paper we give the equations of dynamics in a general form, including the nonminimal coupling of a scalar field with Ricci and Gauss-Bonnet scalars as well as the function that determines the interaction of a field and its kinetic energy. The second part deals with the method of

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generating new exact solutions from those known for models with minimal coupling. In the third part, exact solutions are given for models with scalar-tensor gravity for Higgs potential and the general method for obtaining exact solutions is considered. The fourth part deals with models of inflation with Einstein-Gauss-Bonnet gravity, the influence of non-minimal coupling on the scalar field potential is shown and the parameters characterizing this type of gravity are calculated for the potential $V(\phi) \propto \cosh^2(A\phi)$.

1. Equations of cosmological dynamics in Friedmann universe

First, let us write the action that determines the dynamics of the scalar field at the stage of cosmological inflation in the system of units $8\pi G = c = 1$

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}F(\phi)R - \frac{1}{2}\omega(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{2}\xi(\phi)R^2_{GB}\right], \quad (1)$$

where R is the Ricci scalar, ϕ is the scalar field and $V(\phi)$ is its potential, the function $\omega(\phi)$ determines the interaction of the field and its kinetic energy, $\xi(\phi)$ determines the coupling of the scalar field and Gauss-Bonnet scalar $R^2_{GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$.

For the case of a homogeneous, isotropic, spatially flat Friedmann universe, the geometry of which is determined by the Friedman-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right), \quad (2)$$

we obtain the equations of dynamics in the following form [15]

$$3FH^2 + 3H\dot{F} - \frac{\omega}{2}\dot{\phi}^2 - V(\phi) - 12H^3\dot{\xi} = 0, \quad (3)$$

$$3FH^2 + 2H\dot{F} + 2F\dot{H} + \ddot{F} + \frac{\omega}{2}\dot{\phi}^2 - V(\phi) - 8H^3\dot{\xi} - 8H\dot{H}\dot{\xi} - 4H^2\ddot{\xi} = 0, \quad (4)$$

$$\omega\ddot{\phi} + 3\omega H\dot{\phi} + \frac{1}{2}\dot{\phi}^2\omega'_\phi + V'_\phi - 6H^2F'_\phi - 3\dot{H}F'_\phi + 12H^4\xi'_\phi + 12H^2\dot{H}\xi'_\phi = 0, \quad (5)$$

where the dot denotes the derivative with respect to time, the Hubble parameter $H = \dot{a}/a$, $V'_\phi = dV/d\phi$.

The three equations (3)-(5) are independent only two, so the dynamics of the universe at the stage of inflation can be considered on the basis of a system with two nonlinear differential equations for various cases of the coupling of the scalar field and curvature. Thus, for the analysis of cosmological dynamics at the early stage of the evolution of the universe we will use the first two equations in (3)-(4).

2. Exact solutions in models with minimal coupling

Consider the case $\xi = 0$, $F = 1$ which corresponds to the minimal coupling of the scalar field and curvature or Einstein's theory of gravity.

In this case, the dynamics equations (3)-(4) can be written in the following form

$$3H^2 = \frac{\omega(\phi)}{2}\dot{\phi}^2 + V(\phi), \quad (6)$$

$$\dot{H} = -\frac{\omega(\phi)}{2}\dot{\phi}^2. \quad (7)$$

In the context of models with minimal coupling, as a rule, the case $\omega = 1$ is considered. However we can also consider models with a new scalar field $\varphi = \int\sqrt{\omega(\phi)}d\phi$, thus, we have analogous equations for scalar fields ϕ and φ with different potentials and Hubble parameters.

In many papers the various ways of solving the system of equations (6)-(7) were considered earlier and a large number of exact solutions are given in the review [16], also, the classification of methods of exact solutions was considered in the work [17]. In this case, we use the method associated with the choice of the Hubble parameter (or scale factor), which means the reconstruction of the evolution of scalar field and its potential from given dynamics.

Thus, we write the system of equations (6)-(7) for the case $\omega = 1$ as follows

$$V(\phi) = 3H^2 + \dot{H}, \tag{8}$$

$$\dot{\phi}^2 = -2\dot{H}. \tag{9}$$

Thus, we obtain exact solutions of the dynamics equations by specifying the Hubble parameter corresponding to the accelerated expansion of the universe from the system of equations (8)-(9). Some exact solutions, given in the paper [16], on the basis of which various scenarios of the early universe are proposed, are presented in Table 1.

Table 1

Exact solutions in cosmological models with minimal coupling. The scalar field is written with taking into account its initial value $\phi - \phi_0 \to \phi$

Hubble parameter	Scalar field evolution	Scalar field potential
$H(t) = -At + B$	$\phi(t) = \pm\sqrt{2A}\,t$	$V(\phi) = 3\left(\mp\sqrt{\frac{A}{2}}\phi + B\right)^2 - A$
$H(t) = B\exp(-At)$	$\phi(t) = \sqrt{\frac{8B}{A}}\exp\left(-\frac{A}{2}t\right)$	$V(\phi) = \frac{3A}{8}\phi^2\left(\frac{A}{8}\phi^2 - A\right)$
$H(t) = -\frac{AB}{3}\tan(At)$	$\phi(t) = \sqrt{\frac{2B}{3}}\ln\sqrt{\frac{1+\sin(At)}{1-\sin(At)}}$	$V(\phi) = \frac{A^2B(B-1)}{3}\cosh^2\left(\sqrt{\frac{3}{2B}}\phi\right) - \frac{A^2B^2}{3}$
$H(t) = \frac{AB}{3}\tanh(At)$	$\phi(t) = \sqrt{-\frac{2B}{3}}\arcsin(\tanh(At))$	$V(\phi) = \frac{A^2B}{3}\left(B\sin^2\sqrt{-\frac{3}{2B}}\phi + \cos^2\sqrt{-\frac{3}{2B}}\phi\right)$
$H(t) = A^2B\coth(2Bt)$	$\phi(t) = A\ln(\tanh(Bt))$	$V(\phi) = A^2B^2\left[(3A^2-2)\cosh^2\left(\frac{\phi}{A}\right) + 2\right]$
$H(t) = A^2B\cot(2Bt)$	$\phi(t) = A\arctan(\cos(2Bt))$	$V(\phi) = A^2B^2\left[(3A^2-2)\cosh^2\left(\frac{\phi}{A}\right) - 3A^2\right]$
$H(t) = \frac{B}{3t} - \frac{A}{3B}$	$\phi(t) = \sqrt{\frac{2B}{3}}\ln(t)$	$V(\phi) = \frac{B(B-1)}{3}\left(e^{-2\sqrt{\frac{3}{2B}}\phi} - \frac{2Ae^{-\sqrt{\frac{3}{2B}}\phi}}{B(B-1)}\right) + \frac{A^2}{3B^2}$
$H(t) = A\left[\frac{B+4}{6AB}t\right]^{-\frac{B}{B+4}}$	$\phi(t) = \left[\frac{B+4}{6AB}t\right]^{\frac{2}{B+4}}$	$V(\phi) = 3A^2\phi^{-B}\left(1 - \frac{B^2}{6}\phi^{-2}\right)$
$H(t) = \frac{A^2B}{6}\coth^3(Bt)$	$\phi(t) = \frac{A}{\sinh(Bt)}$	$V(\phi) = \frac{B^2}{12A^2}\phi^2(\phi^2 + A)\left(\frac{\phi^4}{A^2} + 2\phi^2 + A^2 - 6\right)$
$H(t) = C\ln(At + B)$	$\phi(t) = \sqrt{-\frac{8C}{A}(At + B)}$	$V(\phi) = 3C^2\ln^2\left(-\frac{A}{8C}\phi^2\right) - \frac{8C^2}{\phi^2}$

To generate new exact solutions from known ones, we consider the transformations in a general form $(H, V, \phi) \to (\overline{H}, U, \varphi)$ [17].

Further, we consider the following transformation of the Hubble parameter

$$\overline{H} = f(t)H, \tag{10}$$

where $f(t)$ is an arbitrary function of time.

Taking into account the equations (8)-(9), we get

$$U(\varphi) = 3H^2f^2 + \frac{d}{dt}(fH), \tag{11}$$

$$\frac{1}{2}\dot{\varphi}^2 = -\frac{d}{dt}(fH) = \frac{1}{2}\dot{\phi}^2\left(f + 2\dot{f}\frac{H}{\dot{H}}\right) = \frac{\omega(\phi)}{2}\dot{\phi}^2. \tag{12}$$

Thus, the transition from old solutions to new ones is carried out on the basis of the choice of the function $\omega(\phi) \equiv \omega(H(t))$ in the equation

$$f + 2\dot{f}\frac{H}{\dot{H}} = f(H) + f'_H H = \omega(H(t)). \tag{13}$$

For the case of a constant function $\omega(H(t)) = n = const$, from the equation (13), we obtain

$$f(H(t)) = n + \frac{\lambda}{H}, \tag{14}$$

where λ is the constant of integration.

Thus, taking into account the equations (10)-(12), we can write the relations between new and initial exact solutions for the function (14):

$$\overline{H} = nH + \lambda, \tag{15}$$

$$\overline{a}(t) = Ca^n(t)\exp(\lambda t), \quad C = \overline{a}_0/a_0^n, \tag{16}$$

$$U(\varphi) = 3n^2H^2 + 6\lambda nH + n\dot{H} + 3\lambda^2, \tag{17}$$

$$\varphi = \pm\sqrt{n}\phi, \tag{18}$$

where the sign of the parameter n determines the relationship between models with canonical and phantom fields.

Let us demonstrate this method using the example of the model with Higgs potential and with Hubble parameter $H(t) = B\exp(-At)$ to which the scale factor is $a(t) = a_0\exp\left(-\frac{B}{A}e^{-At}\right)$, which implies a double exponential expansion for the case when $A < 0$.

On the basis of the transformations (15)-(18), we write down the new exact solutions

$$\overline{H} = nB\exp(-At) + \lambda, \tag{19}$$

$$\overline{a}(t) = \overline{a}_0\exp\left(\lambda t - \frac{nB}{A}e^{-At}\right), \tag{20}$$

$$\varphi = \pm\sqrt{\frac{8nB}{A}}\exp\left(-\frac{A}{2}t\right), \tag{21}$$

$$U(\varphi) = \frac{3A^2}{64}\varphi^4 + \left(\frac{3A\lambda}{4} - \frac{A^2}{8}\right)\varphi^2 + 3\lambda^2, \tag{22}$$

that is, we get Higgs potential with a different dynamics, also, for the case $\lambda = A/6$, we get the potential

$$U(\varphi) = \frac{3A^2}{64}\varphi^4 + 3\lambda^2, \tag{23}$$

corresponding to chaotic inflation.

We note that, writing the initial equations (8)-(9) in the context of the Ivanov-Salopek-Bond method, the description of which is given in the review [16],

$$V(\phi) = 3H^2 - 2H'^2_\phi, \tag{24}$$

$$\dot{\phi} = -2H'_\phi, \tag{25}$$

it is possible to represent the equation (17) in terms of a scalar field

$$U(\varphi(\phi)) = 3n^2H^2 + 6\lambda nH - 2nH'^2_\phi + 3\lambda^2. \tag{26}$$

Also, based on the equations (11)–(13), one can consider more complicated transformations of exact cosmological solutions in models with minimal coupling by choosing the functions $\omega(H(t))$ or $f(t)$.

3. Cosmological models with scalar-tensor gravity

Scalar-tensor theories of gravitation are a possible alternative of Einstein's gravity for describing the dynamics of the universe, in the context of which, the problem of the interpretation of dark energy in the repeated stage of accelerated expansion is solved by modifying GR.

In this case, we will consider a material scalar fields, this approach is based on the possibility of the conformal transformation of metric $\hat{g}^{\mu\nu} = F(\phi)g^{\mu\nu}$ from the Jordan frame (with the geometrical scalar fields) to the Einstein frame [9].

In the case of cosmological models with scalar-tensor gravity $\xi = 0$, $\omega = \omega(\phi)$, $F = F(\phi)$ we write the dynamical equations (3)–(4) in the form

$$\frac{\omega(\phi)}{2}\dot{\phi}^2 + V(\phi) = 3FH^2 + 3H\dot{F}, \tag{27}$$

$$\omega(\phi)\dot{\phi}^2 = H\dot{F} - 2F\dot{H} - \ddot{F}. \tag{28}$$

Further, on the basis of the following representation of the functions $F(\phi)$ and $\omega(\phi)$

$$F(t)=1-\frac{\beta_{ST}}{a^2(t)},\quad F(\phi)=1-\frac{\beta_{ST}}{a^2(\phi)}, \tag{29}$$

$$\omega(t)=1-\beta_{ST}\left(\frac{3H^2}{\dot{H}a^2}\right),\quad \omega(\phi)=1+3\beta_{ST}\left(\frac{H}{aH'}\right)^2, \tag{30}$$

where β_{ST} is a constant parameter that determines the nonminimal coupling for the case of scalar-tensor gravity theories, we obtain equations similar to (8)–(9)

$$V(t)=3H^2+\dot{H},\quad V(\phi)=3H^2-2H'^2, \tag{31}$$

$$\dot{\phi}^2=-2\dot{H},\qquad \dot{\phi}=-2H'. \tag{32}$$

As an example, we consider the initial model with Einstein's gravity and Higgs potential (19)–(22). From the equations (29)–(30) we obtain

$$F(\varphi)=1-\frac{\beta_{ST}}{a_0^2}\left(\frac{A\varphi^2}{8nB}\right)^{2\lambda/A}\exp\left(\frac{1}{4}\varphi^2\right), \tag{33}$$

$$\omega(\varphi)=1+\frac{3\beta_{ST}}{a_0^2A^2}\left(A\varphi+\frac{8\lambda}{\varphi}\right)^2\left(\frac{A\varphi^2}{8nB}\right)^{2\lambda/A}\exp\left(\frac{1}{4}\varphi^2\right). \tag{34}$$

Thus, based on the equations (29)–(32), the exact solutions can be translated from models with minimal coupling to the case of scalar-tensor gravity theories and one can obtain the corresponding parameters of such theories.

4. Models with nonminimal coupling between a scalar field and Gauss-Bonnet scalar

For the case of models with nonminimal coupling between a scalar field and Gauss-Bonnet scalar, corresponding to $F=1$ in the equations (3)–(4), we obtain the dynamical equations in the following form

$$3H^2=\frac{\omega(\phi)}{2}\dot{\phi}^2+V(\phi)+12\dot{\xi}H^3, \tag{35}$$

$$-2\dot{H}=\omega(\phi)\dot{\phi}^2-4\ddot{\xi}H^2-4\dot{\xi}H(2\dot{H}-H^2). \tag{36}$$

To find the exact solutions we write this system of equations in terms of the generating function $g(t)$

$$V(\phi)=3H^2+5Hg+\dot{H}+\dot{g}, \tag{37}$$

$$\frac{\omega(\phi)}{2}\dot{\phi}^2=Hg-\dot{H}-\dot{g}, \tag{38}$$

$$g=-2\dot{\xi}H^2. \tag{39}$$

We note that the function g determines the difference between Hubble parameters for the Einstein's gravity (or minimal coupling) and Einstein-Gauss-Bonnet gravity $g=H_E-H$, in the case when $g=0$ the equations (37)–(38) are reduced to (8)–(9) and $\xi=const$.

Further, we consider, separately, two cases:

- The case when $\omega(\phi)=1$.

For this case we consider the function g, which is defined from equation

$$Hg-\dot{g}=0, \tag{40}$$

i.e. $g(t)=a(t)\alpha_{GB}$, where α_{GB} is a constant parameter that determines a coupling of the field and Gauss-Bonnet scalar.

In this case, from (37)–(39), we get the system of equations

$$V(\phi) = 3H^2 + \dot{H} + 6\dot{a}\alpha_{GB}, \tag{41}$$

$$\dot{\phi}^2 = -2\dot{H}, \tag{42}$$

$$\dot{\xi} = -\left(\frac{\alpha_{GB}}{2}\right)\frac{a^3}{\dot{a}^2}\,. \tag{43}$$

Thus, in this case, the potential will differ from the model in Einstein's gravity by the term $U_{GB} = 6\dot{a}\alpha_{GB}$, which appears due to the coupling of the field and Gauss-Bonnet scalar.

To illustrate the influence of nonminimal coupling of a scalar field and Gauss-Bonnet scalar, we consider the model with quadratic potential, Hubble parameter $H(t) = -At + B$ from Tab. 1 and with the scale factor

$$a(t) = a_0 \exp\left(Bt - \frac{A}{2}t^2\right). \tag{44}$$

From the equations (41) and (43), for the case $\phi(t) = \sqrt{2A}\,t$, we obtain

$$V(\phi) = 3\left(-\sqrt{\frac{A}{2}}\phi + B\right)^2 + 6a_0\alpha_{GB}\left(-\sqrt{\frac{A}{2}}\phi + B\right)\exp\left(\frac{B\phi}{\sqrt{2A}} - \frac{1}{4}\phi^2\right) - A, \tag{45}$$

$$\xi(t) = \frac{a_0\alpha_{GB}}{2}\left[\frac{\exp\left(\frac{B^2-(At-B)^2}{2A}\right)}{A(At-B)} + \sqrt{\frac{\pi}{2A^3}}e^{\frac{B^2}{2A}}\,erf\left(\frac{At-B}{\sqrt{2A}}\right)\right] + const, \tag{46}$$

$$\xi(\phi) = \frac{a_0\alpha_{GB}}{2}\left[\frac{\exp\left(\frac{1}{2A}\left(B^2 - \left(\sqrt{\frac{A}{2}}\phi - B\right)^2\right)\right)}{A\left(\sqrt{\frac{A}{2}}\phi - B\right)} + \sqrt{\frac{\pi}{2A^3}}e^{\frac{B^2}{2A}}\,erf\left(\frac{\sqrt{\frac{A}{2}}\phi - B}{\sqrt{2A}}\right)\right] + const. \tag{47}$$

The potentials (45) for the case of Einstein gravity theory and Einstein-Gauss-Bonnet gravity are shown in Figure. 1, the nonzero energy of vacuum was taken into account when constructing the graphs, i.e. $V \to V + const$.

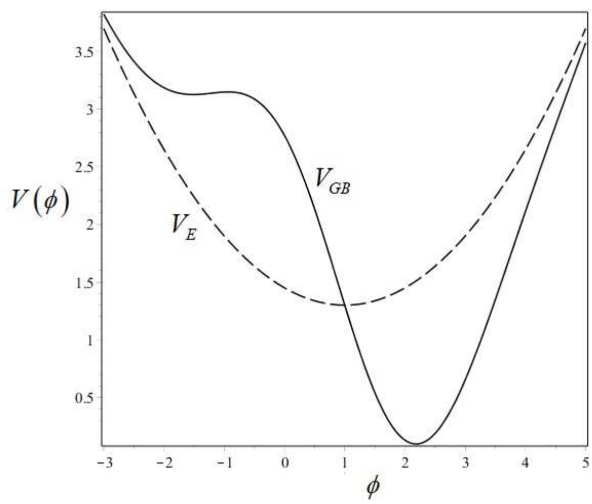

Fig. 1. The potential V_E for Einstein's gravity with parameter $\alpha_{GB} = 0$ and the potential V_{GB} corresponding to Einstein-Gauss-Bonnet gravity with parameter $\alpha_{GB} = 1$.

Thus, the coupling of the scalar field with Gauss-Bonnet scalar creates an additional state of a false vacuum, from which the scalar field passes into the state of a true vacuum. This type of potential is often considered in the context of string theory and supergravity [8] which corresponds to the original assertion that Gauss-Bonnet scalar arises in the low-energy limit for the string's action.

• The case when $\omega(\phi) \neq 1$.

For the present case we consider the function g which we will define from equation

$$5Hg + \dot{g} = 0, \tag{48}$$

i.e. $g = a^{-5}\alpha_{GB}$.

Thus, on the basis of the following representation of the functions $g(t)$ and $\omega(t)$

$$g = a^{-5}\alpha_{GB}, \quad \omega(t) = 1 - \alpha_{GB}\left(\frac{6\dot{a}}{\dot{H}a^6}\right), \tag{49}$$

we obtain the system of equations

$$V(\phi) = 3H^2 + \dot{H}, \tag{50}$$

$$\dot{\phi}^2 = -2\dot{H}, \tag{51}$$

$$\dot{\xi} = -\frac{\alpha_{GB}}{2a^3\dot{a}^2}, \tag{52}$$

which also allows us to use the exact solutions given earlier for cosmological models with Einstein-Gauss-Bonnet gravity.

For the model determined by Hubble parameter $H(t) = A^2 B \coth(2Bt)$ with constant $A = \sqrt{8/5}$ we obtain

$$\xi(t) = \frac{25\,\alpha_{GB}}{128a_0^5 B^3 \tanh(4Bt)} + const, \tag{53}$$

$$\omega(t) = 1 + \frac{3\,\alpha_{GB}\cosh(2Bt)}{a_0^5 B \sinh^3(2Bt)}. \tag{54}$$

After substituting the inverse relation $t = t(\phi)$ we get the functions

$$\xi(\phi) = \frac{25\,\alpha_{GB}}{128a_0^5B^3}\left[\frac{1}{4}\cosh\left(\frac{\phi}{A}\right) + \frac{1}{\cosh\left(\frac{\phi}{A}\right)}\right] + const, \tag{55}$$

$$\omega(\phi) = 1 + \frac{3\,\alpha_{GB}}{8a_0^5B}\left[2\cosh^3\left(\frac{\phi}{A}\right) - 3\cosh\left(\frac{\phi}{A}\right)\right], \tag{56}$$

which correspond to potential $V(\phi) = A^2B^2\left[(3A^2-2)\cosh^2\left(\frac{\phi}{A}\right) + 2\right]$ in cosmological models with Einstein-Gauss-Bonnet gravity.

5. Conclusion

In this paper, the methods for obtaining new exact solutions of cosmological dynamical equations at the early (inflationary) stage of the universe's evolution from known ones are considered. The proposed approach makes it possible to use the solutions obtained for the case of minimal coupling of a scalar field and curvature (Einstein's gravity) to generate new solutions in the same type of models, but with nonminimal coupling involving scalar-tensor gravity theories and the Einstein-Gauss-Bonnet gravity.

Since the physical mechanisms for implementing the inflationary scenario are directly related to the potential of the scalar field, the proposed approach is aimed at using exact solutions for physical potentials obtained in standard cosmological models for generalization to models with modified gravity.

We also note that the proposed approach can be used in approximate methods such as the slow-roll approximation [18], when neglecting the contribution of the field's kinetic energy to the dynamics of the universe at the inflationary stage, or the kinetic approximation implying a quasilinear connection of kinetic energy of a scalar field with state parameter considered in papers [19, 20].

It is obvious that the analysis of cosmological dynamics can be carried out without connection to standard cosmological models, however, the presence of such connection allows one to consider modified gravity theories as a certain parametrized extension of general relativity associated with the values of constants β_{ST} and α_{GB}.

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